

Linear Algebra for Computer Science

Homework 4

Read these first:

- i You may write your solutions on paper, under a word processing software (MS-word, Libre Office, etc.), or under \LaTeX .
- ii If writing on paper, you must use a scanner device or a Camera Scanner (CamScanner) software to scan the document and submit a *single* PDF file.
- iii Up to 15% extra score will be given to solutions written under \LaTeX , provided that you follow either of the following conventions:
 - (a) Represent scalars with normal (italic) letters (a, A), vectors with bold lower-case letters (\mathbf{a} , using `\mathbf{a}`), and matrices with bold upper-case letters (\mathbf{A} , using `\mathbf{A}`), or
 - (b) represent scalars with normal (italic) letters (a, A), vectors with bold letters (\mathbf{a}, \mathbf{A}), and matrices with typewriter upper-case letters (\mathbf{A} , using `\mathtt{A}`).
 - (c) Your latex document must contain a *title*, a *date*, and your name as the author.
 - (d) In all cases, you must submit a *single* PDF file.
 - (e) If writing under \LaTeX , you must submit the *.tex* source (and other necessary source files if there are any) in addition to the PDF file.

Here is a short tutorial on \LaTeX : https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes

Questions

Projections

1. Consider a linear subspace \mathcal{S} and a vector $\mathbf{y} \in \mathcal{S}$. Using the projection formula, show that the projection of \mathbf{y} into \mathcal{S} is itself.

2. For a linear subspace $\mathcal{S} \subseteq \mathbb{R}^n$ its *orthogonal complement* is defined as $\mathcal{S}^\perp = \{\mathbf{y} \in \mathbb{R}^n \mid \mathbf{y}^T \mathbf{x} = 0 \text{ for all } \mathbf{x} \in \mathcal{S}\}$. In other words, \mathcal{S}^\perp comprises all the vectors that are perpendicular to all vectors in \mathcal{S} . Show that the orthogonal complement of a linear subspace is a linear subspace.
3. Prove that the *null space* of a matrix is the orthogonal complement of its *row space*.
4. Let $\mathbf{P} \in \mathbb{R}^{n \times n}$ be the projection matrix into a linear subspace \mathcal{S} . Show that $\mathbf{I} - \mathbf{P}$ represents the projection into the orthogonal complement of \mathcal{S} . Hint: First show that $\mathbf{I} - \mathbf{P}$ is a projection matrix.

Determinant

5. Prove that the determinant of an orthogonal matrix is either equal to 1 or -1 .
6. Show that the determinant of a projection matrix is either equal to 0 or 1. How do you explain this geometrically?