Foundations of Electrical Engineering

Queensland University of Technology

Dr Jasmin Martin

Semester 2, 2021

Tarang Janawalkar

Contents

1 Electrical Circuits

1.1 Fundamental Quantities

Name	**Definition**	**Symbol**	**Unit**
Charge	Electric charge is a fundamental property of matter that governs how particles are affected by an electromagnetic field.	q	Coulomb (C)
Current	$i = \frac{\mathrm{d}q}{\mathrm{d}t} \iff 1\,\mathrm{A} = 1\,\mathrm{C\,s^{-1}}$	i	Ampere (A)
Voltage	$v = \frac{\mathrm{d}w}{\mathrm{d}q} \iff 1\,\mathrm{V} = 1\,\mathrm{J\,C^{-1}}$	v	Volt (V)
Power	$p = \frac{\mathrm{d}w}{\mathrm{d}t} \iff 1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}$	p	Watt (W)

Charge in an electron. $q = 1.6022 \times 10^{-19}\,\mathrm{C}$.

Electric Power. $p = \frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\mathrm{d}w}{\cancel{\mathrm{d}q}}\frac{\cancel{\mathrm{d}q}}{\mathrm{d}t} = vi$.

1.2 Passive Sign Convention

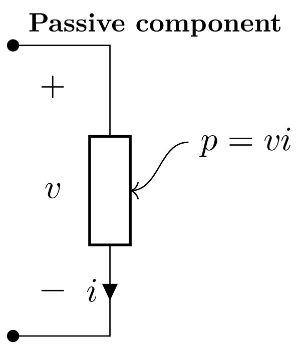

Figure 1: Energy dissipated.

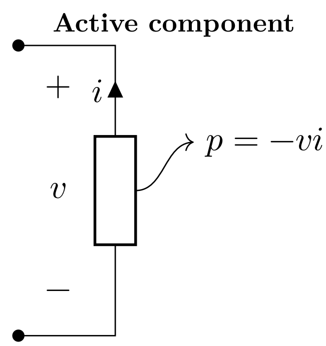

Figure 2: Energy produced.

Theorem 1.2.1 (Power Balance)**.**

$$p_{\mathrm{net}} = 0$$

Theorem 1.2.2 (Energy)**.**

$$w(\tau) = \int_0^{\tau} p(t)\,\mathrm{d}t$$

1.3 Circuits and Sources

Definition 1.3.1 (Circuits)**.** A circuit is a mathematical model that approximates a real system. It is built from ideal circuit elements connected by ideal wires.

Definition 1.3.2 (Voltage Source)**.** Produces or dissipates power at a specified voltage with whatever current is required.

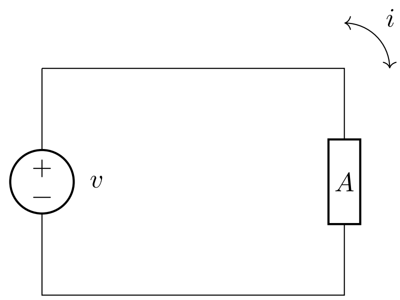

Figure 3: Voltage Source – v is specified, i varies depending on circuit element A.

Definition 1.3.3 (Current Source)**.** Produces or dissipates power at a specified current with whatever voltage is required.

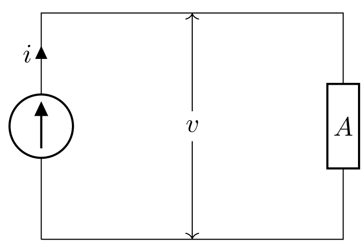

Figure 4: Current Source – i is specified, v varies depending on circuit element A.

1.4 Ground

Definition. The zero volt point is referred to as the circuit ground.

Symbol. 

1.5 Earth

Definition. An earthed ground is literally a connection to the earth.

Symbol. 

1.6 Connected Sources

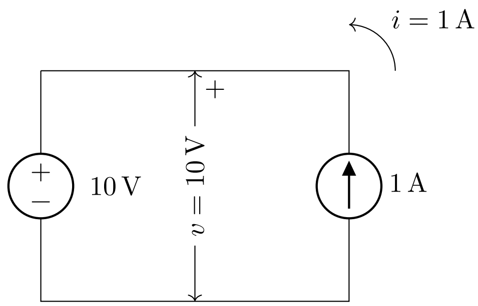

Figure 5: Example of a connected voltage and current source.

- The voltage source sets v to $10\,\mathrm{V}$, with the upper wire being positive.
- The current source sets i to $1\,\mathrm{A}$, flowing anticlockwise.
- Therefore $10\,\mathrm{W}$ of power is produced by the current source and dissipated by the voltage source.

1. Two voltage sources must be connected at the same terminals and supply the same voltage.
2. Two current sources must flow in the same direction and supply the same current.

1.7 Invalid Circuits

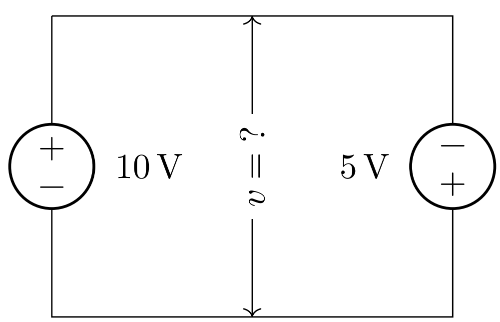

Figure 6: Voltage terminals are incorrect, and voltages are different.

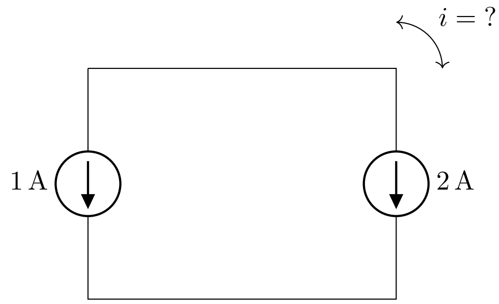

Figure 7: Current flows oppose each other, and currents are different.

1.8 Resistors

Definition 1.8.1 (Resistor)**.** Resistors dissipate power, and the voltage across both terminals is proportional to the current.

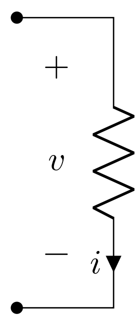

Figure 8:

Theorem 1.8.1 (Voltage through a resistor)**.**

$$v = iR$$

$$R = \frac{\rho L}{A}$$

2 Simple Resistive Circuits

2.1 Ignored Physics

1. Electrical effects occur instantaneously, so there is no time delay along the wires.
2. The net charge on every component is zero. Charge is never lost or gained.
3. There is no magnetic coupling between the components.

2.2 Kirchhoff's Laws

Definition 2.2.1 (Kirchhoff's Current Law (KCL))**.** The sum of all currents into a node equals zero.

$$\sum i_{node} = 0$$

Definition 2.2.2 (Kirchhoff's Voltage Law (KVL))**.** The sum of all voltages around a loop equals zero.

$$\sum v_{loop} = 0$$

2.3 Series and Parallel Circuits

Definition 2.3.1. Elements connected end-to-end are in series. If both ends of an element are connected directly to another element, the two elements are in parallel.

Element	**Series**	**Parallel**
Current Source	$i_{eq} = i_1 = i_2 = \cdots = i_n$	$i_{eq} = \sum_k i_k$
Voltage Source	$v_{eq} = \sum_{k=1}^{n} v_k$	$v_{eq} = v_1 = v_2 = \cdots = v_n$
Resistor	$R_{eq} = \sum_{k=1}^{n} R_k$	$\frac{1}{R_{eq}} = \sum_{k=1}^{n} \frac{1}{R_k}$
Inductor	$L_{eq} = \sum_{k=1}^{n} L_k$	$\frac{1}{L_{eq}} = \sum_{k=1}^{n} \frac{1}{L_k}$
Conductor	$\frac{1}{C_{eq}} = \sum_{k=1}^{n} \frac{1}{C_k}$	$C_{eq} = \sum_{k=1}^{n} C_k$

Table 1: Equivalent values for various components connected in series and parallel.

These equations can be used to simplify a complex circuit.

Proof. Using KVL for voltage sources connected in series

$$\begin{aligned} v_1 + v_2 + \cdots + v_n - iR &= 0 \\ v_1 + v_2 + \cdots + v_n &= iR \end{aligned}$$

Let v_{eq} be the voltage across the resistor so that

$$v_{eq} = iR$$

Therefore

$$v_{eq} = v_1 + v_2 + \cdots + v_n$$

As the current through resistors in series remains the same, using KVL we have

$$\begin{aligned} v - iR_1 - iR_2 - \cdots - iR_n &= 0 \\ v &= i\left(R_1 + R_2 + \cdots + R_n\right) \\ v &= iR_{eq} \end{aligned}$$

Hence the resistance across multiple resistors in series is equivalent to the sum of the resistances. □

2.4 Voltage and Current Dividers

Definition 2.4.1 (Voltage Divider)**.** A voltage divider is a circuit that divides a voltage in the proportion of the series resistances.

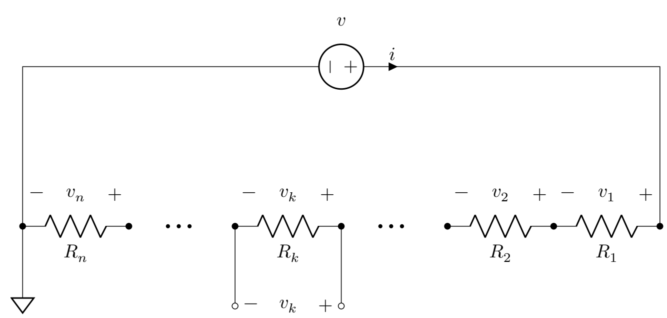

Figure 9: Resistors connected in series to a voltage source.

Theorem 2.4.1.

$$v_k = v\frac{R_k}{R_{eq}}$$

Proof. The current through any resistor is

$$i = \frac{v}{R_{eq}}$$

Therefore the voltage drop in any resistor is

$$\begin{aligned} v_k &= iR_k \\ v_k &= \frac{v}{R_{eq}}R_k \end{aligned}$$

□

Definition 2.4.2 (Current Divider)**.** A voltage divider is a circuit that divides a voltage in the proportion of the series resistances.

Theorem 2.4.2.

$$i_k = i\frac{R_{eq}}{R_k}$$

3 Diodes

4 Mesh Analysis

5 Source Transformation

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10 Frequency Response

11 Filters and Rectifiers

12 Zener Diodes and Voltage Regulators