

# Engineering Mechanics

Semester 1, 2022

*Dr Tuquabo Tesfamichael*

TARANG JANAWALKAR

This work is licensed under a Creative Commons  
“Attribution-NonCommercial-ShareAlike 4.0 International” license.



# Contents

<b>Contents</b>	<b>1</b>
<b>1 Stress and Strain</b>	<b>2</b>
1.1 External Forces . . . . .	2
1.1.1 Types of Forces . . . . .	2
1.2 Internal Loadings . . . . .	2
1.3 Internal Resultant Loadings . . . . .	2
1.3.1 In 3D . . . . .	2
1.3.2 In 2D . . . . .	3
1.3.3 In 1D . . . . .	3
1.4 Stress . . . . .	3
1.5 Average Normal Stress . . . . .	3
1.6 Strain . . . . .	4
<b>2 Tension and Compression Tests</b>	<b>4</b>
2.1 Stress-Strain Diagram . . . . .	5
2.1.1 Conventional Stress-Strain Diagram . . . . .	5
2.1.2 True Stress-Strain Diagram . . . . .	5
2.2 Elastic Behaviour . . . . .	5
2.2.1 Proportional Limit . . . . .	5
2.2.2 Modulus of Elasticity . . . . .	5
2.3 Elastic Limit . . . . .	5
2.4 Plastic Behaviour . . . . .	5
2.4.1 Yielding . . . . .	5
2.4.2 Yield Strength . . . . .	6
2.4.3 Ultimate Tensile Stress . . . . .	6
2.4.4 Strain Hardening . . . . .	6
2.4.5 Necking . . . . .	6
2.4.6 Fracture Stress . . . . .	6
2.5 Ductility . . . . .	6
2.6 Brittleness . . . . .	6
2.7 Poisson's Ratio . . . . .	6

# 1 Stress and Strain

## 1.1 External Forces

Rigid bodies are subjected to external force and couple moment systems that result from the effects of gravitational, electrical, magnetic, or contact forces. Contact forces can be surface, linear, or concentrated forces.

### 1.1.1 Types of Forces

- Compressive (pushing)
- Tensile (pulling)
- Shear (sliding)
- Torsional (twisting)
- Biaxial tension
- Hydrostatic compression
- Bending (induces tension, compression and shear)

## 1.2 Internal Loadings

External forces cause internal loadings that occur in equal and opposite collinear pairs as stresses and strains. Internal loading is associated with **stress** while **strain** is a measure of a body's deformation.

These loadings have no external effects on the body, and are not included on a **Free Body Diagram** (FBD) if the entire body is considered.

To determine the forces in each member, we can use the method of sections to represent the internal loading as external forces.

## 1.3 Internal Resultant Loadings

Although the exact distribution of the internal loading may be *unknown*, we can determine the resultant force  $\mathbf{F}_R$  and resultant moment  $(\mathbf{M}_R)_O$  about a point  $O$  by applying the equations of equilibrium

$$\begin{aligned}\sum \mathbf{F} &= \mathbf{0} \\ \sum \mathbf{M}_O &= \mathbf{0}.\end{aligned}$$

### 1.3.1 In 3D

In 3D, we can represent resultant loadings using four vectors acting over the sectioned area.

**Normal force**  $\mathbf{N}$  force acting perpendicular to the area

**Shear force**  $\mathbf{V}$  force acting on an axis tangent to the area

**Torsional moment  $\mathbf{T}$**  rotation about the perpendicular axis

**Bending moment  $\mathbf{M}$**  rotation about an axis tangent to the area

### 1.3.2 In 2D

In 2D, the body is subjected to a coplanar system of forces, where  $\mathbf{T} = \mathbf{0}$ .

### 1.3.3 In 1D

In 1D, the body is only subjected to axial forces, where  $\mathbf{V} = \mathbf{T} = \mathbf{M} = \mathbf{0}$ .

## 1.4 Stress

The force and moment acting at a specific point on a sectioned area of a body represent the resultant effects of the distribution of internal loading that acts over the sectioned area.

**Definition 1.1** (Stress). Consider the quotient of the force  $\Delta\mathbf{F}$  over an area  $\Delta A$ , then as the  $\Delta A \rightarrow 0$ , so does  $\Delta\mathbf{F}$ , while the quotient approaches a finite limit. This quotient is called the stress at that point.

$$\boldsymbol{\sigma} = \lim_{\Delta A \rightarrow 0} \frac{\Delta\mathbf{F}}{\Delta A}$$

Here the normal and shear stresses can be expressed using  $\sigma_z$  and  $\tau_{zx}$  and  $\tau_{zy}$ .

$$\begin{aligned}\sigma_z &= \lim_{\Delta A \rightarrow 0} \frac{\Delta F_z}{\Delta A} \\ \tau_{zx} &= \lim_{\Delta A \rightarrow 0} \frac{\Delta F_x}{\Delta A} \\ \tau_{zy} &= \lim_{\Delta A \rightarrow 0} \frac{\Delta F_y}{\Delta A}\end{aligned}$$

Stress describes the intensity of the internal force acting on a specific region passing through a point.

The unit for stress is Pascal where 1 Pa or 1 N m<sup>-2</sup> and 1 MPa or 1 N mm<sup>-2</sup>.

## 1.5 Average Normal Stress

To determine the average stress distribution acting over a cross-sectional area of an axially loaded bar, we assume that the material is both *homogeneous* and *isotropic*. This means the load  $P$  applied through the centroid of the cross-sectional area will cause the bar to deform uniformly throughout the central region of its length.

By passing a section through a bar, equilibrium requires the resultant normal force  $N$  at the section to be equal to the external force  $P$ . And because the material undergoes a uniform deformation, it is necessary that the cross section be subjected to a constant normal stress distribution.

As a result, each small area  $\Delta A$  on the cross section is subjected to a force  $\Delta N = \sigma \Delta A$ , where the sum of these forces over the entire cross-sectional area is  $P$ . By letting  $\Delta A \rightarrow dA$  and therefore

also  $\Delta N \rightarrow dN$ , then as  $\sigma$  is a constant, we have

$$\begin{aligned}\int dN &= \int_A \sigma dA \\ N &= \sigma A\end{aligned}$$

Therefore

$$\sigma_{\text{avg}} = \frac{N}{A}$$

where in this case  $N = P$ .

**Theorem 1.5.1** (Equilibrium). *For an uniaxially loaded body, the equation of force equilibrium gives*

$$\begin{aligned}\sigma(\Delta A) - \sigma'(\Delta A) &= 0 \\ \sigma &= \sigma'\end{aligned}$$

hence the normal stress components must be equal in magnitude but opposite in direction.

Under this condition, the material is subjected to **uniaxial stress** and this analysis applies to members subjected to tension or compression.

## 1.6 Strain

**Definition 1.2** (Deformation). Whenever a force is applied to a body, it will tend to change the body's shape and size. These changes are referred to as deformation.

**Definition 1.3** (Strain). To describe the deformation of a body through changes in lengths of line segments on the surface, we will develop the concept of strain. If an axial load  $P$  is applied to a bar, it will change the bar's length  $L_0$  to  $L$ . Then the **average normal strain** of the bar is defined

$$\epsilon_{\text{avg}} = \frac{L - L_0}{L_0}$$

where the numerator is often written as  $\delta = L - L_0$  and is known as elongation or extension.

The **normal strain**  $\epsilon$  at a point in a body with an arbitrary shape is defined similarly. Consider a small line segment  $\Delta s$  which becomes  $\Delta s'$  after deformation. Then the limit of the normal strain is

$$\epsilon = \lim_{\Delta s \rightarrow 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

In both cases normal strain is positive when the initial length elongates, and negative when the length contracts.

Strain is a dimensionless quantity sometimes expressed mm/mm<sup>2</sup> or m/m<sup>2</sup>, or as a percentage.

## 2 Tension and Compression Tests

To determine the strength of a material, we must perform a tension or compression test. This test measures the stress and strain from a load  $P$ , and the results can be used to produce a **stress-strain diagram**. There are two ways in which the stress-strain diagram is normally described.

## 2.1 Stress-Strain Diagram

A stress-strain diagram has the following shape

### 2.1.1 Conventional Stress-Strain Diagram

The engineering stress assumes that the area  $A$  is constant throughout the gauge length

$$\sigma = \frac{P}{A_0}$$

where  $A_0$  is the *original* cross-sectional area of the specimen.

Likewise, the engineering strain uses the specimen's original length  $L_0$

$$\epsilon = \frac{\delta}{L_0}$$

### 2.1.2 True Stress-Strain Diagram

The true stress and true strain use the instantaneous area  $A$  and length  $L$  at each measurement.

## 2.2 Elastic Behaviour

The initial region of the curve is referred to as the **elastic region** where the deformation is *elastic*, meaning it returns to its original shape when the load is removed.

### 2.2.1 Proportional Limit

For the majority of the elastic deformation, the curve is *linear* until the **proportional limit**  $\sigma_{pl}$ .

### 2.2.2 Modulus of Elasticity

As the curve is linear up to  $\sigma_{pl}$ , the stress is proportional to the strain. This is characterised by Hooke's law, and is expressed as

$$\sigma = E\epsilon$$

where  $E$  is the constant of proportionality, also called the **modulus of elasticity** or **Young's modulus**.

## 2.3 Elastic Limit

After reaching the proportional limit, the curve bends slightly until it reaches the **elastic limit**.

## 2.4 Plastic Behaviour

If the stress exceeds the elastic limit, the specimen undergoes *plastic* deformation, meaning any deformation is permanent.

### 2.4.1 Yielding

The behaviour after elastic deformation is called **yielding** and this occurs at the **yield point**  $\sigma_Y$ .

### 2.4.2 Yield Strength

Commonly  $\sigma_{pl}$ , the elastic limit, and  $\sigma_Y$  occur at the same point, so a quantity called the **yield strength**  $\sigma_{YS}$  is defined.

Normally, a 0.2% strain is chosen, and from this point, a line with gradient  $E$  is drawn. The point where this line intersects the curve defines  $\sigma_{YS}$ .

### 2.4.3 Ultimate Tensile Stress

When yielding has ended, any load causing an increase in stress will be supported by the specimen, resulting in a curve that rises until it reaches a maximum stress referred to as the **ultimate stress**  $\sigma_u$ .

### 2.4.4 Strain Hardening

This rise in the curve is called **strain hardening**.

### 2.4.5 Necking

As the specimen elongates up to  $\sigma_u$ , its cross-sectional area will decrease in a *uniform* manner over the gauge length. However after reaching  $\sigma_u$ , the cross-sectional area will begin to decrease in a *localised* region where the stress increases. As a result, a “neck” forms at this region, and the specimen experiences **necking**.

### 2.4.6 Fracture Stress

Finally, the specimen breaks at the **fracture stress**  $\sigma_f$  at the end of the downward curve.

## 2.5 Ductility

**Definition 2.1** (Ductility). Ductility is a measure of the amount of plastic deformation a material can sustain under tensile stress before failure.

Ductility can be measured using its **percent elongation** (in length) or **percent reduction** (in area).

## 2.6 Brittleness

**Definition 2.2** (Brittleness). Brittleness

## 2.7 Poisson’s Ratio

When a deformable body is subjected to a force, it can elongate longitudinally and also contract laterally. The strain in the longitudinal (or axial) direction is given by

$$\epsilon_{\text{long}} = \frac{\delta}{L}$$

and the strain in the lateral (or radial) direction is given by

$$\epsilon_{\text{lat}} = \frac{\delta'}{r}$$

where  $\delta'$  is the change in the radius  $r$ .

Consider the ratio of these two quantities  $\nu$

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}.$$

Within the elastic region, this quantity will be constant, and it is referred to as **Poisson's ratio**.

Note the negative value is introduced as the lateral axis contracts.