MATLAB

```
function [output1, ...]
        = func_name(input1, ...)
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Partial Fraction Decomposition

$$ax+b \to \frac{A}{ax+b}$$
$$(ax+b)^k \to \frac{A_1}{ax+b} + \cdots + \frac{A_k}{(ax+b)^k}$$
$$ax^2+bx+c \to \frac{A}{ax^2+bx+c}$$
$$(ax^2+bx+c)^k \to$$
$$\frac{A_1x+B_1}{ax^2+bx+c} + \cdots + \frac{A_kx+B_k}{(ax^2+bx+c)^k}$$

Differential Equations

Electrical Circuits

$$\sum v_{loop} = 0 \qquad \sum i_{node} = 0$$
$$i = \frac{\mathrm{d}q}{\mathrm{d}t}$$

Voltage drop across various elements:

$$v_R = iR \qquad R: \text{resistance}$$
$$v_C = \frac{q}{C} \qquad C: \text{capacitance}$$
$$v_L = L\frac{\mathrm{d}i}{\mathrm{d}t} \qquad L: \text{inductance}$$

Mechanical Systems

$$\sum F = \frac{\mathrm{d}p}{\mathrm{d}t}$$

where $p = mv$.

$$F_T = (c-v)\,d_f$$
$$F_g = mg$$
$$F_S = -kx$$

Separable ODEs

For $\frac{\mathrm{d}y}{\mathrm{d}t} = p(t)q(y)$:

$$\int \frac{1}{q(y)}\frac{\mathrm{d}y}{\mathrm{d}t}\,\mathrm{d}t = \int p(t)\,\mathrm{d}t\,.$$

Linear ODEs

For $\frac{\mathrm{d}y}{\mathrm{d}t} + p(t)y = q(t)$, use the *integrating factor*: $I(t) = e^{\int p(t)\mathrm{d}t}$, so that

$$y(t) = \frac{1}{I(t)}\int I(t)q(t)\,\mathrm{d}t\,.$$

Linearisation

$$f(t) \approx f(t_0) + f'(t_0)(t-t_0)$$
$$f(y(t)) \approx f'(y(t_0))(y(t)-y(t_0)) + f(y(t_0))$$

Euler's Method

$$y(t+h) = y(t) + hy'(t)$$
$$\boldsymbol{y}(t+h) = \boldsymbol{y}(t) + h\boldsymbol{y}'(t)$$

Modified Euler's Method

$$y(t+h) = y(t) + \frac{h}{2}(y'(t) + y'(t+h))$$
$$\boldsymbol{y}(t+h) = \boldsymbol{y}(t) + \frac{h}{2}(\boldsymbol{y}'(t) + \boldsymbol{y}'(t+h))$$

where $y'(t+h)$ is determined using Euler's method.

Second-Order ODEs

$$ay'' + by' + cy = F(t)$$

Homogeneous: $F(t) = 0$
Nonhomogeneous: $F(t) \neq 0$

Homogeneous ODEs

$$y_H(t) = e^{\lambda t}$$

Real Distinct Roots

$$y_H(t) = c_1e^{\lambda_1 t} + c_2e^{\lambda_2 t}$$

Real Repeated Roots

$$y_H(t) = c_1e^{\lambda t} + c_2te^{\lambda t}$$

Complex Conjugate Roots

Given $\lambda = \alpha \pm \beta i$:

$$y_H(t) = e^{\alpha x}(c_1\cos(\beta t) + c_2\sin(\beta t))$$

Nonhomogeneous ODEs

$$y(t) = y_H(t) + y_P(t).$$

Method of Undetermined Coefficients

See table below. Substitute y_P into the nonhomogeneous ODE, and solve the undetermined coefficients.

System of ODEs

$$\boldsymbol{y}' = \mathbf{A}\boldsymbol{y}$$

Homogeneous System

$$\boldsymbol{y}_H = \boldsymbol{q}e^{\lambda t}$$

λ_i are the eigenvalues of $\boldsymbol{A}$, and $\boldsymbol{q}_i$ are the associated eigenvectors.

Higher-Order ODEs

$$y^{(n)} + a_1y^{(n-1)} + \cdots + a_{n-1}y' + a_ny = 0$$

Let $y_1 = y$, $y_2 = y'$, …, $y_n = y^{(n-1)}$ so that $\boldsymbol{y} = \langle y_1, y_2, \ldots, y_n\rangle$. Then

$$\boldsymbol{y}' = \mathbf{A}\boldsymbol{y}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}$$

Probability

Events

$$\Pr(A) = \sum_{x\in A} p(x)$$
$$\Pr(A^C) = \Pr\left(\overline{A}\right) = 1 - \Pr(A)$$

Disjoint Events (Mutually Exclusive)

Events don't have outcomes in common.

$$A \cap B = \varnothing$$
$$\Pr(A\cap B) = 0$$

Unions

$$\Pr(A\cup B) = \Pr(A) + \Pr(B) - \Pr(A\cap B)$$
$$= 1 - \Pr(A^C \cap B^C)$$

Independent Events

Outcome of events do not influence each other. Joint probability:

$$\Pr(A\cap B) = \Pr(A)\Pr(B)$$

Dependent (Conditional) Events

Outcome of event depends on the outcome of the other. Joint probability of A given B:

$$\Pr(A \mid B) = \frac{\Pr(A\cap B)}{\Pr(B)}$$

Total Probability

For disjoint events B_i:

$$A = \bigcup_{i=1}^{n}(A\cap B_i)$$
$$\Pr(A) = \sum_{i=1}^{n}\Pr(A\cap B_i)$$
$$\Pr(A) = \sum_{i=1}^{n}\Pr(A\mid B_i)\Pr(B_i)$$

Bayes' Theorem

$$\Pr(B\mid A) = \frac{\Pr(A\mid B)\Pr(B)}{\Pr(A)}$$

Summary Statistics

Discrete Random Variables

Has countably many outcomes. Distributed according to a Probability Mass Function (PMF):

$$p(x) = \Pr(X = x)$$

Continuous Random Variables

Has an infinite number of individual outcomes. Distributed according to a Probability Density Function $f(x)$ where

$$\Pr(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f(u)\,\mathrm{d}u\,.$$

The Cumulative Density Function (CDF) is defined as:

$$F(x) = \Pr(X \le x) = \int_{-\infty}^{x} f(u)\,\mathrm{d}u$$

Expectation

$$\mu = \mathrm{E}(X) = \begin{cases} \sum_\Omega xp(x) \\ \int_\Omega xf(x)\,\mathrm{d}x \end{cases}$$

Variation

$$\sigma^2 = \mathrm{Var}(X) = \begin{cases} \sum_\Omega (x-\mu)^2 p(x) \\ \int_\Omega (x-\mu)^2 f(x)\,\mathrm{d}x \end{cases}$$
$$= \mathrm{E}(X^2) - \mathrm{E}(X)^2$$

Standard Deviation

$$\sigma = \sqrt{\mathrm{Var}(X)}$$

General Linear Combinations

Expectation:

$$\mathrm{E}(aX+b) = a\,\mathrm{E}(X) + b$$
$$\mathrm{E}(aX+bY) = a\,\mathrm{E}(X) + b\,\mathrm{E}(Y)$$

Variation:

$$\operatorname{Var}(aX+b) = a^2 \operatorname{Var}(X)$$

$$\begin{aligned}\operatorname{Var}(aX+bY) &= a^2 \operatorname{Var}(X) \\ &\quad + b^2 \operatorname{Var}(Y) \\ &\quad - 2ab \operatorname{Cov}(X,\ Y)\end{aligned}$$

where

$$\begin{aligned}\operatorname{Cov}(X,\ Y) &= \operatorname{E}(XY) - \operatorname{E}(X)\operatorname{E}(Y) \\ &= \rho_{XY}\sqrt{\operatorname{Var}(X)\operatorname{Var}(Y)}.\end{aligned}$$

The correlation $\operatorname{Corr}(X,\ Y)$ or ρ_{XY} is a constant that describes the statistical relationship between X and Y. $-1 \leq \rho_{XY} \leq 1$.

$F(t)$	$y_P(t)$
a constant	A
a polynomial of degree n	$\sum_{i=0}^{n} A_i t^i$
e^{kt}	Ae^{kt}
$\cos(\omega t)$ or $\sin(\omega t)$	$A_0 \cos(\omega t) + A_1 \sin(\omega t)$
a combination of the above	a combination of the above
linearly dependent to $y_H(t)$	multiply $y_P(t)$ by t until linearly independent