

MATLAB

```
% Function declaration
function [output1, ...]
    = function_name(input1, ...)
% Iterate over the vector v
for i = v ... end
% Repeat while condition is true
while condition ... end
% Execute first true alternative
if condition_1 ...
elseif condition_2 ...
else ... end
```

Partial Fraction Decomposition

Given the LHS in the denominator, substitute the RHS.

$$(ax + b)^k \rightarrow \frac{A_1}{ax + b} + \dots + \frac{A_k}{(ax + b)^k}$$

$$(ax^2 + bx + c)^k \rightarrow \frac{A_1x + B_1}{ax^2 + bx + c} + \dots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}$$

Differential Equations

Mechanical Systems

$$\sum F = \frac{dp}{dt}, \quad \sum M = I \frac{d^2\theta}{dt^2}$$

where momentum $p = mv$, moment $M = Fx$ and inertia $I = mr^2$.

$$F_T = (c - v) d_f \quad F_g = -mg$$

$$F_s = -kx \quad F_f = -bv^2 \text{ (or } -bv)$$

Electrical Circuits

$$i = \frac{dq}{dt}, \quad \sum v_{loop} = 0, \quad \sum i_{node} = 0$$

Voltage drop across elements:

$$v_R = iR, \quad v_C = \frac{q}{C}, \quad v_L = L \frac{di}{dt}$$

Separable ODEs

For $\frac{dy}{dt} = p(t)q(y)$:

$$\int \frac{1}{q(y)} \frac{dy}{dt} dt = \int p(t) dt.$$

Linear ODEs

For $\frac{dy}{dt} + p(t)y = q(t)$, use the *integrating factor*: $I(t) = e^{\int p(t)dt}$, so that

$$y(t) = \frac{1}{I(t)} \int I(t)q(t) dt.$$

Linearisation

$$f(t) \approx f(t_0) + f'(t_0)(t - t_0)$$

$$f(y(t)) \approx f(y(t_0)) + f'(y(t_0))(y(t) - y(t_0))$$

Euler's Method

For $y' = f(t, y)$:

$$y_{n+1} = y_n + hf(t_n, y_n)$$

Modified Euler's Method

$$y_{n+1} =$$

$$y_n + \frac{h}{2} (f(t_n, y_n) + f(t_{n+1}, y_{n+1}))$$

where $f(t_{n+1}, y_{n+1})$ is determined using Euler's method.

Second-Order ODEs

$$ay'' + by' + cy = F(t)$$

General Solution

$$y(t) = y_H(t) + y_P(t)$$

Homogeneous Solution

$$y_H(t) = e^{\lambda t}$$

To solve for λ , substitute the homogeneous form into the homogeneous

ODE.

Real distinct roots:

$$y_H(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

Real repeated roots:

$$y_H(t) = c_1 e^{\lambda t} + c_2 t e^{\lambda t}$$

Complex conjugate roots: $\lambda = \alpha \pm \beta i$

$$y_H(t) = e^{\alpha x} (c_1 \cos(\beta t) + c_2 \sin(\beta t))$$

Particular Solution

See table below. Substitute y_P into the nonhomogeneous ODE, and solve the undetermined coefficients.

System of ODEs

Given $y' = Ay$,

$$y_H(t) = q e^{\lambda t}.$$

λ_i are the eigenvalues of A that satisfy

$$\det(A - \lambda I) = 0.$$

q_i are the associated eigenvectors that satisfy

$$(A - \lambda_i I) q_i = 0.$$

For real distinct roots:

$$y_H(t) = c_1 q_1 e^{\lambda_1 t} + c_2 q_2 e^{\lambda_2 t}$$

Higher-Order ODEs

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = 0$$

Let $y_1 = y$, $y_2 = y'$, ..., $y_n = y^{(n-1)}$ so that $y = \langle y_1, y_2, \dots, y_n \rangle$. Then $y' = Ay$, where

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix}$$

$F(t)$	$y_P(t)$
a constant	A
a polynomial of degree n	$\sum_{i=0}^n A_i t^i$
e^{kt}	$A e^{kt}$
$\cos(\omega t)$ or $\sin(\omega t)$	$A_0 \cos(\omega t) + A_1 \sin(\omega t)$
a combination of the above	a combination of the above
linearly dependent to $y_H(t)$	multiply $y_P(t)$ by t until linearly independent

Probability

Events

$$\Pr(A) = \sum_{x \in A} p(x)$$

$$\Pr(A^C) = \Pr(\overline{A}) = 1 - \Pr(A)$$

Unions

$$\begin{aligned} \Pr(A \cup B) &= \Pr(A) + \Pr(B) \\ &\quad - \Pr(A \cap B) \\ &= 1 - \Pr(A^C \cap B^C) \end{aligned}$$

Bayes' Theorem

$$\Pr(B | A) = \frac{\Pr(A | B) \Pr(B)}{\Pr(A)}$$

Disjoint Events

Events don't have outcomes in common. For disjoint events B_i :

$$A \cap B = \emptyset$$

$$\Pr(A \cap B) = 0$$

Independent Events

Outcome of events do not influence each other. Joint probability:

$$\Pr(A \cap B) = \Pr(A) \Pr(B)$$

Dependent (Conditional) Events

Outcome of event depends on the outcome of the other. Joint probability of A given B :

$$\Pr(A | B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Total Probability

$$A = \bigcup_{i=1}^n (A \cap B_i)$$

$$\Pr(A) = \sum_{i=1}^n \Pr(A \cap B_i)$$

Expectation

Average output or mean:

$$\mu = E(X).$$

Variance

Measure of spread from the mean:

$$\sigma^2 = E(X^2) - E(X)^2.$$

Probability Distributions

Discrete Random Variables

Has countably many outcomes. Distributed with a probability mass function $p(x)$.

Continuous Random Variables

Has an infinite number of outcomes. Distributed with a probability density function $f(x)$.

$$\Pr(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f(x) dx$$

$$F(x) = \Pr(X \leq x)$$

	Discrete	Continuous
Valid probabilities	$0 \leq p(x) \leq 1$	$f(x) \geq 0$
Cumulative probability	$\sum_{u \leq x} p(u)$	$\int_{-\infty}^x f(u) du$
Expectation	$\sum_{\Omega} xp(x)$	$\int_{\Omega} xf(x) dx$
Variance	$\sum_{\Omega} (x - \mu)^2 p(x)$	$\int_{\Omega} (x - \mu)^2 f(x) dx$

General Linear Combinations

$$E(aX \pm b) = aE(X) \pm b$$

$$E(aX \pm bY) = aE(X) \pm bE(Y)$$

$$\text{Var}(aX \pm b) = a^2 \text{Var}(X)$$

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X)$$

$$+ b^2 \text{Var}(Y)$$

$$\pm 2ab \text{Cov}(X, Y)$$

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$= \rho_{XY} \sqrt{\text{Var}(X) \text{Var}(Y)}$$

The correlation $\text{Corr}(X, Y)$ or ρ_{XY} is a constant that describes the statistical relationship between X and Y . $-1 \leq \rho_{XY} \leq 1$.

Binomial (Discrete)

Probability of x successes out of n independent trials, each with chance p .

Poisson (Discrete)

Probability of observing x events over an interval t , where events occur at an average rate λ . $\mu = \lambda t$.

Exponential (Continuous)

The time t between events, where the events are independent and occur at an average rate λ .

Uniform (Continuous)

The probability of any value $x \in [a, b]$ is constant.

Normal (Continuous)

Events occur more frequently near μ and less frequently further away from μ .

Standardised Normal (Continuous)

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

Distribution	Probability	Cumulative Probability	μ	σ^2
Binomial: $X \sim \text{bin}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	See table above.	np	$np(1-p)$
Poisson: $X \sim \text{Pois}(\mu)$	$e^{-\mu} \mu^x / x!$	See table above.	λt	λt
Exponential: $T \sim \text{exp}(\lambda)$	$\lambda e^{-\lambda t}$	$1 - \lambda e^{-\lambda t}$	$1/\lambda$	$1/\lambda^2$
Uniform: $X \sim U(a, b)$	$1/(b-a)$	$(x-a)/(b-a)$	$(a+b)/2$	$(b-a)^2/12$
Normal: $X \sim N(\mu, \sigma^2)$	—	—	μ	σ^2

Sample Statistics

Given n samples x_i :

$$\text{Mean: } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\begin{aligned} \text{Variance: } s^2 &= \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \\ &= \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right) \end{aligned}$$

t Distribution

The sample mean is distributed according to the t distribution where

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}$$

$$\Pr(T \leq t_{n-1, 1-\alpha/2}) = 1 - \frac{\alpha}{2}$$

$$\Pr(T > t_{n-1, \alpha/2}) = \frac{\alpha}{2}$$

Confidence Intervals

Given the confidence level c :

$$c = 1 - \alpha.$$

The confidence interval for \bar{x} :

$$CI_c = \bar{x} \pm t_{n-1, 1-\alpha/2} \frac{s}{\sqrt{n}}.$$

Hypothesis Testing

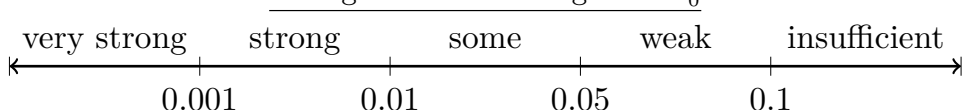
Hypothesis testing assesses the likelihood of observing the sample if the null hypothesis was true.

Hypothesis

$$H_0 : \mu = \mu_0 \quad \text{vs} \quad H_1 : \mu \neq \mu_0$$

Generally there is no evidence supporting H_0 , but only evidence or lack of evidence

Strength of evidence against H_0



Linear Regression

Given a set of n points (x_i, y_i) that are assumed to have a linear relationship, the model for y_i is

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

where $\varepsilon_i \sim N(0, \sigma^2)$ is the residual. s (RMS error) gives an estimate for σ — how close the data is to the model.

Estimates

$$T_{\text{test}} : \frac{\hat{\beta}_0 - \beta_0}{s_{\hat{\beta}_0}} \sim t_{n-2}, \quad \frac{\hat{\beta}_1 - \beta_1}{s_{\hat{\beta}_1}} \sim t_{n-2}$$

Inferences about β_0 and β_1

$$CI_c = \hat{\beta}_0 \pm t_{n-2, 1-\alpha/2} s_{\hat{\beta}_0}$$

$$CI_c = \hat{\beta}_1 \pm t_{n-2, 1-\alpha/2} s_{\hat{\beta}_1}$$

where $s_{\hat{\beta}_0}$ and $s_{\hat{\beta}_1}$ are the standard errors (SE) for $\hat{\beta}_0$ and $\hat{\beta}_1$. $H_{0, \beta_0} : \beta_0 = 0$ tests whether the model crosses the origin. $H_{0, \beta_1} : \beta_1 = 0$ tests whether the model is constant.

R-squared

The percentage of the observed variance in y that is explained by the model.

Residual Plots

Test the following two assumptions:

1. The relationship between X and Y is best modelled linearly — clear indication of a non-linear trend suggests assumption is not valid.
2. The variance of residuals is the same for all observations, (not affected by y_i) — uneven width of residual suggests assumption is not valid. This may lead to inaccurate inferences.