$$V_i = \begin{cases} h_1/2, & i = 1 \\ (h_{i-1} + h_i)/2, & 2 \leqslant i \leqslant N-1 \\ h_{N-1}/2, & i = N \end{cases}$$

$$h_i = x_{i+1} - x_i$$
$$\Omega_i = (w_i, e_i)$$

(mesh diagram with nodes $w_1, w_2, w_3, \dots, w_{N-1}, w_N$; $x_1, x_2, x_3, \dots, x_{N-2}, x_{N-1}, x_N$; $e_1, e_2, \dots, e_{N-2}, e_{N-1}, e_N$; volumes $V_1, V_2, \dots, V_{N-1}, V_N$; spacings $h_1, h_2, \dots, h_{N-2}, h_{N-1}$)

## Finite Differences

$$u(w_i, t) = (1-\sigma)u_{i-1} + \sigma u_i \qquad \partial_x u(w_i, t) = (u_i - u_{i-1})/h_{i-1} \quad \text{(west node)}$$
$$u(e_i, t) = (1-\sigma)u_i + \sigma u_{i+1} \qquad \partial_x u(e_i, t) = (u_{i+1} - u_i)/h_i \quad \text{(east node)}$$
$$\partial_x u(w_1, t) = \partial_x u(0, t) \qquad \partial_x u(e_1, t) = (u_2 - u_1)/h_1 \quad \text{(node 1)}$$
$$\partial_x u(w_N, t) = (u_N - u_{N-1})/h_{N-1} \qquad \partial_x u(e_N, t) = \partial_x u(L, t) \quad \text{(node } N)$$

$D(u(x_i, t)) = D(u_i)$, $D(u(w_i, t)) = \frac{D(u_{i-1}) + D(u_i)}{2}$, $D(u(e_i, t)) = \frac{D(u_i) + D(u_{i+1})}{2}$
Flow left to right ($v > 0$): $\sigma = 0$. Flow right to left ($v < 0$): $\sigma = 1$.

**Time Discretisation (integrate between $t_n$ and $t_{n+1}$)**

$$(\mathbf{I} - \delta t \theta_1 \mathbf{A})\mathbf{u}^{(n+1)} = [\mathbf{I} + \delta t(1 - \theta_1)\mathbf{A}]\mathbf{u}^{(n)} + \delta t \left[\mathbf{b}_1 + (1 - \theta_2)\mathbf{b}_2^{(n)} + \theta_2 \mathbf{b}_2^{(n+1)}\right]$$

using $\int_{t_n}^{t_{n+1}} f(t)\,dt \approx \delta t[(1-\theta)f(t_n) + \theta f(t_{n+1})]$, $\tilde{\mathbf{A}}\mathbf{u}^{(n+1)} = \tilde{\mathbf{B}}\mathbf{u}^{(n)} + \tilde{\mathbf{c}} = \tilde{\mathbf{b}}$

**FE** ($\theta_1 = \theta_2 = 0$), **BE** ($\theta_1 = \theta_2 = 1$), **C-N** ($\theta_1 = \theta_2 = \frac{1}{2}$). **Dirichlet BCs** replace row of $\tilde{\mathbf{A}}$ with $\mathbf{e}_1$ or $\mathbf{e}_N \in \mathbb{R}^{1 \times N}$ and row of $\tilde{\mathbf{b}}$ with Dirichlet BC.

**Krylov Methods** $\mathbf{K}_m(\mathbf{A}, \mathbf{b}) = \text{span}\{\mathbf{b}, \mathbf{Ab}, \mathbf{A}^2\mathbf{b}, \dots, \mathbf{A}^{m-1}\mathbf{b}\}$

**Hessenberg factorisation**: $\mathbf{AQ} = \mathbf{QH}$ for $\mathbf{Q}, \mathbf{H} \in \mathbb{R}^{n \times n}$. Reduced factorisation: $\mathbf{AQ}_m = \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m$ for $\mathbf{Q}_m \in \mathbb{R}^{n \times m}$, $\bar{\mathbf{H}}_m \in \mathbb{R}^{m+1 \times m}$.

$$\mathbf{q}_{j+1} = (\mathbf{Aq}_j - h_{1j}\mathbf{q}_1 - h_{2j}\mathbf{q}_2 - \cdots - h_{jj}\mathbf{q}_j)/h_{j+1,j} = \frac{1}{h_{j+1,j}}\left(\mathbf{Aq}_j - \sum_{i=1}^{j} h_{ij}\mathbf{q}_i\right)$$

**Arnoldi's method** apply Gram-Schmidt process to $\mathcal{K}_m(\mathbf{A}, \mathbf{b})$.

$$\mathbf{q}_{j+1} = \frac{1}{\|\mathbf{v}_{j+1}\|}\left(\mathbf{Aq}_j - \sum_{i=1}^{j}(\mathbf{q}_i^\top \mathbf{Aq}_j)\mathbf{q}_i\right), \quad \mathbf{Aq}_j = \sum_{i=1}^{j}(\mathbf{q}_i^\top \mathbf{Aq}_j)\mathbf{q}_i + \|\mathbf{v}_{j+1}\|\mathbf{q}_{j+1}$$

$$\bar{\mathbf{H}}_1 = \begin{bmatrix} \mathbf{q}_1^\top \mathbf{Aq}_1 \\ \|\mathbf{v}_2\| \end{bmatrix}, \quad \bar{\mathbf{H}}_2 = \begin{bmatrix} \mathbf{q}_1^\top \mathbf{Aq}_1 & \mathbf{q}_1^\top \mathbf{Aq}_2 \\ \|\mathbf{v}_2\| & \mathbf{q}_2^\top \mathbf{Aq}_2 \\ & \|\mathbf{v}_3\| \end{bmatrix}, \quad \dots, \quad \bar{\mathbf{H}}_m = \begin{bmatrix} \mathbf{H}_m \\ h_{m+1,m}\mathbf{e}_m^\top \end{bmatrix}$$

Left-multiply $\bar{\mathbf{H}}_m$ by $\mathbf{Q}_{m+1}$ to show $\mathbf{Q}_{m+1}\bar{\mathbf{H}}_m = \mathbf{Q}_m \mathbf{H}_m + h_{m+1,m}\mathbf{q}_{m+1}\mathbf{e}_m^\top$, where $\mathbf{H}_m \in \mathbb{R}^{m \times m}$. Left-multiply this result by $\mathbf{Q}_m^\top$ to show $\mathbf{Q}_m^\top \mathbf{AQ}_m = \mathbf{H}_m$.

**Sparse Linear Systems**

- Assume $\mathbf{x}^{(m)} \in \mathbf{x}^{(0)} + \mathcal{K}_m(\mathbf{A}, \mathbf{r}^{(0)})$, for initial residual $\mathbf{r}^{(0)} = \mathbf{b} - \mathbf{Ax}^{(0)} = \beta \mathbf{q}_1$ where $\mathbf{q}_1$ is taken from the Gram-Schmidt process and $\beta = \|\mathbf{r}^{(0)}\|$.
- Solution form: $\mathbf{x}^{(m)} = \mathbf{x}^{(0)} + \mathbf{Q}_m \mathbf{y}_m$, solve $\mathbf{y}_m$ using $\mathbf{r}^{(m)} = \mathbf{b} - \mathbf{Ax}^{(m)} \perp \mathcal{W}_m$.
- FOM: $\mathcal{W}_m = \mathcal{K}_m(\mathbf{A}, \mathbf{r}^{(0)})$.
- GMRES: $\mathcal{W}_m = \mathbf{A}\mathcal{K}_m(\mathbf{A}, \mathbf{r}^{(0)})$.

**Left preconditioning** ($\mathbf{M}^{-1}\mathbf{Ax} = \mathbf{M}^{-1}\mathbf{b}$ where $\tilde{\mathbf{r}}^{(0)} = \mathbf{M}^{-1}\mathbf{r}^{(0)}$):

- Assume $\mathbf{x}^{(m)} \in \mathbf{x}^{(0)} + \mathcal{K}_m(\mathbf{M}^{-1}\mathbf{A}, \tilde{\mathbf{r}}^{(0)})$, with $\mathbf{q}_1 = \tilde{\mathbf{r}}^{(0)}/\beta$ for $\beta = \|\tilde{\mathbf{r}}^{(0)}\|$.
- Arnoldi decomposition: $(\mathbf{M}^{-1}\mathbf{A})\mathbf{Q}_m = \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m$.
- Solution form: $\mathbf{x}^{(m)} = \mathbf{x}^{(0)} + \mathbf{Q}_m \mathbf{y}_m$, solve $\mathbf{y}_m$ using $\tilde{\mathbf{r}}^{(m)} = \mathbf{M}^{-1}\mathbf{r}^{(m)} \perp \mathcal{W}_m$.
- FOM: $\mathcal{W}_m = \mathcal{K}_m(\mathbf{M}^{-1}\mathbf{A}, \tilde{\mathbf{r}}^{(0)})$.
- GMRES: $\mathcal{W}_m = (\mathbf{M}^{-1}\mathbf{A})\mathcal{K}_m(\mathbf{M}^{-1}\mathbf{A}, \tilde{\mathbf{r}}^{(0)})$.

**Right preconditioning** ($\mathbf{AM}^{-1}\tilde{\mathbf{x}} = \mathbf{b}$ where $\mathbf{M}\tilde{\mathbf{x}} = \mathbf{x}$):

- Assume $\mathbf{x}^{(m)} \in \mathbf{x}^{(0)} + \mathbf{M}^{-1}\mathcal{K}_m(\mathbf{AM}^{-1}, \mathbf{r}^{(0)})$.
- Arnoldi decomposition: $(\mathbf{AM}^{-1})\mathbf{Q}_m = \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m$.
- Solution form: $\mathbf{x}^{(m)} = \mathbf{x}^{(0)} + \mathbf{M}^{-1}\mathbf{Q}_m \mathbf{y}_m$, solve $\mathbf{y}_m$ using $\mathbf{r}^{(m)} \perp \mathcal{W}_m$.
- FOM: $\mathcal{W}_m = \mathcal{K}_m(\mathbf{AM}^{-1}, \mathbf{r}^{(0)})$.
- GMRES: $\mathcal{W}_m = (\mathbf{AM}^{-1})\mathcal{K}_m(\mathbf{AM}^{-1}, \mathbf{r}^{(0)})$.

No/right preconditioning GMRES solution minimises $\|\mathbf{r}^{(m)}\|$ over the affine space:
$$\mathbf{y}_m = \arg\min_{\mathbf{y}}\|\beta \mathbf{e}_1 - \bar{\mathbf{H}}_m \mathbf{y}\| \implies \mathbf{y}_m = \beta \bar{\mathbf{H}}_m^\dagger \mathbf{e}_1$$

## Finite Volume Method

$$\underbrace{\frac{\partial u}{\partial t}}_{\text{unsteady}} + \underbrace{\boldsymbol{\nabla}\cdot(\mathbf{v}u)}_{\text{advection}} = \underbrace{\boldsymbol{\nabla}\cdot(\mathbf{D}\boldsymbol{\nabla}u)}_{\text{diffusion}} + \underbrace{R}_{\text{reaction}}$$

$$\partial_t u = -\boldsymbol{\nabla}\cdot\mathbf{q} + R, \quad \mathbf{q} = \mathbf{v}u - \mathbf{D}\boldsymbol{\nabla}u$$

$$\frac{du_i}{dt} \approx \frac{d\bar{u}_i}{dt} = \frac{1}{V_i}\left(q_{w_i} - q_{e_i}\right) + \bar{R}_i$$

$$\frac{d\mathbf{u}}{dt} = \mathbf{Au} + \mathbf{b}(t) \text{ or } \mathbf{G}(t, \mathbf{u}(t))$$

**Newton Methods ($\mathbf{u}^{(n+1)} := \mathbf{x}^{(k)}$)**

$$\frac{d\mathbf{u}}{dt} = \mathbf{G}(\mathbf{u}) \implies \mathbf{F}(\mathbf{u}^{(n+1)}) = \mathbf{0}$$
$$\mathbf{F}(\mathbf{x}^{(k)}) + \mathbf{J}(\mathbf{x}^{(k)})(\mathbf{x} - \mathbf{x}^{(k)}) = \mathbf{0}$$
$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \mathbf{J}(\mathbf{x}^{(k)})^{-1}\mathbf{F}(\mathbf{x}^{(k)})$$

**Newton (quad.)**: $m = 1$
**Shamanskii (suplin.)**: $m$
**Chord (lin.)**: $m = \infty$
**Convergence Theory**

$$\lim_{k \to \infty}\|\mathbf{x}^{(k)} - \mathbf{x}^*\| = 0 \implies \{\mathbf{x}^{(k)}\}_{k=0}^{\infty} \to \mathbf{x}^*$$

Cauchy if for all $\epsilon > 0$
$$\exists M : \|\mathbf{x}^{(k)} - \mathbf{x}^{(m)}\| < \epsilon : \forall k, m > M$$

Sequence converges ($\exists K > 0$):

**Quad.** $\|\mathbf{x}^{(k+1)} - \mathbf{x}^*\| \leqslant K\|\mathbf{x}^{(k)} - \mathbf{x}^*\|^2$
**Suplin.** $\|\mathbf{x}^{(k+1)} - \mathbf{x}^*\| \leqslant K\|\mathbf{x}^{(k)} - \mathbf{x}^*\|^\alpha$
**Linear** $\|\mathbf{x}^{(k+1)} - \mathbf{x}^*\| \leqslant \alpha\|\mathbf{x}^{(k)} - \mathbf{x}^*\|$

order $\alpha \in (0, 1)$, factor $\sigma \in (0, 1)$, $k \gg 1$.

**Lipschitz** continuous $\mathbf{J} \in \text{Lip}_\gamma(D)$ if
$$\|\mathbf{J}(\mathbf{x}) - \mathbf{J}(\mathbf{y})\| \leqslant \gamma\|\mathbf{x} - \mathbf{y}\| : \forall \mathbf{x}, \mathbf{y} \in D$$

**Useful Identities**

$$\|\mathbf{x} + \mathbf{y}\| \leqslant \|\mathbf{x}\| + \|\mathbf{y}\|, \ \|\alpha\mathbf{x}\| = |\alpha|\|\mathbf{x}\|$$
$$\|\mathbf{A} + \mathbf{B}\| \leqslant \|\mathbf{A}\| + \|\mathbf{B}\|, \ \|\mathbf{Ax}\| \leqslant \|\mathbf{A}\|\|\mathbf{x}\|$$

$$\mathbf{x} = \int_0^1 \mathbf{x}\,dt$$

$$\left\|\int_a^b \mathbf{F}(t)\,dt\right\| \leqslant \int_a^b \|\mathbf{F}(t)\|\,dt.$$

$$\mathbf{F}(\mathbf{x} + \mathbf{h}) - \mathbf{F}(\mathbf{x}) = \int_0^1 \mathbf{J}(\mathbf{x} + t\mathbf{h})\mathbf{h}\,dt$$

$$\int_a^b \frac{\partial}{\partial t}\mathbf{F}(t)\,dt = \mathbf{F}(b) - \mathbf{F}(a)$$

$$\frac{d}{dx}\left(\int_a^b f(x, t)\,dt\right) = \int_a^b \frac{\partial f(x, t)}{\partial x}\,dt$$

**Inexact Newton Method ($\approx$quad.)**

$$\tilde{\mathbf{J}}(\mathbf{x})\mathbf{e}_j = (\mathbf{F}(\mathbf{x} + \varepsilon\mathbf{e}_j) - \mathbf{F}(\mathbf{x}))/\varepsilon$$

**FOM Solution and Residual Norm Derivations**

**No preconditioning**

$$\mathbf{Q}_m^\top \mathbf{r}^{(m)} = \mathbf{0}$$
$$\mathbf{Q}_m^\top \left( \mathbf{b} - \mathbf{A}\mathbf{x}^{(m)} \right) = \mathbf{0}$$
$$\mathbf{Q}_m^\top \left( \mathbf{b} - \mathbf{A}\mathbf{x}^{(0)} - \mathbf{A}\mathbf{Q}_m\mathbf{y}_m \right) = \mathbf{0}$$
$$\mathbf{Q}_m^\top \left( \mathbf{r}^{(0)} - \mathbf{A}\mathbf{Q}_m\mathbf{y}_m \right) = \mathbf{0}$$
$$\mathbf{Q}_m^\top \mathbf{r}^{(0)} - \left( \mathbf{Q}_m^\top \mathbf{A}\mathbf{Q}_m \right) \mathbf{y}_m = \mathbf{0}$$
$$\beta \mathbf{Q}_m^\top \mathbf{q}_1 - \mathbf{H}_m \mathbf{y}_m = \mathbf{0}$$
$$\mathbf{H}_m \mathbf{y}_m = \beta \mathbf{e}_1$$

$$
\begin{aligned}
\left\| \mathbf{r}^{(m)} \right\| &= \left\| \mathbf{b} - \mathbf{A}\mathbf{x}^{(m)} \right\| \\
&= \left\| \mathbf{b} - \mathbf{A}\mathbf{x}^{(0)} - \mathbf{A}\mathbf{Q}_m\mathbf{y}_m \right\| \\
&= \left\| \mathbf{r}^{(0)} - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right\| \\
&= \left\| \mathbf{Q}_m\mathbf{Q}_m^\top \mathbf{r}^{(0)} - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right\| \\
&= \left\| \beta \mathbf{Q}_m\mathbf{Q}_m^\top \mathbf{q}_1 - \left( \mathbf{Q}_m\mathbf{H}_m + h_{m+1,m}\mathbf{q}_{m+1}\mathbf{e}_m^\top \right) \mathbf{y}_m \right\| \\
&= \left\| \beta \mathbf{Q}_m\mathbf{e}_1 - \mathbf{Q}_m\mathbf{H}_m\mathbf{y}_m - h_{m+1,m}\mathbf{q}_{m+1}\mathbf{e}_m^\top\mathbf{y}_m \right\| \\
&= \left\| \beta \mathbf{Q}_m\mathbf{e}_1 - \beta \mathbf{Q}_m\mathbf{H}_m\mathbf{H}_m^{-1}\mathbf{e}_1 - h_{m+1,m}\mathbf{q}_{m+1}\mathbf{e}_m^\top\mathbf{y}_m \right\| \\
&= h_{m+1,m}\left\| \mathbf{q}_{m+1}\mathbf{e}_m^\top\mathbf{y}_m \right\| = h_{m+1,m}\left| \mathbf{e}_m^\top\mathbf{y}_m \right|
\end{aligned}
$$

**Left preconditioning**

Same solution $\mathbf{y}_m$, with appropriate substitutions.

$$
\begin{aligned}
\left\| \mathbf{r}^{(m)} \right\| &= \left\| \mathbf{M}\tilde{\mathbf{r}}^{(m)} \right\| \\
&= \left\| \mathbf{M}\mathbf{M}^{-1} \left( \mathbf{b} - \mathbf{A}\mathbf{x}^{(0)} - \mathbf{A}\mathbf{Q}_m\mathbf{y}_m \right) \right\| \\
&= \left\| \mathbf{M} \left( \mathbf{M}^{-1}\mathbf{r}^{(0)} - \left( \mathbf{M}^{-1}\mathbf{A} \right) \mathbf{Q}_m\mathbf{y}_m \right) \right\| \\
&= \left\| \mathbf{M} \left( \tilde{\mathbf{r}}^{(0)} - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right) \right\| \\
&= h_{m+1,m}\left\| \mathbf{M}\mathbf{q}_{m+1}\mathbf{e}_m^\top\mathbf{y}_m \right\|
\end{aligned}
$$

**Right preconditioning**

Same solution $\mathbf{y}_m$ and residual norm $\left\| \mathbf{r}^{(m)} \right\|$, with appropriate substitutions.

**GMRES Solution and Residual Norm Derivations**

**No preconditioning**

$$\left( \mathbf{A}\mathbf{Q}_m \right)^\top \mathbf{r}^{(m)} = \mathbf{0}$$
$$\left( \mathbf{A}\mathbf{Q}_m \right)^\top \left( \mathbf{b} - \mathbf{A}\mathbf{x}^{(m)} \right) = \mathbf{0}$$
$$\left( \mathbf{A}\mathbf{Q}_m \right)^\top \left( \mathbf{b} - \mathbf{A}\mathbf{x}^{(0)} - \mathbf{A}\mathbf{Q}_m\mathbf{y}_m \right) = \mathbf{0}$$
$$\left( \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m \right)^\top \left( \mathbf{r}^{(0)} - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right) = \mathbf{0}$$
$$\bar{\mathbf{H}}_m^\top \mathbf{Q}_{m+1}^\top \left( \beta \mathbf{q}_1 - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right) = \mathbf{0}$$
$$\beta \bar{\mathbf{H}}_m^\top \mathbf{Q}_{m+1}^\top \mathbf{q}_1 - \bar{\mathbf{H}}_m^\top \bar{\mathbf{H}}_m\mathbf{y}_m = \mathbf{0}$$
$$\bar{\mathbf{H}}_m^\top \bar{\mathbf{H}}_m\mathbf{y}_m = \beta \bar{\mathbf{H}}_m^\top \mathbf{e}_1$$

$$
\begin{aligned}
\left\| \mathbf{r}^{(m)} \right\| &= \left\| \mathbf{b} - \mathbf{A}\mathbf{x}^{(m)} \right\| \\
&= \left\| \mathbf{b} - \mathbf{A}\mathbf{x}^{(0)} - \mathbf{A}\mathbf{Q}_m\mathbf{y}_m \right\| \\
&= \left\| \mathbf{r}^{(0)} - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right\| \\
&= \left\| \mathbf{Q}_{m+1}\mathbf{Q}_{m+1}^\top \mathbf{r}^{(0)} - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right\| \\
&= \left\| \beta \mathbf{Q}_{m+1}\mathbf{Q}_{m+1}^\top \mathbf{q}_1 - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right\| \\
&= \left\| \beta \mathbf{Q}_{m+1}\mathbf{e}_1 - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right\| \\
&= \left\| \mathbf{Q}_{m+1} \left( \beta \mathbf{e}_1 - \bar{\mathbf{H}}_m\mathbf{y}_m \right) \right\| = \left\| \beta \mathbf{e}_1 - \bar{\mathbf{H}}_m\mathbf{y}_m \right\|
\end{aligned}
$$

**Left preconditioning**

Same solution $\mathbf{y}_m$, with appropriate substitutions.

$$
\begin{aligned}
\left\| \mathbf{r}^{(m)} \right\| &= \left\| \mathbf{M}\tilde{\mathbf{r}}^{(m)} \right\| \\
&= \left\| \mathbf{M}\mathbf{M}^{-1} \left( \mathbf{b} - \mathbf{A} \left( \mathbf{x}^{(0)} + \mathbf{Q}_m\mathbf{y}_m \right) \right) \right\| \\
&= \left\| \mathbf{M}\mathbf{M}^{-1} \left( \mathbf{b} - \mathbf{A}\mathbf{x}^{(0)} - \mathbf{A}\mathbf{Q}_m\mathbf{y}_m \right) \right\| \\
&= \left\| \mathbf{M} \left( \mathbf{M}^{-1}\mathbf{r}^{(0)} - \left( \mathbf{M}^{-1}\mathbf{A} \right) \mathbf{Q}_m\mathbf{y}_m \right) \right\| \\
&= \left\| \mathbf{M} \left( \tilde{\mathbf{r}}^{(0)} - \mathbf{Q}_{m+1}\bar{\mathbf{H}}_m\mathbf{y}_m \right) \right\| \\
&= \left\| \mathbf{M}\mathbf{Q}_{m+1} \left( \beta \mathbf{e}_1 - \bar{\mathbf{H}}_m\mathbf{y}_m \right) \right\|
\end{aligned}
$$

**Right preconditioning**

Same solution $\mathbf{y}_m$ and residual norm $\left\| \mathbf{r}^{(m)} \right\|$, with appropriate substitutions.