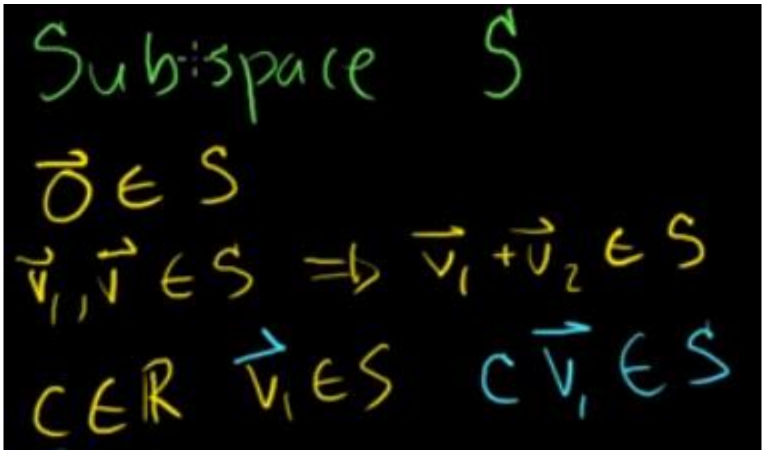
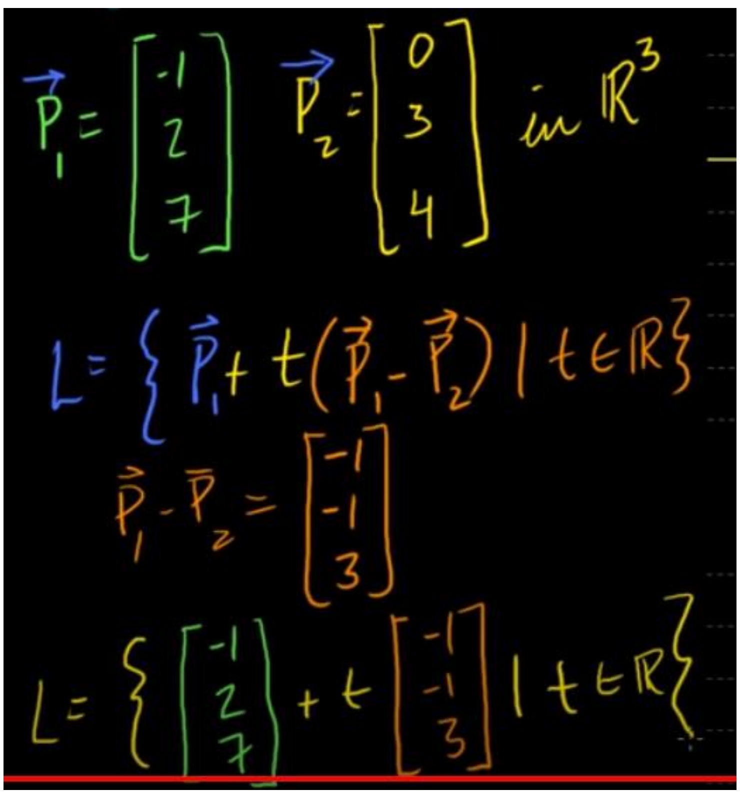
Properties of a subspace



Equation of a line that passes through P1 and P2 in R3



This implies a line in R3 can be represented using the following 3 equations.

$$x = -1 + -1t$$
$$y = 2 + -1t$$
$$z = 7 + 3t$$

https://www.youtube.com/watch?v=hWhs2cIj7Cw

Square of the length of a vector is dot product of the vector with itself

$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} \qquad \|\vec{a}\| = \sqrt{\vec{a}\cdot\vec{a}}$$

$$\vec{a}\cdot\vec{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = a_1^2 + a_2^2 + \cdots a_n^2 \qquad \boxed{\|\vec{a}\|^2 = \vec{a}\cdot\vec{a}}$$

https://www.youtube.com/watch?v=WNuIhXo39_k

Cauchy Swarz inequality

$$\|\vec{y}\|\,\|\vec{x}\| \geq |\vec{x}\cdot\vec{y}|$$

If x = cy, then this turns to an equality.

Watch the entire proof at https://www.youtube.com/watch?v=r2PogGDl8_U

Triangle inequality

$$\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$$

If x = cy, then this turns to an equality.

Watch the entire proof at https://www.youtube.com/watch?v=PsNidCBr5II

Angle between two vectors of any dimension.

$$(\vec{a} \cdot \vec{b}) = \|\vec{a}\|\|\vec{b}\| \cos\theta$$

This implies if two vectors are perpendicular/orthogonal, dot product of the two vectors is 0.

Watch the entire proof at https://www.youtube.com/watch?v=5AWob_z74Ks

Given an R3 plane containing the point (x0,y0,z0) and normal to the plane is represented by (n1,n2,n3), equation of the plane is given as:

$$n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$$

Watch the entire proof at https://www.youtube.com/watch?v=UJxgcVaNTqY

Cross-product of two vectors in R3 is defined as

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

The cross-product is orthogonal to both vectors a and b.

Watch the entire proof at https://www.youtube.com/watch?v=pJzmiywagfY

Properties of dot product and cross product

$$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta$$

$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\| \sin\theta$$

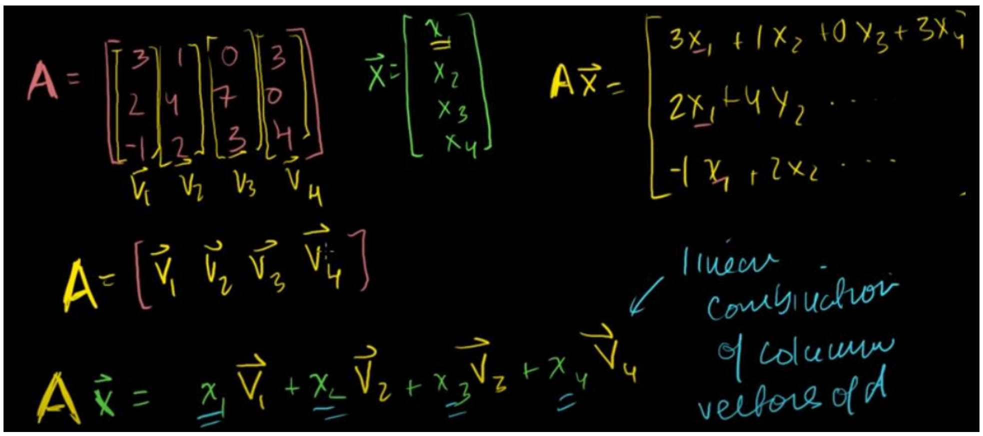
Watch the entire proof at https://www.youtube.com/watch?v=7MKA2QlKvHc

In the reduced row echelon form,

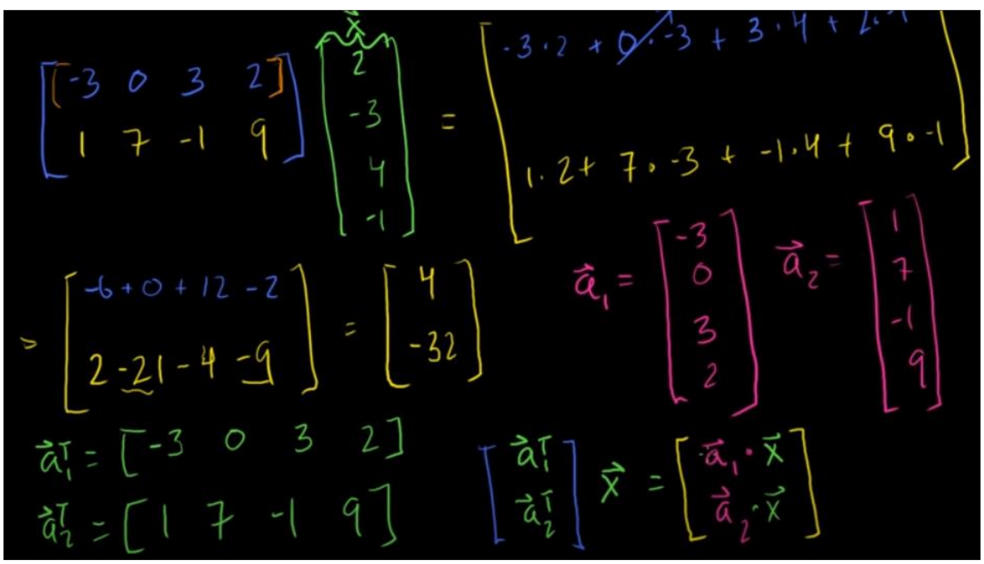
- if the number of pivot entries is equal to the number of columns, then there's a unique solution.
- If there are free (independent) variables, then there are infinite solutions.
- If you get 0 = <a>, then there's no solution.

Watch the video here https://www.youtube.com/watch?v=JVDrlTdzxiI

Matrix multiplication Ax can be interpreted as a linear combination of the column vectors of A, using the weights in x.

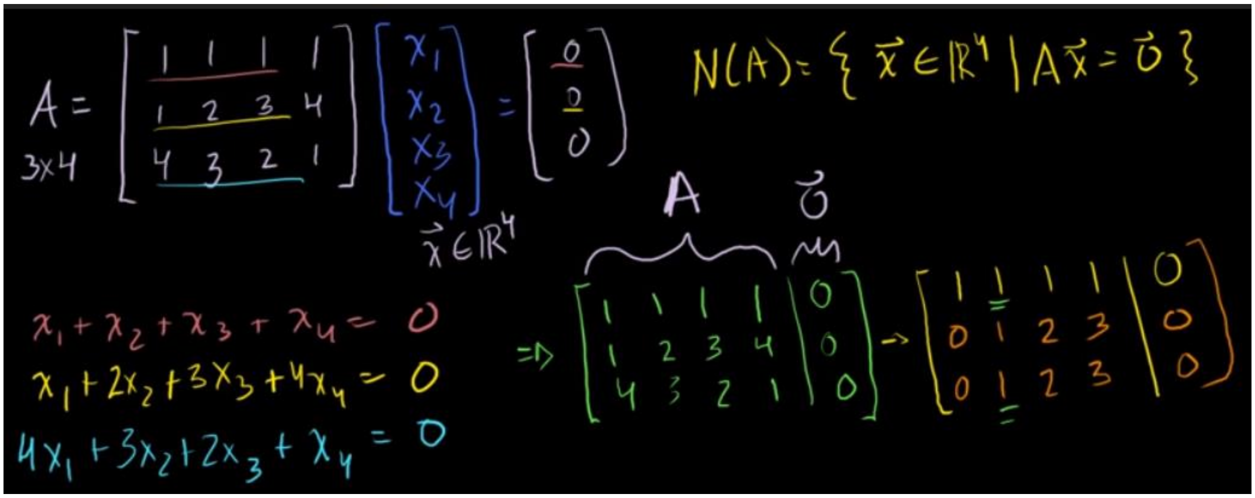


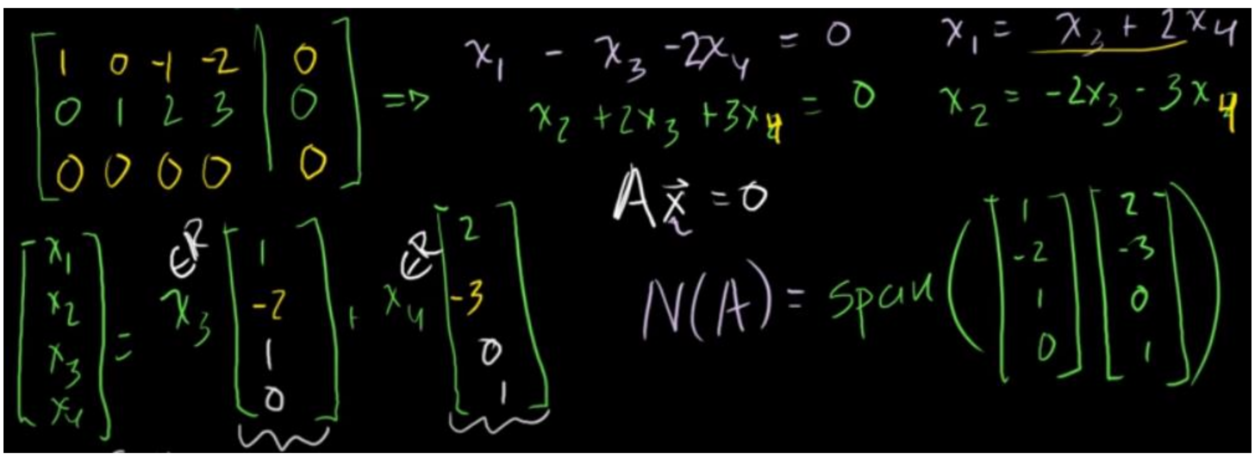
Or as dot product of transpose of the row vectors in A with x.



Watch here at https://www.youtube.com/watch?v=7Mo4S2wyMg4

Null space is defined as that subspace of the vector space A, such that Ax = 0

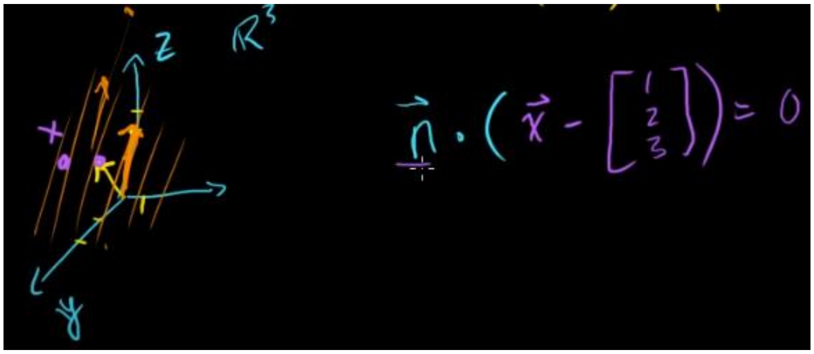




Null space of a vector space A is zero matrix, if and only if A is linearly independent, in which case it has zero free variables in RREF.

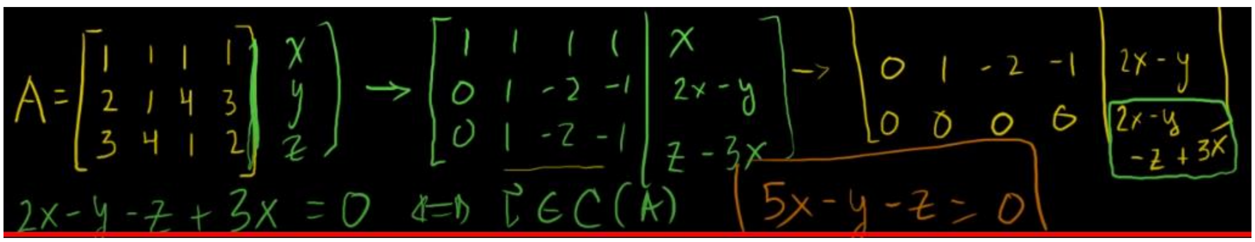
Two different ways to find the equation of an R3 plane.

1. Using the normal vector, and two vectors in the plane like this:



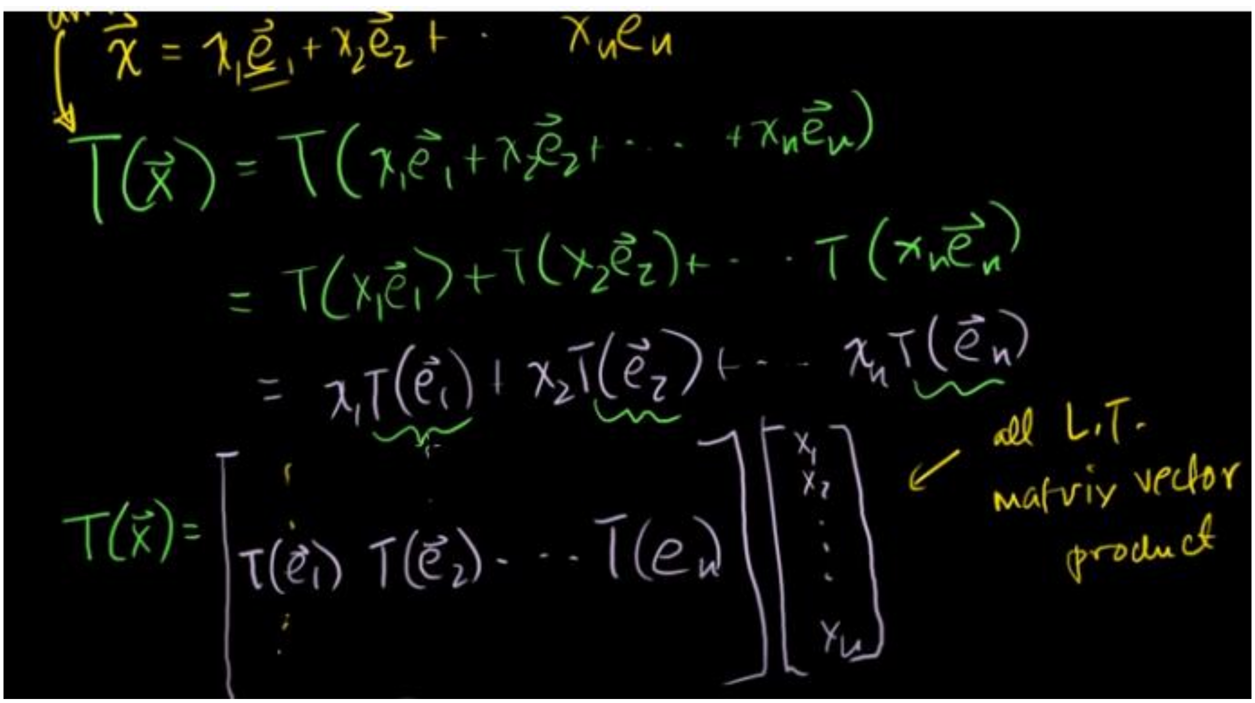
where n is the normal vector. To get n, find the cross product of the given vectors.

2. Using the generic vector [x,y,z] and applying rref.



Watch the entire video at https://www.youtube.com/watch?v=EGNlXtjYABw

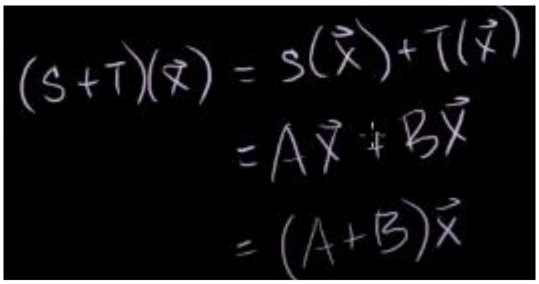
Any arbitrary linear transformation of a vector can be represented as a product of a matrix where each column is the transformation of the basis vectors, and the given vector.



Watch the entire working at https://www.youtube.com/watch?v=PErhLkQcpZ8

Linear transformation of a shape is demoed at https://www.youtube.com/watch?v=MIAmN5kgp3k

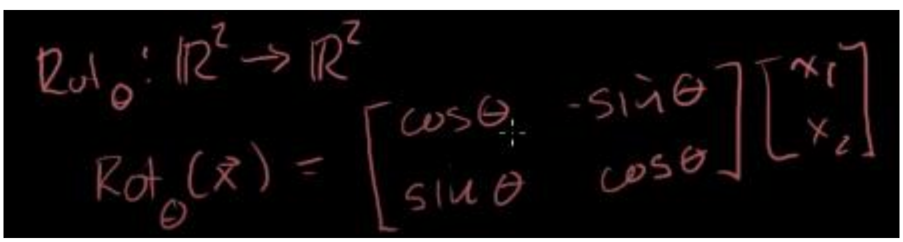
Sum of linear transformation S and T (each represented by matrices A and B). can be represented by A + B.



Similarly, scalar multiple of the transformation S (represented by A) can be represented as scalar times A.

Watch the full proof at https://www.youtube.com/watch?v=wHuY97vss18

Rotating a vector in R2 is a linear transformation. Assuming the angle of rotation is θ, the transformation matrix is



Watch the full proof at https://www.youtube.com/watch?v=IPWfIq5DzqI

Projection of a vector x on another vector L can be derived as follows.

$\mathrm{Proj}_L(\vec{x}) =$ <u>some vector in L</u>

where <u>$\vec{x} - \mathrm{Proj}_L(\vec{x})$</u>

is <u>orthogonal</u> to L

$$= c\vec{v}$$

$$(\vec{x} - \underbrace{c\vec{v}}_{\mathrm{proj}}) \cdot \vec{v} = 0$$

$$\vec{x} \cdot \vec{v} - c\,\vec{v} \cdot \vec{v} = 0$$

$$\vec{x} \cdot \vec{v} = c\,\vec{v} \cdot \vec{v}$$

NOTE: L can be represented as a scalar multiple of v.

Hence, projection is $\mathrm{Proj}_L(\vec{x}) = c\vec{v} = \left(\frac{\vec{x}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$

Watch details of the proof at https://www.youtube.com/watch?v=27vT-NWuw0M

Projection of a vector x onto another vector v can be rewritten as $\mathrm{Proj}_L(\vec{x}) = (\vec{x}\cdot\hat{u})\,\hat{u}$, given u is the unit vector in the direction of v. It can also be proved that this projection is a linear transformation, represented by the matrix below.

$$A = \begin{bmatrix} \underbrace{\left(\begin{bmatrix}1\\0\end{bmatrix}\cdot\begin{bmatrix}u_1\\u_2\end{bmatrix}\right)}_{u_1}\begin{bmatrix}u_1\\u_2\end{bmatrix} & \left(\begin{bmatrix}0\\1\end{bmatrix}\cdot\begin{bmatrix}u_1\\u_2\end{bmatrix}\right)\begin{bmatrix}u_1\\u_2\end{bmatrix} \end{bmatrix}$$

that simplifies to

$$\begin{bmatrix} u_1^2 & u_2 u_1 \\ u_1 u_2 & u_2^2 \end{bmatrix}$$

Watch the full proof at https://www.youtube.com/watch?v=JK-8XNIoAkI

Composition of two linear transformations can be represented by the matrix obtained by multiplying the individual matrices that represent each transformation. Thus, if B represents the T and A represents S, then the composition of the two transformations is $S_0T(x) = AB$.

Watch the full proof at https://www.youtube.com/watch?v=BuqcKpe5ZQs

If f is invertible, then f(x) = y has a unique solution for x. This implies that there's only one x-value that produces the specified y-value.

Watch the full explanation at https://www.youtube.com/watch?v=7GEUgRcnfVE

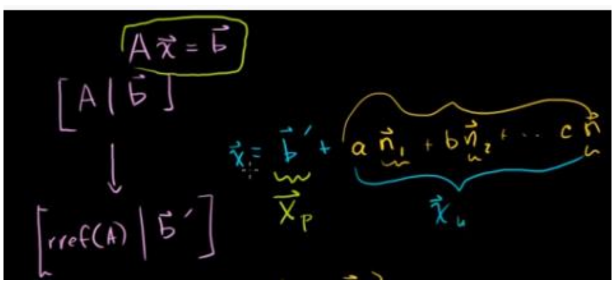
A function f is invertible, only if it's both surjective and injective.

Watch the full explanation at https://www.youtube.com/watch?v=QIU1daMN8fw

A linear transformation from Rn to Rm is onto(surjective) only if the column space of the transformation matrix A equals the co-domain. In other words, rref(A) has a pivot entry for every row in A. This means, the given transformation is surjective only if rank of the transformation matrix is m.

Watch the full explanation at https://www.youtube.com/watch?v=eR8vEdJTvd0

For Ax = b, the solution set comprises of a particular vector, combined with a null-space.



A linear transformation is one-to-one only if the null space is empty. In this case, column vectors of matrix A are linearly independent.

Watch the full explanation at https://www.youtube.com/watch?v=M3FuL9qKTBs

For a linear transformation to be invertible, dimensions of the input and output space must be same. This means, the transformation matrix A must be a square matrix. Also implies that the rref(A) is the identity matrix.

Watch more explanation at https://www.youtube.com/watch?v=Yz2OosyMTmY