

1. In any undirected graph, the number of vertices with odd degree is even. This statement (as well as the degree sum formula) is known as the [handshaking lemma](#). The latter name comes from a popular mathematical problem, to prove that in any group of people the number of people who have shaken hands with an odd number of other people from the group is even.

$$\sum_{v \in V} \deg(v) = 2|E|.$$

Reference: [Degree \(graph theory\) - Wikipedia](#)

2. Number of edges of a graph can vary between $(n - 1)$ to $n * (n - 1)/2$, where n is the number of vertices. In the former case, the graph is minimally connected tree (DAG), and in the latter case it's a complete graph.
3. Sum of degrees of all vertices in a graph is twice the number of edges.
4. Sum of degrees of all vertices in a graph is even.
5. Sum of degrees of all vertices with odd degree is always even. It follows that number of vertices with odd degrees is also even.
6. If more than one vertex in a graph has $(n - 1)$ degree, it means there cannot be a vertex with degree 1.
7. Maximum number of edges a graph with 4 vertices can have is $4C2 = 6$. Maximum sum of degrees is $6 * 2 = 12$.
8. The maximum number of non-zero entries in an adjacency matrix of a simple graph having n vertices can be $n(n - 1)$. This is obtained by nPn .

NOTE: All above observations are valid for undirected graphs

9. In a directed graph, sum of indegree of all vertices = sum of outdegree of all vertices.
10. BFS uses queue to store its visited nodes; DFS uses stack to store its visited nodes
11. BFS and DFS can be used to identify cycles in a graph, but BFS is less efficient.

Reference: [algorithm - Why DFS and not BFS for finding cycle in graphs - Stack Overflow](#)

12. BFS tree will try to find the shortest path between two vertices. So, in order to find an edge that can't appear in the original graph, use [data structures - BFS tree to graph - Stack Overflow](#)
13. Any two vertices present on different branches on the DFS tree cannot be an edge in the original graph.

Reference: [algorithm - DFS Tree of an Undirected Graph - Stack Overflow](#)

14. Time complexity of BFS/DFS on a graph with n vertices is n^2 , when using adjacency matrix.
15. When using adjacency list, time complexity of BFS/DFS on a graph with n vertices and m edges is $n + 2m$ (considered $n + m$)
16. Independent set is that set (of vertices) that have at most one edge from its elements. There must be at least 2 independent edges.
17. Independent set of a graph includes at most one endpoint of every edge of the graph. Some edges may be excluded, since we want to maximize the number of vertices.
18. A maximal independent set is that set to which no edge can be added in order to make it non-independent.
19. A maximum independent set is that maximal independent set with maximum edges.
20. Vertex cover of a graph is a set of vertices that includes at least one endpoint of every edge in the graph. Our goal is to minimize the vertices, but cover all edges.
21. Number of spanning trees possible in a graph with n vertices is n^{n-2} . This is known as Cayley's formula.

1. Path of length n between two nodes consists of $(n + 1)$ nodes.
2. Isolated nodes can be found using BFS
3. Graphs can belong to one of the following categories.
 - a. Directed
 - i. Acyclic (DAG)
 - ii. Trees.
 - b. Undirected
 - i. Connected
 - ii. Complete
 - iii. Biconnected
4. Trees are DAGs where each node can have only one parent.
5. DAG has at least one vertex with indegree 0 and at least one vertex with outdegree 0.
6. Adding an edge to a tree creates a cycle
7. In a tree, every pair of vertices is connected by a unique path, else there's bound to be a cycle somewhere.
8. Any two of the following facts about a graph G , implies the third.
 - a. G is connected
 - b. G is acyclic
 - c. G has $n-1$ edges

Thus, if G is connected and acyclic, it has $n-1$ edges. If G is connected and acyclic and has $n-1$ edges, it's acyclic. Similarly, if G is acyclic and has $n-1$ edges, it's connected.

9. Connected graph has a path between every pair of vertices. There's no unreachable node.
10. Path graph is represented as a path that joins the vertices. In a path graph, every vertex other than the two end vertices have degree 2. End vertices have degree 1.
11. Complete graph has $n(n - 1)/2$ edges, since every node must connect to every other node.
12. A graph in which each edge connects two **different** vertices and where no two edges connect the same pair of vertices is called a simple graph.
13. A graph in which multiple edges may connect the same pair of vertices is called a multigraph.

14.

Algorithm	Objective	Weights	Iterations	Remarks
Dijkstra	Single-source shortest path	+	1	
Bellman-Ford	Single-source shortest path	+/-	n	
Floyd-Warshall	Allpairs shortest path	+/-	n	A0, A1, A2..An matrices
Prim	Minimum spanning tree	+/-	1	
Kruskal	Minimum spanning tree	+/-	1	

15. For a weighted graph $G(V,E)$, the shortest path doesn't change if the weight of its edges are multiplied by a constant value. However, it changes if the weight of its edges are incremented by a constant value.

16.