- Consider the following data set where each data point consists of three features x_1, x_2 and x_3:

x_1	x_2	x_3
10	10	9
13	12	13
5	5	4
8	7	7

Consider two encoder functions f and $\tilde{f}$ with decoders g and $\tilde{g}$ respectively aiming to reduce the dimensionality of the data set from 3 to 1:

Pair 1: $f(x_1, x_2, x_3) = x_1 - x_2 + x_3$ and $g(u) = [u, u, u]$

Pair 2: $\tilde{f}(x_1, x_2, x_3) = \dfrac{x_1 + x_2 + x_3}{3}$ and $\tilde{g}(u) = [u, u, u]$

The reconstruction loss of the encoder decoder pair is the mean of the squared distance between the actual input and reconstructed input.

- Consider the following input data points:

X	y
[2]	5.8
[3]	8.3
[6]	18.3
[7]	21
[8]	22

What will be the amount of loss when the functions $g = 3x_1 + 1$ and $h = 2x_1 + 2$ are used to represent the regression line. Consider the average squared error as loss function.

- Consider the following input data points:

X	y
[4, 2]	+1
[8, 4]	+1
[2, 6]	−1
[4, 10]	−1
[10, 2]	+1
[12, 8]	−1

What will be the average misclassification error when the functions $g(X) = sign(x_1 - x_2 - 2)$ and $h(X) = sign(x_1 + x_2 - 10)$ are used to classify the data points into classes $+1$ or -1.

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A probability model $P(X) = \begin{cases} \frac{1}{5}, & \text{if } x \in [0,5] \\ 0, & \text{otherwise} \end{cases}$ is obtained by the density estimation algorithm for the data points $x_1 = 2.5, x_2 = 1, x_3 = 3, x_4 = 4.5$ and $x_5 = 4.95$. Compute the negative log likelihood loss of the model.

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Consider the following function:

$$f(x) = \begin{cases} 7x + 2, & \text{if } x > 1 \\ 9, & \text{if } x \leq 1 \end{cases}$$

Is $f(x)$ continuous?

- What is the linear approximation of $f(x, y) = x^2 + y^2$ around (1, 1)?
-

The directional derivative of $f(x, y, z) = x^2 + 3y + z^2$ at (1, 2, 1) along the unit vector in the direction of [1, -2, 1] is (correct upto three decimal places)

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Let $g(x) = 2.5e^{-x^2+0.2x+2}$. Determine the equation of the tangent line at $x = 0.5$ and using it estimate the value of $g(1.5)$.

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9) A function $f(x, y) = x^2 + 2xy^3$ is approximated linearly in the neighbourhood of (2,-2). Use the approximation to approximate $f(2.3, -2.2)$.

-

Find the direction of steepest ascent for the function $x^2 + y^3 + z^4$ at point (1, 1, 1).

-

Approximate $\sqrt{3.9}$ by linearizing $\sqrt{x}$ around $x = 4$.

-

Consider a set of 3 paired observations on $(x_i, b_i), i = 1, 2, 3$ as $((1, 6), (-1, 3), (3, 15))$. For the closest line b to go through these points, which of the following is the least squares solution $(\hat{\theta})$?

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Find projection of [5,-4,1] along [3,-2,4]

-

The Leaning Tower of Pisa is represented as a vector, $u = \begin{bmatrix} 2 \\ 10 \\ 25 \end{bmatrix}$ and the ground is represented as a vector, $v = \begin{bmatrix} 35 \\ 50 \\ 0 \end{bmatrix}$. Find the length of the tower's shadow when the Sun is shining bright at 12:00 PM.

-

Find the direction in which the function $f(x, y) = x^2 + xy + y^2$ decreases most rapidly at a point $a = (-1, 1)$. Also find the directional derivative of the function in this direction.

Hint: Give direction in (a + ib) and directional derivative as a scalar value.

- The projection matrix for the matrix $v = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ is

- The trace of a 2×2 matrix A is 4, and its determinant is 3. The eigenvalues of A are

- If the eigenvalues of a matrix A are 0 -1 and 5, then the eigenvalues of A^3 are

Hint: Eigen-values are cubed as well, but eigen-vectors remains same.

- If the matrix $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ x & \frac{4}{5} \end{bmatrix}$ is orthogonal, then the value of x is

Hint: For orthogonal matrix A, $AA^T = I$

- The best second degree polynomial that fits the data set

x	y
0	0
1.5	1.5
4	1

- For the matrix, $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$, the eigen values are 3 and 7. Find $x + y$.

- The two eigenvalues of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & p \end{bmatrix}$ have a ratio 2:1 for $p = 3$. What is another value of p for which eigenvalues have same ratio 2:1?

- Suppose that A, P are 3 × 3 matrices, and P is an invertible matrix.

If $P^{-1}AP = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 3 & 8 \\ 0 & 0 & 4 \end{bmatrix}$, then the eigenvalues of the matrix A^2 are

Hint: $P^{-1}AP = D \Rightarrow A = PDP^{-1}$. As per spectral theorem, diagonal elements of D is composed of A's eigen-values and columns of P is composed of A's eigen-vectors.

-

The inner product of $x = \begin{bmatrix} 1-i \\ 2i \end{bmatrix}$ and $y = \begin{bmatrix} -1-i \\ i \end{bmatrix}$ is

Hint: Take the conjugate of x before multiplying with y

- True or False?

The matrix $A = \begin{bmatrix} \frac{(1+i)}{\sqrt{3}} & \frac{(1+i)}{\sqrt{6}} \\ \frac{i}{\sqrt{3}} & \frac{2i}{\sqrt{6}} \end{bmatrix}$ is unitary.

Hint: $UU^* = U^*U = I$

-

The set containing all the eigenvalues corresponding to the unitary matrix
$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i & 0 \\ 0 & 0 & i \end{bmatrix}$ is.

Hint: All (a + ib) where magnitude of the vector is 1 are valid solutions.

-

The matrix $A = k \begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$ is unitary if k is

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11) Let $A = \begin{bmatrix} 2 & 1+i \\ 1-i & 3 \end{bmatrix}$. If A can be factorized as $A = UDU^*$, with U denoting a unitary matrix, and D denoting a diagonal matrix, then, U and D are

Hint: Use spectral theorem; eigen-values make the diagonal entries of D. Normalized eigen-vectors make the columns in U.

Hint: To normalize complex vectors, multiply with conjugate.

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The SVD of matrix $A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ is

Hint: Q_2 should be transposed.

- 2) A matrix S is decomposed using Singular Value Decomposition (SVD) as given below

$$S = \begin{bmatrix} -0.9739 & -0.1146 & 0.1961 \\ -0.1539 & -0.3023 & -0.9407 \\ 0.1671 & -0.9463 & 0.2768 \end{bmatrix} \begin{bmatrix} 3.8287 & 0 & 0 \\ 0 & 1.6607 & 0 \\ 0 & 0 & 1.2582 \end{bmatrix} \begin{bmatrix} -0.2578 & 0.6829 & 0.6835 \\ -0.4651 & -0.7078 & 0.5317 \\ -0.8469 & 0.1808 & -0.5001 \end{bmatrix}$$

Then the absolute value of the determinant of the matrix SS^T is?

- 2

Decompose the unitary matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i & 0 \\ 0 & 0 & i \end{bmatrix}$ using Singular Value Decomposition (SVD) and enter the sum of singular values.

- The eigenvalues and corresponding eigenvectors of a 2×2 matrix A are given by

Eigenvalue	Eigenvector
-1	$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
-2	$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

What will be the matrix A?

Hint:

Solve $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 8 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. It can also be solved by using the formula A = SλS^{-1}

- 4) The inner product of two distinct vectors x and y that are from $\mathbb{C}^{100}$ is $0.8 - 0.37i$. The vector x is scaled by a scalar $1 - 2i$ to obtain a new vector z, then the inner product between z and y is

Hint: 3 $(cx) \cdot y = \bar{c}(x \cdot y)$

- The matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is

Is it positive/negative definite/semi-definite?
Hint: Find the eigen values.

- A 2x2 diagonal matrix A has determinant 21 and trace 10.

Is it positive/negative definite/semi-definite?
Hint: Find the eigen values

- The function $f(x, y) = 4 + x^3 + y^3 - 3xy$ has a stationary point at

- Given a function $f(x, y) = -3x^2 - 6xy - 6y^2$, the point $(0, 0)$ is a ______

Hint: Find the Hessian matrix

- Given $f(x, y) = 5x^2 + 8xy - 4y^2$, the point $(0, 0)$ is a _____

- Consider the data points x_1, x_2, x_3 to answer the following questions.

$$x_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$
$$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$x_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

Do PCA, and project these data onto a single-dimension space. Find the reconstruction error and projected variance.

- The point on $y = x^2 + 1$ closest to (0,1.5) is

- The volume of the largest cone that can be inscribed in a sphere of radius 6m is

- An open box is to be made from a 12cm by 18cm rectangular piece of cardboard by cutting equal squares from each corner and turning up the sides. Find the volume in cubic cm of the largest box that can be made in this manner.

- The value of a function at point 10 is 100. The values of the function's first and second order derivatives at this point are 20 and 2 respectively. What will be the function's approximate value correct up to two decimal places at the point 10.5

Hint: Use Taylor's expansion

[redacted] For the function $f(x) = \frac{x sin x - 1}{2}$, with an initial guess of $x_0 = -7$, and step size of 0.25, the value of the function after two iterations is

[redacted] The value of $f(x_1, x_2) = 4x_1^2 - 4x_1 x_2 + 2x_2^2$ with an initial guess of (2, 3) after two iterations of gradient descent algorithm will be .... Take the step size $\eta = \frac{1}{t}$, where t= no. of iteration.

What is the value of a, the function $f : \mathbb{R} \to \mathbb{R}, f(x, y) = ax^4 + 8y$ is a convex function

Hint: Find the Hessian matrix

[redacted] If objective function which is to be minimised is $f(x, y, z) = x + z$ and the constrained equation is $g(x, y, z) = x^2 + y^2 + z^2 = 1$. The point where minimum value occurs will be

Let $f(x) = -2x^2 + 5$. At $x = -3$, is $f(x)$ increasing or decreasing?

Consider two functions $g(x) = 2x - 3$ and $f(x) = x - 10ln(5x)$. Is the function convex or concave?

Given below is a set of data points and their labels.

X	y
$[1, 0]$	1.5
$[2, 1]$	2.9
$[3, 2]$	3.4
$[4, 2]$	3.8
$[5, 3]$	5.3

To perform linear regression on this data set, the sum of squares error with respect to w is to be minimized.

Find the optimal w*.

[redacted] Let w^1 be initialized to $\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$. Gradient descent optimization is used to find the value of optimal w^*. For the first iteration $t = 1$, which of the following is the gradient computed with respect to w^1?

Further, Using the gradient descent update equation with a learning rate $\eta_t = 0.1$, compute the value of w at $t = 2$.

[redacted] Minimize the function $f = x_1^2 + 60x_1 + x_2^2$ subject to the constraints $g_1 = x_1 - 80 \geq 0$ and $g_2 = x_1 + x_2 - 120 \geq 0$ using KKT conditions. Which of the following is the optimal solution set?

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Find the largest interval of x in which a function $f(x) = xe^{x^2}$ is convex.

-

Let the composition of two functions $f(x) = sin(x) - 2x^2 + 1$ and $g(x) = e^x$ be $h = fog$. At a point $x = 5$, is h convex or concave? Is the function increasing or decreasing at the given point?

-

[redacted] [redacted]

Let a set of data points with five samples and two features per sample be $X = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 2.5 \\ 6 & 4 \\ 7.5 & 5 \end{bmatrix}$ and the corresponding labels be $y = \begin{bmatrix} 1.5 \\ 2 \\ 2.5 \\ 3 \\ 4 \end{bmatrix}$. Perform linear regression on this data set and choose the optimal solution for w^* to minimize the sum of

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[redacted] Consider a vector $\hat{w} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$ in $\mathbb{R}^3$. In $\mathbb{R}^3$, there are many unit vectors. Use Lagrange method to find the unit vector which gives the minimum dot product.

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Solve the following linear program using KKT conditions.

$$\text{minimize} \quad v = 24y_1 + 60y_2$$

subject to

$$0.5y_1 + y_2 \geq 6$$
$$2y_1 + 2y_2 \geq 14$$
$$y_1 + 4y_2 \geq 13$$
$$y_1 \geq 0, y_2 \geq 0$$

Find the optimal solution for $[y_1^*, y_2^*]$ and the minimum value of v.