- 1. If an action A can occur in  $n_1$  different ways, another action, B can occur in  $n_2$  different ways, then the total number of occurrences of the actions A or B is  $n_1 + n_2$ .
- 2. If an action A can occur in n<sub>1</sub> different ways, another action, B can occur in n<sub>2</sub> different ways, then the total number of occurrences of the actions A and B together is n<sub>1</sub> \* n<sub>2</sub>.
- 3. Suppose that r actions are to be performed in a definite order. Further suppose that there are n1 possibilities for the first action and that corresponding to each of these possibilities are n2 possibilities for the second action, and so on. Then there are n1 \* n2 \* ... \* nr possibilities altogether for the r actions.
- 4. The product of the first n positive integers (counting numbers) is called n factorial and is denoted n!. In symbols, n! = n \* (n-1) \* (n-2) \* ... \* 1
- 5. n! = n \* (n 1)!
- 6. Permutation is used when order matters, and combination is used when order doesn't matter.
- 7. A permutation is an ordered arrangement of all or some of n objects.
- 8. The number of possible permutations of r objects from a collection of n distinct objects is given by the formula n \* (n 1) \* (n 2) \* ... \* 1 is denoted by nPr
- 9. nP0 = 1. There is only one ordered arrangement of 0 objects.
- 10. nC1 = n. There are n ways of choosing one object from n objects
- 11. nPn = n. There are n ways of arranging n objects from n objects, without replacement.
- 12. Number of possible permutations or r objects from a collection of n distinct objects when repetition is allowed is n \* n \* ... \* n (r times) =  $n^r$
- 13. The number of permutations of n objects when p1 of them are of one kind, p2 of another, p3 of a third and rest distinct is equal to n!/(p1!p2!p3!)
- 14. The number of ways n distinct objects can be arranged in a circle (clockwise and anticlockwise are same) is equal to (n 1)!/2
- 15. Each combination of r objects from n objects can give rise to r! arrangements. So, number is combinations is obtained by dividing the permutations by r!
- 16. The number of possible combinations of r objects from a collection of n distinct objects is denoted by nCr and is given by n!/(n-r)!r! = nPr/r!. This is also called binomial coefficient.
- 17. nCr = nC(n-r). In other words, selecting r objects from n objects is same as rejecting (n r) objects from n objects.
- 18. nCn = nC0 = 1 for all values of n
- 19. nCr = (n 1)C(r 1) + (n 1)Cr
- 20. With repetitions, the number of permutations is n^r. Without repetitions, it's nPr.
- 21. With repetitions, the number of combinations is (n+r-1)Cr. Without repetitions, it's nCr.
- 22. Choose and then arrange.
- 23. Choosing r objects out of n distinct objects and then arranging is same as directly arranging r out of n objects. Thus 5C3 \* 3P3 = 5P3
- 24. Here're some strategies.
  - a. Identify cases. handle separately, and then add.
  - b. add factors when dealing with OR (different cases), and multiply when dealing with AND.
  - c. Create Partitions, and then choose how to move around the partitions.
  - d. Fill what can occur together first, and then fill rest around them. For example, whoever wants to sit together, arrange them first.
  - e. If certain objects can't occur together, arrange the rest and make gaps while doing so, so as to fill them in. For example, in order to find how many ways can 26 alphabets be arranged, so that no vowels occur together arrange consonants (21!) first with gaps in between and then arrange vowels (22p5) in between them.

- f. nCr \* r! = nPr
- g. Number of ways of arranging n people into r groups = (n+r-1)C(r-1) \* n! (book and bookshelves problem, 5.3 .q4)
- h. Number of ways of arranging n identical people into r groups = (n+r-1)C(r-1) [ Football problem. week 5 challenge question 4].

(Hint: Number of ways in which n identical objects can be divided into r groups, where

each group can have any number of objects, is  $^{n+r-1}C_{r-1}$  or  $\frac{^{n+r-1}P_n}{n!}$  )

(Hint: Number of ways in which n distinct objects can be divided into r groups, where each group can have any number of objects, is  $^{n+r-1}P_n$ ) k.