Consider function $f(x,y) = x^2 y$, $\vec{v} = \cancel{\begin{bmatrix}1\\1\end{bmatrix}} \begin{bmatrix}\sqrt{2}/2\\ \sqrt{2}/2\end{bmatrix}$ and a vector v (normalized)

Directional derivative at a specified point, say (-1, -1) is given by the formula $\nabla_{\vec{v}} f(-1,-1) = \nabla f \cdot \vec{v}$

Gradient is computed using the formula $\nabla f = \begin{bmatrix}\partial f/\partial x\\ \partial f/\partial y\end{bmatrix}$. For the given function, the gradient at (-1, -1) is

$$= \begin{bmatrix}2xy\\ x^2\end{bmatrix} = \begin{bmatrix}2\\ 1\end{bmatrix}$$

Now, substituting in the second equation above, we get the directional derivative as $= \begin{bmatrix}2\\1\end{bmatrix} \cdot \begin{bmatrix}\sqrt{2}/2\\ \sqrt{2}/2\end{bmatrix} = \sqrt{2} + \frac{\sqrt{2}}{2}$

For more details, watch https://www.youtube.com/watch?v=4tdyIGIEtNU

Multi-variable chain rule for differentiation

$$\frac{d}{dt} f(x(t), y(t)) = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

Watch the details at https://www.youtube.com/watch?v=NO3AqAaAE6o