MATRICES EXERCISES

Exercise 1:

1. $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 6 & 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 1 \\ 3 & 0 & 0 \end{bmatrix}$

for matrix A,
→ Size : 3 X 3 (3 rows & 3 columns)

for matrix B,
→ Size : 3 X 4 (3 rows & 4 columns)

for matrix C,
→ Size : 2 X 4 (2 rows & 4 columns)

for matrix D,
→ Size : 3 X 3

for matrix E,
→ Size : 3 X 3

2. for matrix A, 3 X 3 : square matrix & non-symmetric ($\because A_{ij} \neq A_{ji}$ for all $i \neq j$)

for matrix B, 3 X 4 : rectangular matrix + sparse matrix ($\because$ many 0 entries) for matrix C, 2×4 : rectangular matrix & sparse matrix (∵ many 0 entries)

for matrix D, : zero matrix & square matrix

for matrix E, 3×3 : square matrix & non-symmetric matrix.

Exercise 2 :

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 8 & 3 \\ 3 & 8 & 7 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 8 & 3 \\ 1 & 8 & 3 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 8 & 4 & 1 \\ 3 & 3 & 3 & 3 \\ 4 & 8 & 7 & 6 \\ 5 & 1 & 3 & 9 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 1 \\ 3 & 0 & 0 \end{bmatrix}$$

1. $$A + B = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 5 & 8 & 3 \\ 3 & 8 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 5 \\ 7 & 9 & 7 \\ 5 & 9 & 11 \end{bmatrix}$$

2. $$A - C = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 1 & 8 & 3 \\ 1 & 8 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 \\ 1 & -7 & 1 \\ 1 & -7 & 1 \end{bmatrix}$$

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3. $2A + B - 3C$

$$2A = 2\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 2 & 1 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 6 & 8 \\ 4 & 2 & 8 \\ 4 & 2 & 8 \end{pmatrix}$$

$$3C = 3\begin{pmatrix} 1 & 2 & 1 \\ 1 & 8 & 3 \\ 1 & 8 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 3 \\ 3 & 24 & 9 \\ 3 & 24 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 6 & 8 \\ 4 & 2 & 8 \\ 4 & 2 & 8 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 1 \\ 5 & 8 & 3 \\ 3 & 8 & 7 \end{pmatrix} - \begin{pmatrix} 3 & 6 & 3 \\ 3 & 24 & 9 \\ 3 & 24 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 4-3 & 7-6 & 9-3 \\ 9-3 & 10-24 & 11-9 \\ 7-3 & 10-24 & 15-9 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 6 \\ 6 & -14 & 2 \\ 4 & -14 & 6 \end{pmatrix}$$

4. $A \cdot B$

$$\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 5 & 8 & 3 \\ 3 & 8 & 7 \end{pmatrix} = \begin{pmatrix} 29 & 57 & 38 \\ 21 & 42 & 33 \\ 21 & 42 & 33 \end{pmatrix}$$

5. $B \cdot C$

$$\begin{pmatrix} 2 & 1 & 1 \\ 5 & 8 & 3 \\ 3 & 8 & 7 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 8 & 3 \\ 1 & 8 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 20 & 8 \\ 16 & 98 & 38 \\ 18 & 126 & 48 \end{pmatrix}$$

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6. A . B . C

$$\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 4 \\ 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 5 & 8 & 3 \\ 3 & 8 & 7 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 8 & 3 \\ 1 & 8 & 3 \end{pmatrix} = \begin{pmatrix} 124 & 818 & 314 \\ 96 & 642 & 246 \\ 96 & 642 & 246 \end{pmatrix}$$

7. $|A|, |B| + |C|$

$$|A| = 1(4-4) - 3(8-84) + 4(0) = -12$$

$$|B| = 2(56-24) - 1(35-9) + 1(40-24)$$
$$= 2(32) - 26 + 1(16) = 64 - 26 + 16 = 54$$

8. $$|C| = 1(24-24) - 2(3-3) + 1(8-8) = 0$$

9. $$(A|I) = \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 2 & 1 & 4 & 0 & 0 & 1 \end{array}\right)$$

Step 1 → $R_2 \to R_2 - 2R_1$

$$\begin{pmatrix} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -4 & -2 & 1 & 0 \\ 2 & 1 & 4 & 0 & 0 & 1 \end{pmatrix}$$

Step 2 → $R_3 \to R_3 - 2R_1$

$$\begin{pmatrix} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -4 & -2 & 1 & 0 \\ 0 & -5 & -4 & -2 & 0 & 1 \end{pmatrix}$$

Step 3 : $R_3 - R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -4 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right)$$

Since third row is mostly zeroes, matrix A is singular (non-regular). Hence, A^{-1} does not exist.

for E^{-1},

$$E = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 2 & 1 \\ 3 & 0 & 0 \end{pmatrix}$$

$$(E|I) = \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R_3 \rightarrow R_3 - 3R_1$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & -9 & -12 & -3 & 0 & 1 \end{array}\right)$$

$R_2 \rightarrow R_2/2$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & -9 & -12 & -3 & 0 & 1 \end{array}\right)$$

$$R_3 \rightarrow R_3 + 9R_2$$

$$\begin{pmatrix} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & -7.5 & -3 & 4.5 & 1 \end{pmatrix}$$

$$R_3 \rightarrow \frac{R_3}{-7.5}$$

$$\begin{pmatrix} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 2/5 & -3/5 & \frac{-1}{7.5} \end{pmatrix}$$

$$R_2 \rightarrow R_2 - R_3/2$$

$$\begin{pmatrix} 1 & 3 & 0 & 3/5 & 12/5 & 4/5 \\ 0 & 1 & 0 & -1/5 & 4/5 & 1/15 \\ 0 & 0 & 1 & 2/5 & -3/5 & -1/7.5 \end{pmatrix}$$

$$R_1 \rightarrow R_1 - 3R_2$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1/3 \\ 0 & 1 & 0 & -1/5 & 4/5 & 1/15 \\ 0 & 0 & 1 & 2/5 & -3/5 & -1/7.5 \end{pmatrix}$$

$$\therefore E^{-1} = \begin{pmatrix} 1 & 0 & 1/3 \\ -1/5 & 4/5 & 1/15 \\ 2/5 & -3/5 & -1/7.5 \end{pmatrix}$$

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Exercise - 3 :

$$A = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 3 & 4 \\ 5 & 8 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 8 \\ 10 & 16 \\ 3 & 8 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 8 & 7 \end{pmatrix}$$

1. A+B : Here addition is not possible because the order is not compatible. (3×3 , 3×2)

2. A + C : Can't add due to different dimensions. (3×3 , 2×3)

3. A. B + C : multiplying A & B will give 3×2 matrix, which is not possible to add with C due to different dimensions (2×3).

4. B . A + C : product of B & A ~~will~~ is ~~give 3 X 3 matrix~~ not possible due to different orders (3×3 , 3×2)

5. ABC , BAC

We can multiply A & B but not with C because (2×3) dimensions don't match.

6. $|A| + |B|$

$$|A| = 2(3-32) - 1(1-32) + 4(8-12)$$
$$= 2(-29) - 1(-31) + 4(-4) = -43$$

$|B|$ = Since B is 3×2 matrix, we can't get determinant.

7. A^2, & C^2

$$A^2 = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 3 & 4 \\ 5 & 8 & 1 \end{pmatrix} = \begin{pmatrix} 24 & 35 & 14 \\ 23 & 41 & 19 \\ 21 & 33 & 41 \end{pmatrix}$$

C^2 is not a square matrix, so can't do C^2.

8. C^{-1}: C isn't a square matrix, so can't do C^{-1}.

9. rank(A), rank(B), rank(C)

rank (A): A is a full rank 3X3 matrix & the rank is 3.

rank (B): B is a 3X2 matrix

$$B = \begin{pmatrix} 5 & 8 \\ 10 & 16 \\ 3 & 8 \end{pmatrix}$$

$R_1 \rightarrow R_1/5$: $\begin{pmatrix} 1 & 8/5 \\ 10 & 16 \\ 3 & 8 \end{pmatrix}$

$R_2 \rightarrow R_2 - 10R_1$: $\begin{pmatrix} 1 & 8/5 \\ 0 & 0 \\ 3 & 8 \end{pmatrix}$

$R_3 \rightarrow R_3 - 3R_1$: $\begin{pmatrix} 1 & 8/5 \\ 0 & 0 \\ 0 & 16/5 \end{pmatrix}$

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Row echelon form of B : $\begin{pmatrix} 1 & 8/5 \\ 0 & 0 \\ 0 & 16/5 \end{pmatrix}$

$\therefore$ Rank (B) = 2

For matrix C : $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 8 & 7 \end{pmatrix}$

$R_2 \to R_2 - 3R_1$: $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 4 \end{pmatrix}$

$R_2 \to R_2/2$: $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

Row echelon form : $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

$\therefore$ Rank (C) = 2

10. A^T, B^T, C^T

$$A^T = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 3 & 8 \\ 4 & 4 & 1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 5 & 10 & 3 \\ 8 & 16 & 8 \end{pmatrix}$$

$$C^T = \begin{pmatrix} 1 & 3 \\ 2 & 8 \\ 1 & 7 \end{pmatrix}$$

11. A matrix is symmetric if $A = A^T$

→ A is not symmetric because $A \neq A^T$

→ B is not symmetric because $B \neq B^T$

→ C is not symmetric because $C \neq C^T$.