Report on

Ion Trapped Quantum Computing

PH441 : Quantum Computation and Cryptography

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Ref. : A. Steane, Invited Paper[1]

1 Introduction

Here we will discuss various issues of trapped ion quantum computing. We would like to consider the question, what might be the ideal system for a future quantum computer? We would focus on what system we might choose, if we are guided only by basic physical principles and the properties of systems which, in some useful sense, Nature provides. We would like our ideal quantum computer to have the highest quality memory, logic gates, and readout. For a quantum memory, we want a system of qubits which does not interact with anything else, while for logic gates we want a coupling between qubits which is fast, and not coupled to anything other than the bits. These two demands are almost, but not quite, contradictory. They imply that a quantum computer should be composed of entities q which are, in their passive state when no gates are switched on, almost completely isolated, and yet which can be coupled rapidly. This means they must have a strong coupling to something, G. The conflicting demands are met if G has the property that it can be made to be wholly absent when it is not wanted, and introduced rapidly when it is wanted. Furthermore we would like G to interact only with the entities q and with nothing else. It turns out that Nature does provide a physical entity q which meets the contradictory demands of memory and logic gates better than we might have imagined possible. This entity is the nuclear spin. The advantages of nuclear spins for quantum computing are already well recognised. A spin has a smaller coupling to its environment than any degree of freedom based on charge or the motion of particles; a nuclear spin has a particularly small magnetic moment, and this tiny magnet comes ready-packaged in an electron cloud with highly useful properties for logic gates. The atomic electrons provide the means to take hold of the atom and place it where we wish, and they also provide a ready-made very strong and very stable magnetic field on the nucleus. This results in the hyperfine splitting. The stability of this splitting for isolated atoms or ions is well documented, and is in fact used to provide our standards of time and frequency. The existence of hyperfine structure makes it possible to couple to the nuclear spin via the electronic state. This provides the handle whereby logic gates can be achieved. Ion trap experiments up till now have used a light-induced coupling between the electronic state and the vibrational motion (phonon) of relatively heavy charged particles (ions) in a trap in vacuum. There are proposals in which light alone is used to couple the electronic state of one atom and another, and a realisation of this concept (in an experiment not based on nuclear spin or hyperfine interaction) using a beam of neutral atoms.

2 Minimum requirements for quantum information processing

It can be shown that to produce arbitrary unitary transformations of the state of a set of qubits, which is what one wants for information processing, it is sufficient to be able to produce arbitrary rotations in Hilbert space of any individual qubit[2], i.e. the propagator

$$exp(-i\theta.\sigma/2) = \begin{pmatrix} cos(\theta/2) & -e^{-i\phi}sin(\theta/2) \\ e^{i\phi}sin(\theta/2) & cos(\theta/2) \end{pmatrix} \tag{1}$$

and to be able to carry out the 'controlled-rotation' operation $CROT = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|$ between any pair of qubits. The notation used here is standard, the kets $|0\rangle$ and $|1\rangle$ refer to two orthogonal states of a qubit. This basis is referred to as the 'computational basis', since this aids in designing useful algorithms for the QC. an operation like CROT is a propagator acting on the state of a pair of qubits. In matrix form it is written

$$U_{CROT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

in the basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, where matrix elements that are zero have not been written. The appellation 'controlled rotation' comes from the fact that if the first qubit is in the state $|0\rangle$, CROT has no effect, whereas if the first qubit is in the state $|1\rangle$, CROT rotates the state of the second by the Pauli σ_z operator.
The two operators just described form a universal set[2].

3 Ion trapped method

Here we will see into linear ion trapped systems, i.e, a line of N trapped ions. Each ion has two stable or metastable states, for example two hyperfine components of the electronic ground state (which usually requires an odd isotope), or two Zeeman sublevels of the ground state, separated by applying a magnetic field. The ground state and a metastable electronic exited state (e.g. a D state for ions of alkaline earth elements) might also be used, but this is a poor choice since the laser linewidth and frequency, as well as most of the mirrors etc. on the optical bench, will have to be very precisely controlled for such an approach to work. There have been optimistic estimates of the computational abilities of an ion trap processor, based on the use of such optical transitions, but one should beware of the lack of realism in such estimates.

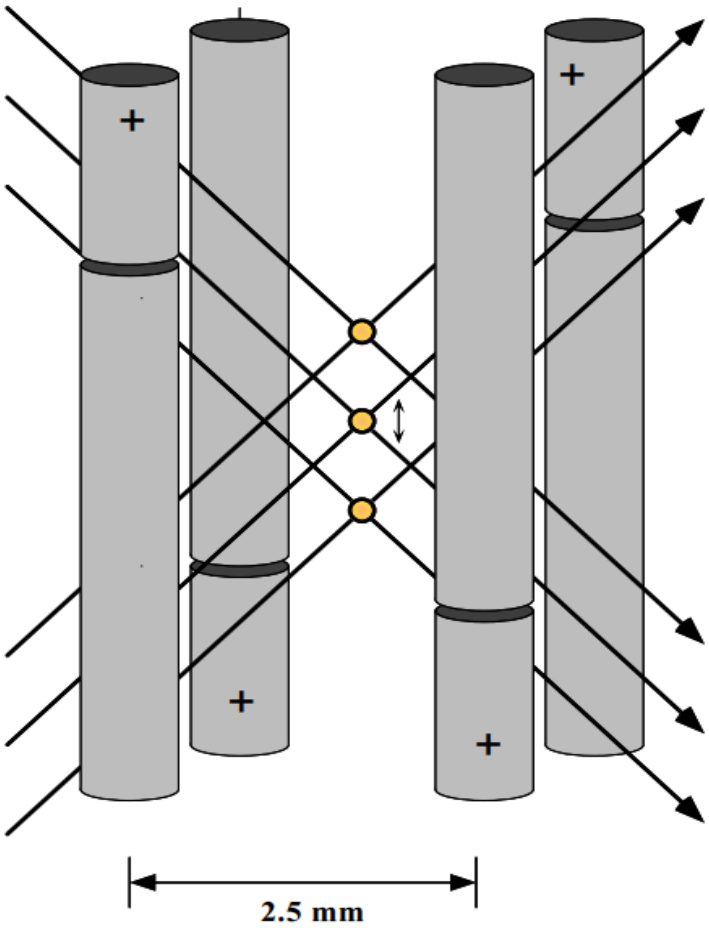

Fig 1: Experimental arrangement. A line of three ions sits between cylindrical electrodes, here seen sideways on. Pairs of laser beams excite Raman transitions, which impart momentum changes to the ions along the axial direction of the trap. The double-ended arrow indicates the direction of the resulting oscillations, it can be regarded as a pictorial representation of the fourth 'qubit' in the system. The electrodes are split in order to allow a constant voltage to be applied between their ends, so that an axial potential minimum occurs in the region where the long electrode segments overlap. Radial confinement is provided by alternating voltages

There are N laser beam pairs, each interacting with one of the ions, (or a single beam which can be directed at will to any chosen ion), see Fig 1. Each ion provides one qubit, the two dimensional Hilbert space being spanned by two of the ion's internal energy eigenstates. A further (N +1)th qubit acts as a 'bus' enabling the crucial CROT operations. This qubit is the vibrational motion of the whole ion string in the trap potential. This motion must be quantised, in other words the ion cloud temperature must be reduced well below the 'quantum limit' defined by the axial vibrational frequency in the ion trap:

$$k_B T << \hbar\omega_z \tag{3}$$

The first major experimental challenge (after making a trap and catching your ions) is to cool the ions down to this quantum regime. So far the quantum regime has been achieved only for a single ion of either mercury in two dimensions[3] or beryllium in three dimensions[4]. Both experiments used optical sideband cooling in the resolved-sideband (tight-trapping) limit.

To get to the quantum regime, it appears to be neccessary to use a Paul rather than Penning trap, since rf technology allows tighter confinement than does high-magnetic-fieldtechnology. Therefore only the Paul trap (rf trap) [Charged particles cannot be trapped in 3D by just electrostatic forces because of Earnshaw's theorem. Instead, an electric field oscillating at radio frequency (RF) is applied, forming a potential with the shape of a saddle spinning at the RF frequency] will be considered from now on, although we may permit ourselves to add a magnetic field if we wish, for some other reason such as to enhance the stability or split the Zeeman levels. In any case, tighter confinement enables a faster 'switching-time' for quantum gates such as CROT, so, as a general rule, tight traps are the best option.
Note that if several ions are in a three-dimensional rf trap of standard geometry (with the rf voltage between end caps and a ring), then matters are complicated since no more than one ion can be at the centre of the trap potential. Away from the centre, ions undergo rf micromotion and this causes heating if there is more than one ion, owing to collisions (Coulomb repulsion) which force the micromotion out of quadrature with the rf field. To avoid this, one must use a linear or ring geometry. The confinement along the axis is then either due to a static field from end cap electrodes (linear case), or to repulsion between ions combined with their confinement to a ring shape. In this case, only radial micromotion is present, but this vanishes for all the ions if they lie along the axis at the centre of the radial potential, so rf heating is avoided.

3.1 Average motion

We can model a row of N ions in a trap as a system of N point charges in a harmonic potential well of tight radial confinement, i.e. $\omega_x, \omega_y >> \omega_z$ [Fig 2]; The total Hamiltonian is

$$H = \sum_{n=1}^{N} \frac{1}{2} M(\omega_x^2 \hat{X_i^2} + \omega_y^2 \hat{Y_i^2} + \omega_z^2 \hat{Z_i^2} + \frac{\hat{P_i^2}}{2}) + \sum_{i=1}^{N} \sum_{j>i} \frac{e^2}{4\pi\varepsilon_0 |\hat{R}_i - \hat{R}_j|} \tag{4}$$

For $\omega_x, \omega_y >> \omega_z$ and at low temperatures, the ions all lie along the z axis, so we can take $|\hat{R}_i - \hat{R}_j| \simeq |\hat{Z}_i - \hat{Z}_j|$, and the radial and axial motion can be separated. The axial motion interests us, so the problem is one-dimensional. A length scale is given by

$$z_s = (\frac{e^2}{4\pi\varepsilon_0 M \omega_z^2})^{\frac{1}{3}} \tag{5}$$

which is of the order of the separation between the ions (typically 10 to 100 μm).

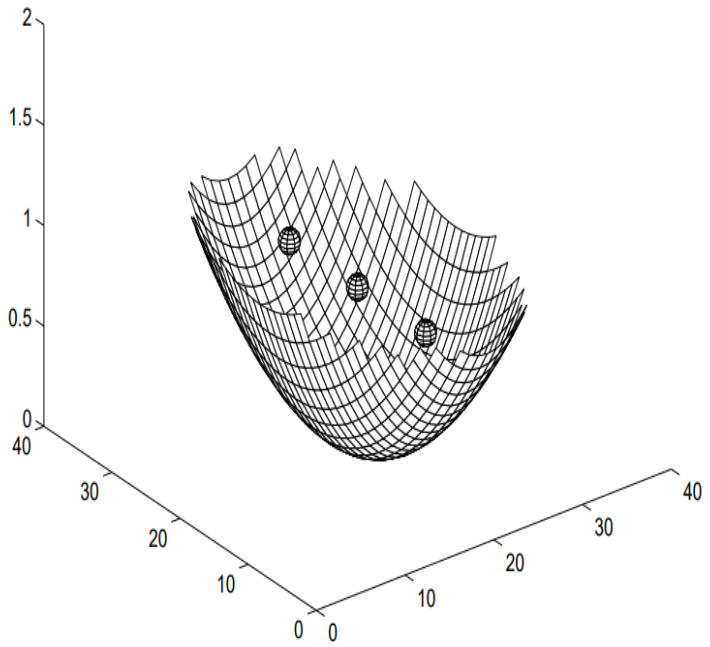

Fig 2: Schematic illustration showing an anisotropic harmonic potential with the positions of three trapped ions indicated

Solving the classical equations of motion (i.e. the operators $\hat{Z}$, $\hat{P}_z$ become classical variables z, p_z), one obtains the equilibrium positions shown in Fig. 3.

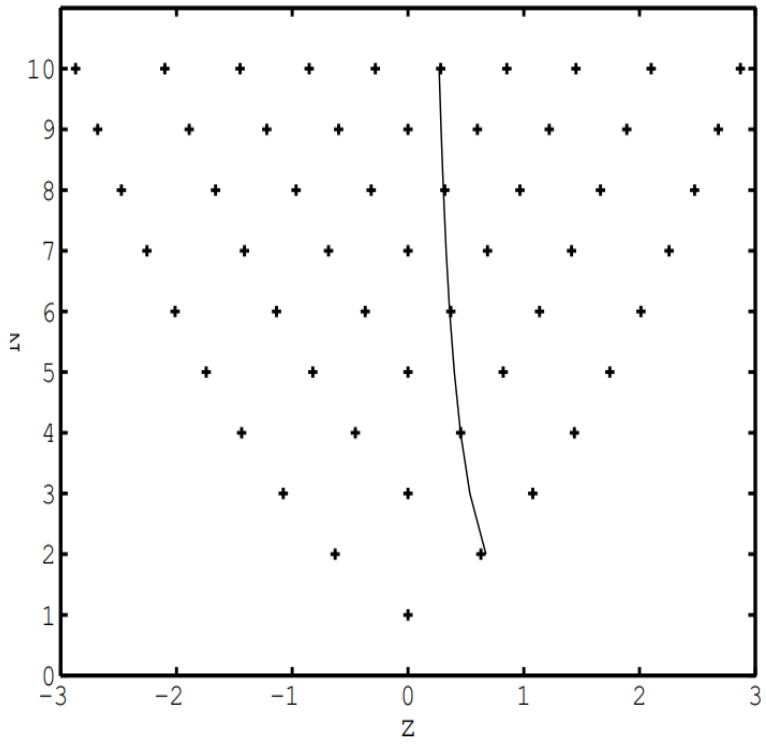

Fig 3: Equilibrium positions for a line of point charges in a quadratic potential, as a function of the number of charges N. The positions are in units of z_0 defined by (5).

With more than two trapped ions, the outer ions tend to push the inner ones closer together, so the ion positions depend on N. Remarkably, however, the frequencies of the first two normal modes of oscillation about these equilibrium positions are independent of N (for small oscillations)[5], and those of higher modes are nearly independent of N. The frequencies of the first two modes are ω_z and $\sqrt{3}\omega_z$, and those of higher modes are given approximately by the list $\{1, \sqrt{3}, \sqrt{29/5}, 3.051, 3.671, 4.272, 4.864, 5.443, 6.013, 6.576\}$, which gives the frequency of the highest mode, in units of ω_z, for N = 1 to 10. The near independence of N of the mode frequencies is illustrated by Fig 4.

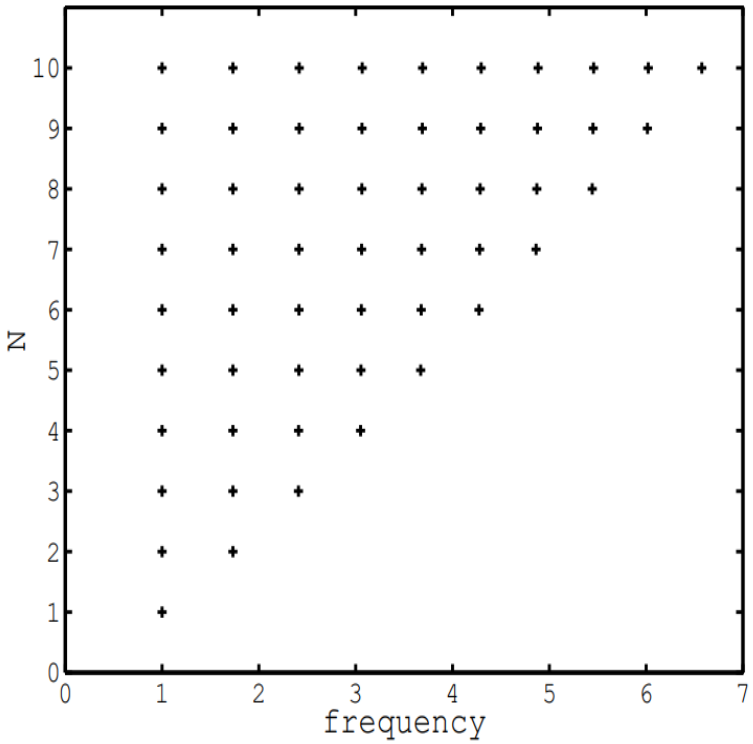

Fig 4: Normal mode frequencies for a line of point charges in a quadratic potential, as a function of the number of charges N. The frequencies are in units of ω_z).

The lowest mode of oscillation corresponds to harmonic motion of the centre of mass of the ion string. In this mode, all the ions move to and fro together. It is important that the frequency of this mode is significantly different from that of any other mode, since this means that experimentally one can excite the centre-of-mass mode without exciting any of the others. We can now proceed directly to a quantum mechanical treatment, simply by treating the centre-of-mass coordinate z_{cm}as a harmonic oscillator. The classical result that the centre-of-mass normal mode has frequency ω_z remains valid even though the ion wavefunctions may now overlap, since all the internal interactions among the ions cancel when one calculates the centre-of-mass motion. Since we have an oscillator of mass NM and frequency ω_z, the energy eigenfunctions are

$$\Psi_n(z_{cm}) = (\frac{NM\omega_z}{\pi\hbar 2^{2n}(n!)^2})^{\frac{1}{4}} H_n(z_{cm}\sqrt{NM\omega_z/\hbar}) e^{-NM\omega_z z_{cm}^2/2\hbar} \quad (6)$$

The spatial extent of the Gaussian ground state probability distribution is indicated by its standard deviation

$$\Delta z_{cm} = \sqrt{\hbar/2NM\omega_z} \quad (7)$$

Since we want a different laser beam to be able to address each of the ions, we require Δz_{cm} to be small compared to the separation between ions. The closest ions are those at the centre of the line. A numerical solution of (4) yields the following formula for the separation of the central ions:

$$\Delta z_{min} \simeq 2.0 z_s N^{-0.57} \tag{8}$$

This formula is plotted for N $\leq$ 10 in Fig. 3. An approximate analytical treatment for N $\gg$ 1 does not predict a power-law dependence of Δz_{min} on N, but rather $\Delta z_{min} \propto z_s(log(N)/N^2)^{1/3}$[6]. However, (8) is more accurate for N $<$ 10 and remains accurate for the range of N that interests us (up to, say, N = 1000). Setting $\Delta z_{cm} \ll \Delta z_{min}$ yields

$$\frac{\omega_z}{M} \ll \frac{32N^{1.86}}{\hbar^3}(\frac{e^2}{4\pi\varepsilon_0})^2 \simeq 2.4 \times 10^{21} N^{1.86} Hz/u, \tag{9}$$

where u is the atomic mass unit 1.66057×10^{-27} kg. This condition is easily fulfilled in practice, with ω_z no greater than a GHz, and M between 9 and 200 u. Therefore it is legitimate to picture the ions as strung out in a line, each sitting in a small wavepacket centred at its classical equilibrium position, not overlapping the others. Note that (9) does not guarantee that the ions are sufficiently separated to be addressed by different laser beams, only that their wavefunctions do not overlap.
In the above, it was assumed that the radial confinement was sufficient to cause the ions to lie along the z axis, rather than form a zigzag or helix about it.

3.2 Principle of operation

The principle of operation of an ion trap 'information processor' was described by Cirac and Zoller[7], and the most important elements of such a system were first realised in the laboratory by Monroe et al.[8].
The basic form of the operators is independent of the type of excitation used, however. The method may be understood by reference to Fig. 5, which shows the relevant energy levels for one of the ions in the trap.
We consider three of the ion's internal energy eigenstates$|F_1, M_1\rangle, |F_2, M_2\rangle and |F_{aux}, M_{aux}\rangle$, and various excitations of the centre-of-mass motion. The ion's internal energy levels are separated in frequency by ω_0 and ω_{aux} as indicated on Fig. 5.
Note that all these levels are low-lying, separated from the ground state only by hyperfine and Zeeman interactions, so their natural lifetime against spontaneous emission of rf photons is essentially infinite. Figure 5 shows the lowest-lying excitations of the second, third, and fourth normal modes as well as the first, to act as a reminder of the location of the closest extraneous levels whose excitation we wish to avoid. The energy eigenstates of the vibrational motion may be written as $|n1, n2, n3, ...\rangle$, where the n_i are the excitations of the various normal modes. Onlythe ground state $|0, 0, 0, ...\rangle$ and first excited state of the centre of mass $|1, 0, 0, ...\rangle$ will be involved in the operations we wish to invoke. This centre-of-mass vibrational degree of freedom is often referred to somewhat loosely as a 'phonon'. The 'computational basis' consists of the states

$$\begin{aligned} |0,0\rangle &\equiv |F_1, M_1\rangle \otimes |0, 0, 0, ...\rangle \\ |0,1\rangle &\equiv |F_1, M_1\rangle \otimes |1, 0, 0, ...\rangle \\ |1,0\rangle &\equiv |F_2, M_2\rangle \otimes |0, 0, 0, ...\rangle \\ |1,1\rangle &\equiv |F_2, M_2\rangle \otimes |1, 0, 0, ...\rangle \end{aligned} \tag{10}$$

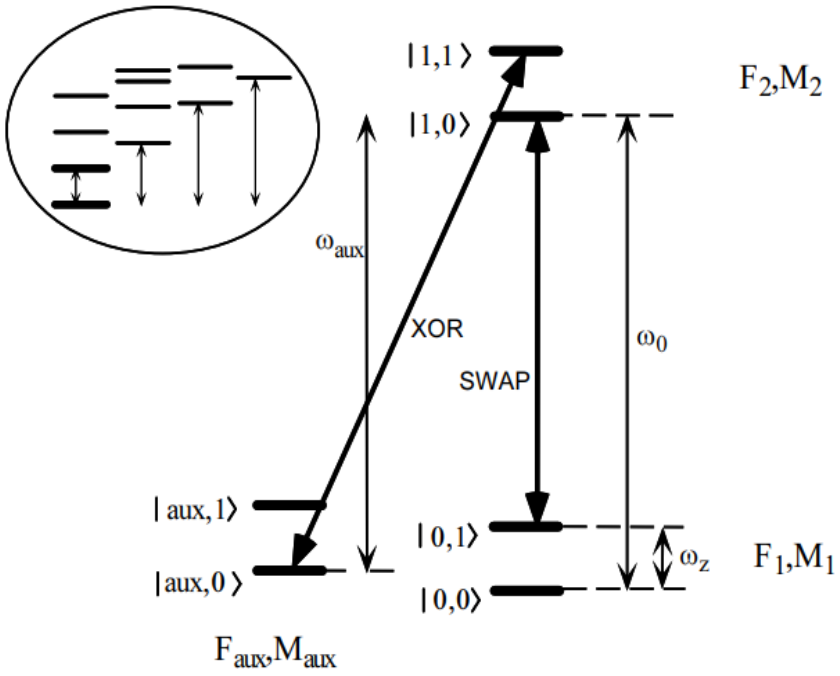

Fig. 5. Energy levels and transitions in a single ion significant for information processing with a line of trapped ions. The labels F, M indicate different internal states of the ion. Each internal state has an associated set of vibrational levels for each of the vibrational modes. Here, just the ground and first levels of the lowest mode (spacing ω_z), are shown in the main diagram, and the insert shows the further low-lying vibrational levels whose excitation we wish to avoid. The full arrows indicate transitions at frequencies $\omega_0 - \omega_z$ and $\omega_{\text{aux}} + \omega_z$, which are used in the $\text{SWAP}_{(-\text{i})}$ and CROT operations described in the text. Note that radiation at a given frequency couples not only the levels at the two ends of the relevant arrow on the diagram, but also other pairs of levels with the same difference of vibrational quantum number. The figure shows ω_{aux} to be of the same order as ω_0, because this is what typically occurs when alkali-like ions are used. The vibrational frequency is smaller, $\omega_0 \simeq 1000\omega_z$

We will now shown (briefly) how to carry out CROT between any single ion's internal state and the bus (phonon) bit, then how to carry out arbitrary rotations of the internal state of an ion, then how to carry out SWAP between any ion and the bus bit. From the discussion in Sect. 2, these three operations form a universal set and so allow arbitrary transformations of the stored qubits in the processor.
The auxillary states $|aux, i\rangle \equiv |F_{aux}, M_{aux}\rangle \otimes |i, 0, 0, ...\rangle$ (i = 0,1) are available as a kind of 'shelf' by means of which useful state-selective transformations can be carried out among the computational basis states.
If one applies radiation at the frequency $\omega_{aux} + \omega_z$, then inspection of Fig. 5 will reveal that only transitions between $|1, 1\rangle$ and $|aux, 0\rangle$ will take place (if we assume that unwanted levels such as $|1, 2\rangle$ are unoccupied). [Cirac and Zoller originally proposed to produce the selective effect of this CROT operation by means of a chosen laser polarisation rather than frequency. However, frequencies can be experimentally discriminated more precisely than polarisations, which explains why Monroe et al. chose to use a frequencyselective rather than polarisation-selective method]
If one applies a 2π pulse at this frequency, then the state $|1, 1\rangle$ is rotated through 2π radians, and therefore simply changes sign. In the computational basis, the effect is equal to that of the CROT operator.

A 2π pulse at frequency$\omega_0 - \omega_{aux} - \omega_z$ also produces a controlled rotation, only now the minus sign appears on the second element down the diagonal of the unitary matrix, rather than on the fourth, causing a sign change of the component $|0,\rangle$ rather than of $|1,1\rangle$. This case will be called C¬ROT, the negation symbol ¬ referring to the fact that here the second qubitis rotated if the first is in the state $|0\rangle$ rather than $|1\rangle$.

To rotate an ion's internal state without affecting the centre-of-mass motion, one applies radiation of frequency ω_0. If such radiation has phase ϕ with respect to some defined origin of phase, and duration sufficient to make a pπ pulse, then the effect in the computational basis is

$$V^p(\phi) = \begin{pmatrix} cos(p\pi/2) & -e^{-i\phi}sin(p\pi/2) \\ e^{i\phi}sin(p\pi/2) & cos(p\pi/2) \end{pmatrix}_{ion} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{cm} \tag{11}$$

where we have followed the notation of [7], but used p instead of k to avoid confusion with the wave vector. Note that to apply such rotations succesfully, it is necessary to have the phase of the radiation under experimental control. That means under control at the position of the ion, not just in some stable reference cavity. This constitutes a severe experimental constraint, which makes computational basis states separated by radio frequencies highly advantageous compared to states separated by optical frequencies. On the other hand, in order to have the right phase experimentally, note that one need not worry about the continuous precession at frequency ω_0 caused by the internal Hamiltanian of each ion. The laser field keeps step with this precession, as becomes obvious when one uses the interaction picture, which we have done implicitly in writing (11). A possible problem arises when different ions have different internal energies, owing to residual electric and magnetic fields in the apparatus. However, this particular problem has a fairly simple experimental solution: since each ion is addressed by a different laser beam pair, one can independently tune the laser beams driving each ion, by means of acoustooptic modulators in the beam paths. Such modulators will be required in any case to allow the laser beam intensities to be switched. So far we have described operations on the ion trap by means of pπ pulses. A complimentary technique is that of adiabatic passage, in which a quantum system is guided from one state to another by a strongly perturbing Hamiltonian applied slowly.

The details are described for a related system in [9].
This method has experimental advantages in being insensitive to features such as the timing and interaction strength. However, it can be more sensitive to off-resonant coupling to other levels, compared with the pulse technique, and this consideration led Monroe et al. to favour rf pulses. Both pπ pulses and adiabatic passage will probably have their uses in a practical QC (whether based on an ion trap or some other system).

The laser pulses described provide the universal set of 'quantum logic gates' for the linear ion trap. To complete the operation of our processor or QC, we require that the final state of the quantum information processor can be measured with high accuracy. This is possible for trapped ions by means of the 'electron-shelving' or 'quantum-jumps' technique [10, 11].

That is, one may measure whether a given ion is in state $|F_1, M_1\rangle$ or $|F_2, M_2\rangle$ by illuminating it with radiation resonant with a transition from $|F_1, M_1\rangle$ to some high-lying level, whose linewidth is small enough so that transitions from $|F_2, M_2\rangle$ are not excited. If fluorescence is produced (which may be detected with high efficiency), the

ion state has collapsed to $|F_1, M_1\rangle$, if none is produced, the ion state has collapsed to $|F_2, M_2\rangle$. This method requires that spontaneous decay from the high-lying level to $|F_2, M_2\rangle$ is forbidden, which is possible in this case through the angular momentum selection rules for electric dipole radiation. Note that typically a quantum process may need to be prepared, run, and measured several times in order to gather more information about the processor's final state than is available from a single measurement of all the qubits. This is related to the ideas of quantum state tomography, which have recently been demonstrated in an ion trap experiment.

4 Cooling

To make the quantum information processor, the main initial requirements of an experimental system are cooling to the quantum regime (3). The ions must be separated by at least several times the laser wavelength [see (8)], but this is automatically the case, for small numbers of trapped ions, since with current technology the ions are always separated by many times the width of their vibrational ground state wavefunction [inequality (9)], which is itself approximately equal to the laser wavelength.

For cooling to the quantum regime, there are two possible approaches. Either one may cool to the ground state of a tight trap having $\eta \ll 1$, then adiabatically open the trap to $\eta \sim 1$, or one may apply cooling to a trap already at $\eta \sim 1$. The advantage of the former approach is that one does not require cooling below the recoil limit $k_B T_R = E_R$. The advantage of the latter is that strong confinement is not necessary, but attaining the quantum regime with $\eta \sim 1$ requires sub-recoil cooling. Cooling to the quantum regime has so far been demonstrated for trapped ions by means of sideband cooling in the resolved-sideband limit. The physics of sideband cooling is very closely related to that involved in information processing in the ion trap. This is no coincidence, and a similar link will probably be found in all physical implementations of quantum information processing. The relationship is sufficiently close that one may say that once the goal of laser cooling to the motional ground state is achieved in any given experimental ion trap, a primitive form of quantum information processing can proceed immediately, since all the required experimental components will be in place.

4.1 Sideband cooling

Sideband cooling is the name for the simplest type of laser cooling of a confined ion. The name comes from how the photon-scattering process looks in the resolved-sideband limit. In the case of free ions, the corresponding process is radiation pressure or Doppler cooling. There are several significant frequencies or energies. First, we have the vibrational frequency in the ion trap potential, ω_z. Next, we have the radiative width of the transition used to do the cooling, Γ . Either a single photon transition is used, in which case Γ is its natural width (or possibly its broadened width if another laser is used to broaden a very narrow level), or a stimulated Raman transition is used, in which case Γ is some combination of the inverse of the duration of the Raman pulses, and the time for optical pumping out of one of the states linked by the Raman transition. The physics in the Raman case and single-photon case is very similar. The Raman method is a way of providing a very narrow transition when one is not already available. It also combines the advantages of precise frequency

control (in the rf regime) with large photon recoil (optical regime), which permits fast cooling, for the same reason that the switching rate for information processing is faster. One could instead use an rf or microwave transition, but then the cooling would be a lot slower and may not compete well enough with heating processes.

Laser cooling of atoms is often done quite happily by using strong, resonant transitions. Indeed, such transitions are eagerly sought out. Why the talk of narrow transitions in the previous paragraph? It is because simple Doppler cooling leads to the well-known Doppler cooling limit $k_B T_D \simeq \hbar\Gamma/2$, when the recoil energy is small compared to $\hbar\Gamma$ (this applies in a trap as well as to free atoms). However, we want to get to the quantum limit (3), so we require

$$\omega_z \gg \Gamma \tag{12}$$

This equation is a further constraint on the performance of the trap. It says the cooling transition must be narrow enough, or the trap confinement tight enough, to resolve the motional sidebands. In the resolved sideband limit, radiation pressure cooling is called sideband cooling. A nice way of understanding it is to consider it as a form of optical pumping towards the state of lowest vibrational quantum number; see Fig. 6.

Note that the recoil after spontaneous emission produces heating.

The measurement probes the high-energy part of the distribution of the ion's population amongst the quantum states of the trap. The observation of no fluorescence, i.e. a null detection, implies that the ion's state has collapsed onto the relatively lower energy part of its initial vibrational distribution. Further such measurements provide opportunities for further cooling. On anygiven application of this method, the measurement may heat or cool the ion, but cooling is more likely and one knows when it has occurred. This idea is reminiscent of forced evaporative cooling, only it is applied to the probability distribution of a single confined particle rather than an ensemble.

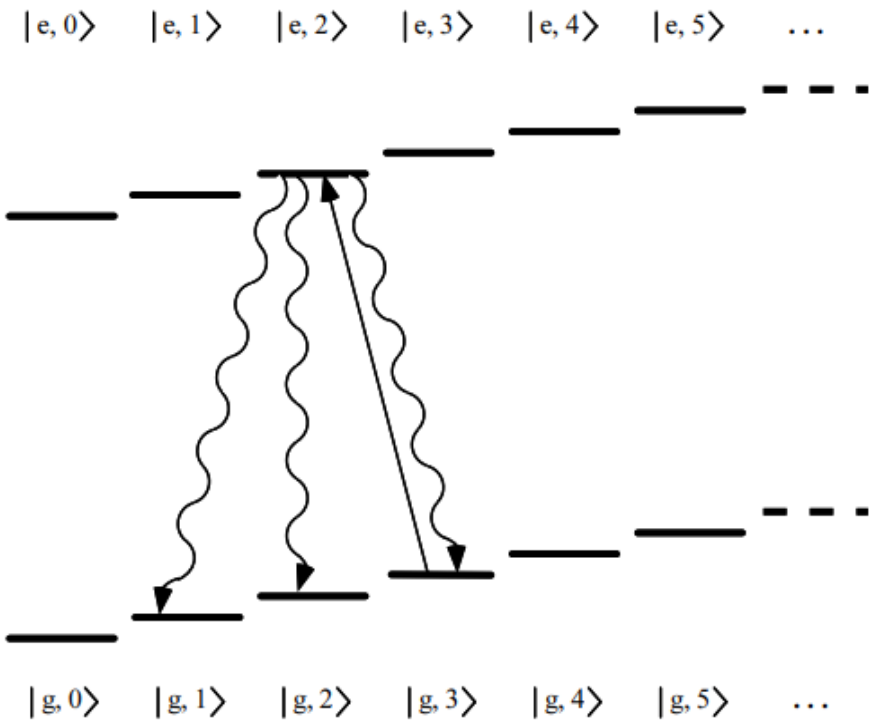

Fig. 6. Sideband cooling. A laser excites transitions in which the vibrational quantum number of a confined ion falls by 1 (or a higher integer). Spontaneous transitions bring the ion's internal state back to the ground state, with the vibrational quantum number changing by ± 1 or 0. On average the vibrational quantum number is reduced, until the vibrational ground state is reached. The internal ground and excited states are $|g\rangle$ and $|e\rangle$, and the figure shows the different vibrational levels spread out horizontally for clarity. When both $|g\rangle$ and $|e\rangle$ are long lived (for example they may be the computational basis states: cf Figures 5 and 8), the $|g,n\rangle \rightarrow |e,n-1\rangle$ transition is driven by a π pulse, ($U^1(0)$ operator), and the spontaneous transition is a Raman transition via an unstable excited state (optical pumping). Note that such experimental techniques are identical to those required for information processing and error correction. To cool a crystal of several ions, it is sufficient for the laser to interact with only one ion since the Coulomb coupling between ions causes rapid thermalisation of their motional state. However, the coupling between different normal modes is weaker, so these may need to be cooled separately by tuning the laser to the various normal mode sideband frequencies

5 Candidate ion

Table 1 gives a list of ions suitable for information processing. The list consists of ions whose electronic structure is sufficiently simple to allow laser cooling without the need for too many different laser frequencies. The list is not intended to be exhaustive, but contains most ions that have been laser cooled in the laboratory.

Table 1. List of candidate ions for information processing. Only singly-charged ions are considered, although some ions of higher charge may also be interesting. For each element, only the most abundant isotope, and those having non-zero nuclear spin are shown. Unstable isotopes are not shown, although most elements in the list (all but Mg and Li) have further isotopes of half-life longer than one week. Thalium and indium are omitted since their ground states have $J = 0$ and so lack hyperfine structure. The hyperfine splittings are for the ground state in all but helium-like lithium; they are taken from G. Werth in [30], and from [76]. The S–D wavelength is only shown when the D level lies below the P level. The recoil energy is based on the S–P wavelength. For Li the S,P,D labels do not apply; the transitions are from the metastable triplet state. Note that the fine structure splitting in the excited state (not shown) is also relevant to the Raman transition rate (see text)

Element, isotopes	Natural abundance (%)	Nuclear spin ($\hbar$)	Hyperfine splitting (GHz)	λ S–P (nm)	λ S–D (nm)	Recoil energy (kHz)
Be 9	100	3/2	1.25001767	313		226
Mg 24	79	0		280		106
25	10	5/2	1.7887631			
Ca 40	97	0		397	730	30
43	0.14	7/2	3.25560829			
87	7	9/2	5.00236835			
Sr 88	83	0		422	674	12.7
135	6.6	3/2	7.18334024			
137	11	3/2	8.03774167			
Ba 138	72	0		493	1760	5.94
199	17	1/2	40.507348			
201	13	3/2	30.16			
Hg 202	30	0		194	282	26.6
171	14	1/2	12.6428121			
173	16	5/2	10.4917202			
Yb 174	32	0		369	411	8.42
6	7.5	1	3.0018			
Li* 7	92.5	3/2	11.8900	539		94.7

Since we require at least three long-lived low lying states of the ion (the states $|0\rangle$, $|1\rangle$ and $|aux\rangle$), this implies that the existence of hyperfine structure (i.e. a nonzero nuclear spin isotope), although it complicates the cooling process, may be advantageous. Indeed, for alkali-like ions (such as singly charged ions from group 2 of the periodic table), the electronic ground state is J = 1/2, so if the nuclear spin is zero there are only two long-lived internal states (the Zeeman components$|J, M\rangle = |1/2, 1/2\rangle$), which is not sufficient. Having said this, we are not necessarily forced to choose $|aux\rangle$ to be a third internal state of the ion (i.e. the choice implied by Fig. 5). One could make use of the second normal mode of oscillation of the ion string instead, choosing $|aux, 0\rangle = |F_1, M_1\rangle \otimes |0, 1, 0, ...\rangle$ [cf equation (10)].
The other major consideration is the difficulty in generating the light required for cooling and information processing.

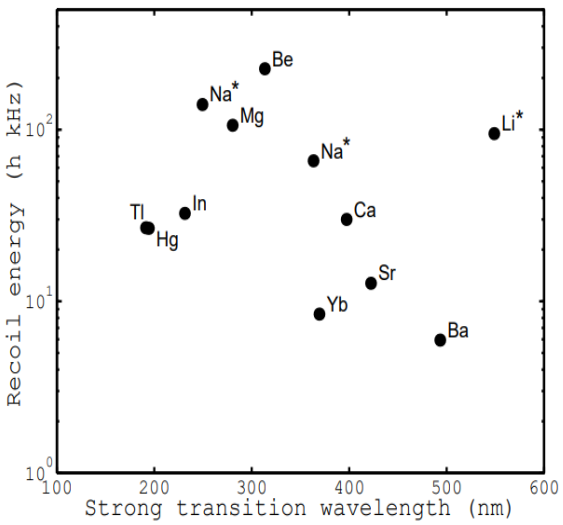

Fig. 7. The recoil energies and main (typically S–P) transition wavelengths for ions that may be amenable to quantum information processing (cf Table 1). A high recoil energy is advantageous for a high switching rate, but tends to be associated with a short wavelength. A rough rule is that the shorter the wavelength, the more complicated and therefore less stable is the laser system. The starred symbols are singly ionised ions from group 1, in which a metastable manifold is used for computing, making them unattractive in the long term

Examining Fig. 7 and Table 1, we see that $_9Be$ is an attractive choice, in that it allows the fastest switching rate and it requires only one laser wavelength for cooling, and the hyperfine splitting frequency of 1.25 GHz is accesible to electro-optic modulators. However, the wavelength of 313 nm requires the use of a dye laser (frequency doubled) which is disadvantageous. The next most promising candidate appears to be ^{43}Ca. It requires two laser wavelengths for cooling, 397 (or 393) nm and 866 (or 850) nm (Fig. 8), but both can be produced by diode lasers (one frequency doubled), which makes this ion very attractive (strontium has similar advantages). Diode lasers can be made very stable in both frequency and power. If more laser power is needed than is possible with diode lasers, then a titanium–sapphire laser can be used, which is also advantageous compared with dye lasers. The hyperfine splitting of 3.26 GHz is accessible to electro-optic modulators, though less easily than the smaller splitting in beryllium. The obvious difficulty in working with ^{43}Ca is that it is a rare isotope, having a natural abundance of only 0.14% or 1 part in 700, making an isotopically enriched sample that much more expensive. However, one could carry out preliminary experiments using the 97% abundant ^{40}Ca.

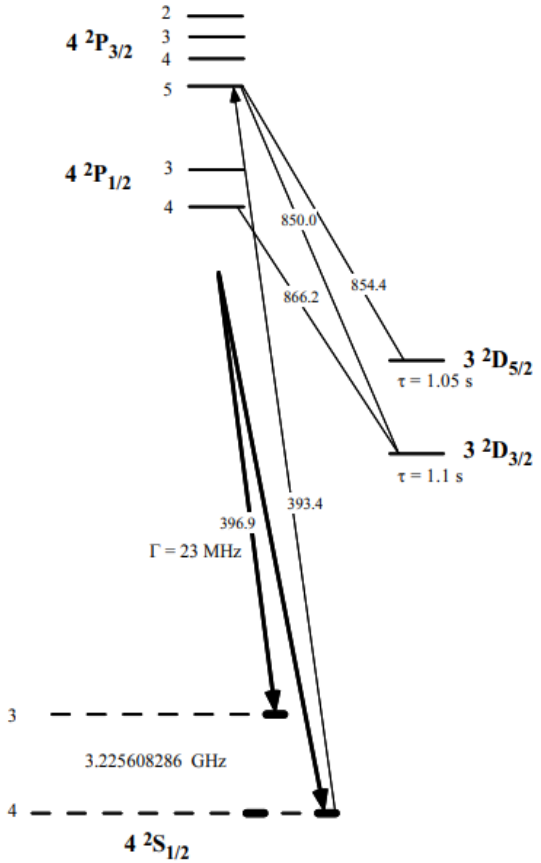

Fig. 8. Low-lying energy levels of the $^{43}Ca^+$ ion, of electronic structure $1s^22s^22p^63s^23p^6nl$. The hyperfine structure of the D states is omitted to keep the diagram uncluttered. The other levels are labelled by the total angular momentum quantum number F (nuclear spin $I = 7/2$). The Zeeman sublevels are shown for the ground state hyperfine manifold, and a possible choice of computational and auxilliary levels is shown by the thickened sublevels. An example Raman transition is shown, for use both in sideband cooling and for the operation of quantum gates. The $3D$ levels are shown because they are involved in the state preparation (including cooling) and measurement, both of which are an integral part of a complete quantum 'computation'. The level separations and lifetimes are taken from references [62]

The conclusion is that for the moment the limitations of the ion trap are associated

with the vibrational degrees of freedom, and with experimental instabilities. It is here that experimental and theoretical work must concentrate if progress is to be made. It remains misleading at present to talk of quantum 'computations' taking place in the lab.

6 Error correction

Although it is important to build an information processor with as much precision and stability as possible, in the longer term the aim of significant computations is almost certainly unrealisable without something that goes beyond such 'passive' stabilisation.

It was initially thought that anything like active stabilisation of a quantum computer would be impossible, since it would rely on a means of monitoring the quantum state of the computer, which would irreversibly destroy the computation. However, the union of information theory with quantum mechanics has lead to another powerful concept, that of quantum error correction [12, 13] . The essential idea is that K qubits of quantum information in the quantum computer can be stored ('encoded') in a carefully chosen way among N > K two-state systems. The simplest case is to store a single qubit in three two-state systems, as follows:

$$\begin{aligned} |0_L\rangle &\equiv (|000\rangle + |011\rangle + |101\rangle + |110\rangle)/2 \\ |1_L\rangle &\equiv (|111\rangle + |100\rangle + |010\rangle + |001\rangle)/2 \end{aligned} \tag{13}$$

All we have done here is written down two orthogonal states, calling them $|0_L\rangle$ and $|1_L\rangle$. With two states we have a single logical qubit, K = 1, but it is stored physically in N = 3 separate ions. A general state of the logical qubit is just a$|0_L\rangle$+b$|1_L\rangle$. The logical qubit inhabits a twodimensional subspace of the total Hilbert space of eight dimensions. The computation is carried out in the specially chosen 2^K -dimensional subspace of the total Hilbert space (2^N dimensions) of the enlarged computer. For example, to apply the logical state rotation $|1_L\rangle\langle 0_L|$-$|0_L\rangle\langle 1_L|$, we must apply
$V^1(-\pi/2) = |1\rangle\langle 0| - |0\rangle\langle 1|$ seperately to each ion.
The encoding is chosen so that the most likely errors cause the computer's state to go out of the special subspace (whereas computing operations keep the state within the special subspace). One can detect such departures, and force the computation back on track, without corrupting the stored quantum information, by making well-chosen joint measurements on the three qubits. The exact corrective procedure is deduced from the error correction methods which are a central part of classical information theory. This is a rather subtle and beautiful link between classical information theory and quantum mechanics.
The above 'simplestcase', (13), is fully analysed there. [14]

7 About IonQ

IonQ is based on trapped ion quantum computing.
They consider individual atoms as a naturally occuring quantum system. These atoms are the heart of IonQ quantum processing units. IonQ trap them in 3D space, and

then use lasers to do everything from initial preparation to final readout. It requires counter intuitive physics, precision optical and mechanical engineering, and fine-grained firmware control over a variety of components.

7.1 The Atom: Nature's Qubit

The most important part of any quantum computer are its quantum bits, or qubits. IonQ's qubits are ionized ytterbium atoms, a silvery rare-earth metal. Each ytterbium atom is perfectly identical to every other ytterbium atom in the universe.

Moreover, once prepared in a particular stable quantum state, they can remain in that state for very long periods of time — they're so consistent they're used in one of the most accurate atomic clocks ever built. The only difference between a neutral ytterbium atom (Yb) and an ytterbium ion (Yb+) is one electron, which we remove with lasers as a part of the trapping process. This process, called ionization leaves the atom with a positive electrical charge and only one valence electron.

7.2 Trapping An Ion

Once they've turned their atom into an ion, they use a specialized chip called a linear ion trap to hold it precisely in 3D space. This small trap features around 100 tiny electrodes precisely designed, lithographed, and controlled to produce electromagnetic forces that hold the ions in place, isolated from the environment to minimize environmental noise and decoherence.

It's impossible to create a trapping force with electrodes at fixed voltages that can hold an ion in a fixed position. Instead, they use rapidly oscillating voltages, such that the average field traps the ions in all three dimensions. As an analogy, imagine placing a ball on top of a saddle, and then spinning the saddle very quickly — it's the same basic idea.

7.3 One Qubit, Or One Hundred

IonQ don't stop at one ion; that wouldn't make a very useful quantum processor. IonQ can load any number of ions into a linear chain.

This on-demand reconfigurability allows it to theoretically create anything from a one-qubit system to a 100+ qubit system without having to fabricate a new chip or change the underlying hardware. To date, IonQ run single-qubit gates on a 79 ion chain, and complex algorithms on chains of up to 11 ions.

7.4 Computing With Atoms

They compute using a series of operations called gates to manipulate the qubits' state, first encoding and then operating on the information we want to calculate.

To perform these gates, they use an array of individual laser beams, each imaged onto an individual ion, plus one "global" beam. The interference between the two beams produces a beat note that is at exactly the necessary energy to kick the qubits into a different state.

7.5 Reading The Answer

Once the computation has been performed, reading the answer from the ions is done by shining a resonant laser on all of them at the same time. This process collapses any complex quantum information IonQ created and forces each qubit into one of two states.

Collecting and measuring this light allows us to simultaneously read the collapsed state of every ion — one of the states glows in response to the laser light, the other does not. IonQ interpret this as a binary string, where each glowing atom is a one, and each dark atom is a zero.

7.6 Isolating the Ions

To successfully hold complex quantum information, qubits can't interact with anything. A single stray hydrogen atom colliding with an ion can spoil the whole thing, collapsing delicate states, making ions switch places, or knocking the chain out of the trap entirely.

So, IonQ put the trap inside an ultra-high vacuum chamber, pumped down to around 10-11 Torr. At this pressure, there are about one hundred trillion times fewer molecules per cubic inch than the air we're currently breathing.

7.7 Putting it all together

Once everything's in the chamber, the whole assembly goes in an even larger enclosure with a variety of electrical, mechanical, and optical control systems.

Then, it all gets hooked to a classical computer running IonQ custom control software, and after a great deal of calibration, it's a fully functioning quantum computer, ready to perform gates and run algorithms with world-leading fidelity.

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