Metrology : Optical Frequency Comb

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1 Introduction

An optical frequency comb is a type of laser that emits a spectrum of light consisting of a series of equally spaced, discrete frequencies. These frequencies are typically spaced at intervals of tens of gigahertz to terahertz, depending on the specific application. The term "comb" refers to the appearance of the spectrum, which resembles the teeth of a comb.

Frequency comb are specific lasers that act as rulers for light. They measure exact frequencies of light — from the invisible infrared and ultraviolet to visible red, yellow, green and blue light — quickly and accurately.[1]

One of the most common methods for generating optical frequency combs is using a mode-locked laser, which produces a train of ultrashort pulses of light that are evenly spaced in time. The spectrum of this train of pulses is a comb of discrete frequencies.

These Nobel Prize-winning devices (the Nobel Prize in Physics being shared by John L. Hall and Theodor W. Hänsch in 2005) fill an important technological gap. Optical frequency combs have a wide range of applications in areas such as spectroscopy, telecommunications, and timekeeping. They can be used to measure the frequencies of light with extreme precision, making them valuable tools for studying the properties of atoms and molecules. They can also be used to generate ultrafast pulses of light, which have applications in high-speed communications and laser machining. Optical frequency combs have revolutionized many areas of science and technology, and their use continues to expand as new applications are discovered. Some of the current status and developments in the field of optical frequency comb are:

- **Frequency comb for optical clocks:** The optical frequency comb has been used in the development of optical clocks, which are some of the most accurate timekeeping devices ever created. These clocks have applications in navigation, geodesy, and fundamental physics.

- **Advances in Microresonator-based combs:** There have been significant advances in the development of microresonator-based combs, which are compact and low-power devices that can generate frequency combs. They have the potential to revolutionize applications such as spectroscopy, communications, and sensing.

- **Application in quantum technology:** The optical frequency comb has been used in various applications of quantum technology, such as quantum communication and quantum computing. They have the potential to improve the precision and stability of quantum devices.

- **Combining optical frequency combs with other technologies:** Researchers are also exploring the use of optical frequency combs in combination with other technologies, such as frequency synthesizers, terahertz radiation, and femtosecond lasers, to create new capabilities and applications.

The optical frequency comb has become a vital tool in various fields and continues to be a subject of active research and development. Its wide-ranging applications and continued advancements make it a promising technology for the future.

2 Methodology

The principle behind an optical frequency comb is based on the phenomenon of mode-locking in lasers. In a laser, light is amplified by stimulated emission and reflected back and forth between two mirrors. This results in the buildup of coherent light waves that have a well-defined frequency and phase.

In a mode-locked laser, the laser cavity is designed in such a way that the various longitudinal modes of the laser cavity are locked together in phase, resulting in a train of ultra-short optical pulses that are separated in time by a fixed time interval.

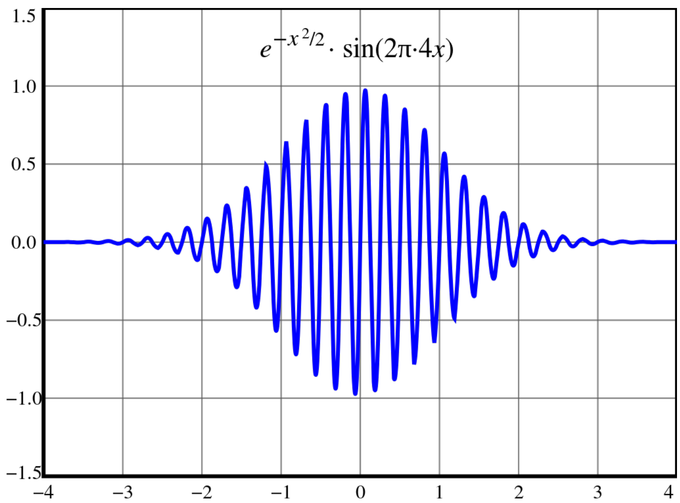

Fig 1: An ultrashort pulse of light in the time domain. The electric field is a sinusoid with a Gaussian envelope. The pulse length is on the order of a few 100 fs.

This pulse train has a very wide spectral bandwidth, which means that it contains many discrete frequencies that are evenly spaced. This spectrum of discrete frequencies appears like a comb of equally spaced 'teeth' when viewed on a spectrum analyzer, hence the name 'optical frequency comb'.

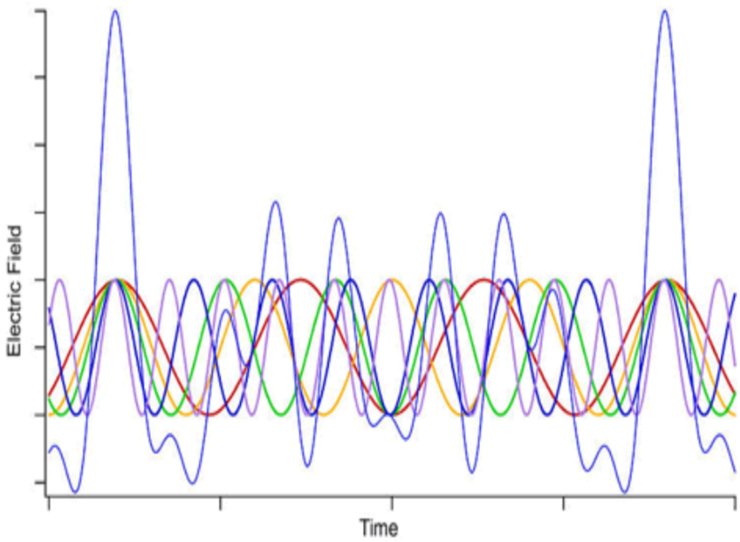

Fig 2: Mode locking forces all the colors in each pulse to start out in phase with each other.

By using high-quality laser resonators, it is possible to generate very stable and precise frequency combs with narrow linewidths. This makes them valuable tools for applications such as optical metrology, spectroscopy, and telecommunications.

2.1 Self - referencing technology

Proposed by Jones et al., in 2000,[2] the f-2f self-reference frequency locking method to measure the absolute frequency of a mode-locked laser. Mode-locked lasers generate periodic laser pulse trains. The periodic laser pulse sequence in the time domain is converted into the frequency space, and it can be expressed as a spectrum containing equally spaced frequencies. The overall intensity of the spectrum is adjusted by the envelope of the laser pulse. Therefore, it is similar to an optical ruler in the frequency domain, and the minimum scale is equal to the repetition frequency of the mode-locked laser[2].
The frequency of each comb line of OFC can be expressed as :

$$f_n = f_{CEO} + nf_{rep} \tag{1}$$

Where f_{CEO} is the carrier envelope offset frequency, f_{rep} is the repetition frequency of the optical pulse, and n is the order of OFC.
In large number studies, there are three types of frequency combs: Incoherent combs where neither f_{CEO} nor f_{rep} is stabilized, coherent combs where only f_{rep} is locked, and fully-stabilized combs where both f_{rep} and f_{CEO} are locked with one frequency reference[2].

When the spectral width of OFC reaches one octave, the nth order comb is frequency multiplied, and the frequency of the new light wave after frequency multiplication can be expressed as

$$2f_n = 2f_{CEO} + 2nf_{rep} \tag{2}$$

The 2nth order comb line frequency is

$$f_{2n} = f_{CEO} + 2nf_{rep} \tag{3}$$

The cavity length of the laser is controlled through the feedback network, thereby locking the carrier envelope offset frequency f_{CEO}, that is, the locking of the carrier envelope phase is realized. At the same time, the feedback network is used to lock the repetition frequency f_{rep} of the laser, so as to realize the frequency lock of each comb line of OFC.

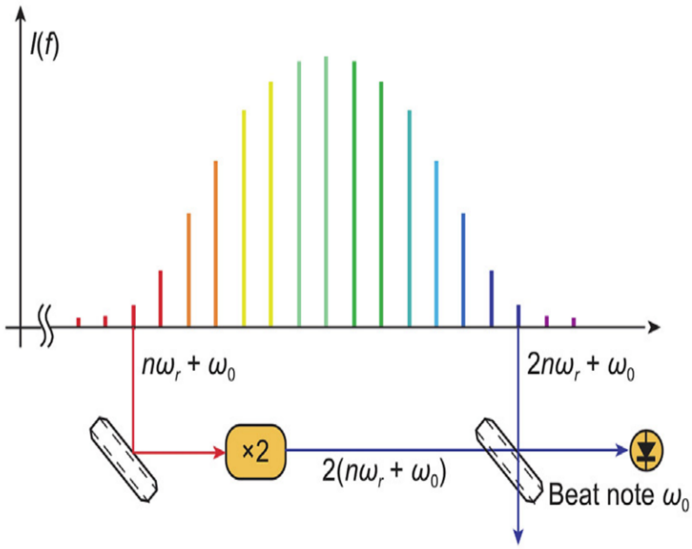

Fig 3: f-2f self-reference frequency locking principle [2].

In summary, self-referencing is a technology used to stabilize the carrier-envelope offset (CEO) frequency of an optical frequency comb. It involves splitting a portion of the comb into two paths, frequency shifting one path, and then recombining the two paths. The interference between the two paths produces a beat note at the CEO frequency, which can be measured and controlled using a f-2f interferometer. This technique enables the stabilization of the CEO frequency with high precision, which is critical for many applications of optical frequency combs. The self-referencing technique allows for the stabilization of the CEO frequency with a precision of a few parts per billion, which is critical for many applications such as precision spectroscopy and optical frequency metrology.

2.2 Four wave mixing effect for optical frequency comb's generation

Four-wave mixing (FWM) is a nonlinear optical process that can occur in optical fibers and other waveguides, which can affect the spectral properties of optical frequency combs. FWM is a type of intermodulation distortion that arises from the interaction between multiple optical waves of different frequencies traveling in a medium with a non-linear response.

In optical frequency combs, FWM can cause the generation of additional frequency components in the comb spectrum, leading to a distortion of the spectral shape. This can affect the accuracy of optical frequency measurements and can limit the precision of applications such as spectroscopy.

FWM arises due to the third-order nonlinearity of the medium, which means that the response of the medium to the interaction of three waves is non-linear. In FWM, two pump waves at frequencies f_1 and f_2 interact to produce two new waves at frequencies f_3 and f_4, such that $f_1 + f_2 = f_3 + f_4$. These new waves can add spectral components to the optical frequency comb[2].

FWM can be used to generate new frequency components at regular intervals, resulting in a comb-like spectrum of frequencies. This can be achieved by launching a pump laser into a highly nonlinear fiber or waveguide, along with a weak continuous-wave (CW) laser or another frequency comb. The nonlinear response of the fiber or waveguide causes the FWM process to occur, leading to the generation of new frequencies at intervals determined by the pump and CW laser frequencies.

The spacing between the comb teeth can be controlled by adjusting the wavelength and power of the pump laser, as well as the dispersion properties of the fiber or waveguide. The spectral bandwidth of the comb can also be controlled by adjusting the power and spectral properties of the pump and CW lasers.

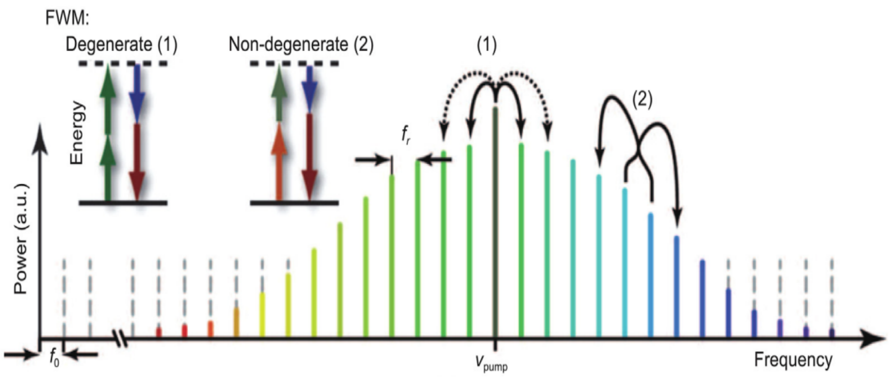

Fig 4: generation of OFC by the FWM effect.

In some cases when $f_1 \neq f_2$, it is also called non-degenerate FWM. In the case of degenerate FWM, only one beam of pump light is needed to realize the FWM process. When the appropriate dispersion conditions (usually anomalous dispersion) are met, it will generate an efficient FWM effect between the longitudinal modes in the microcavity. Utilizing single-frequency continuous light to pump the microcavity, the primary sideband can be formed by degenerate FWM when the pump power exceeds a certain threshold. The primary sideband and the pump light further realize the broadening of the spectrum by cascading FWM to form OFC.[2]

FWM-based OFC generation has several advantages, such as simplicity, scalability, and low cost. However, it also has limitations, such as a limited bandwidth and sensitivity to environmental fluctuations.

3 Optical Frequency Comb based on mode-locked lasers

Optical frequency combs(OFCs) based on mode-locked lasers are one of the most widely used techniques for generating frequency combs. Mode-locked lasers produce a train of ultrashort optical pulses with a repetition rate that is directly related to the laser frequency. By spectrally broadening the pulses using nonlinear effects such as self-phase modulation, the resulting spectrum can be made to resemble a comb of evenly spaced frequency components.

There are several different types of mode-locked lasers that can be used to generate OFCs, which includes Ti:sapphire lasers, fiber lasers, and semiconductor lasers. Each type of laser has its own advantages and limitations, and the choice of laser depends on the specific application requirements.

3.1 Advantages of OFC based on mode-locking

- Mode-locked lasers produces stable and well-defined frequency combs. The repetition rate of the laser can be precisely controlled using electronic feedback loops, and the spectral characteristics of the comb can be optimized by adjusting the laser parameters and using nonlinear effects.

- It can also produce very high repetition rates, which enables the generation of OFCs with very narrow spacing between the comb teeth. This is important for applications such as optical atomic clocks, where a high density of comb teeth is required for precise frequency measurements.

3.2 Limitations

Mode-locked lasers are susceptible to environmental fluctuations and noise, which can degrade the quality of the comb and limit the accuracy of frequency measurements.

To address this issue, a variety of stabilization techniques have been developed, such as self-referencing and active stabilization. These techniques allow for precise control of the comb frequency and phase, and have enabled high-precision measurements of optical frequencies over extended periods of time.

4 Applications in metrology

Optical frequency combs have revolutionized the field of metrology. OFCs enable the measurement of optical frequencies with unprecedented precision and accuracy, which has enabled a wide range of applications in fields such as timekeeping, spectroscopy, and fundamental physics.

4.1 Timekeeping

Optical frequency combs (OFCs) have transformed timekeeping by enabling the development of optical atomic clocks, which are the most precise timekeeping devices ever built. Optical atomic clocks use the frequency of light emitted by atoms as a reference for timekeeping, and they rely on OFCs to measure the frequency of the atomic transition with extremely high accuracy.

In a typical optical atomic clock, atoms of a specific element are trapped and cooled to very low temperatures to minimize their thermal motion. The atoms are then excited with a laser to a specific energy level, and the frequency of the laser is adjusted to match the frequency of the atomic transition. The frequency of the laser is then compared to the frequency of an OFC, which acts as a stable frequency reference. By counting the number of OFC cycles that occur between two atomic transitions, the frequency of the atomic transition can be measured with extremely high accuracy.The accuracy of optical atomic clocks is determined by the stability and accuracy of the OFC.

For example, a strontium-based optical atomic clock developed at the National Institute of Standards and Technology (NIST) has achieved an accuracy of 1 part in 10^{18}, which corresponds to a timekeeping error of less than one second in 15 billion years. This level of precision has enabled a wide range of scientific applications, such as tests of fundamental physics theories and the search for variations in fundamental constants.

The teeth of an OFCs are evenly spaced and precise, the comb acts like the gears of a clock, taking the faster optical frequencies and dividing them down to the lower-frequency microwave signals used by electronics and current atomic clocks.[2]

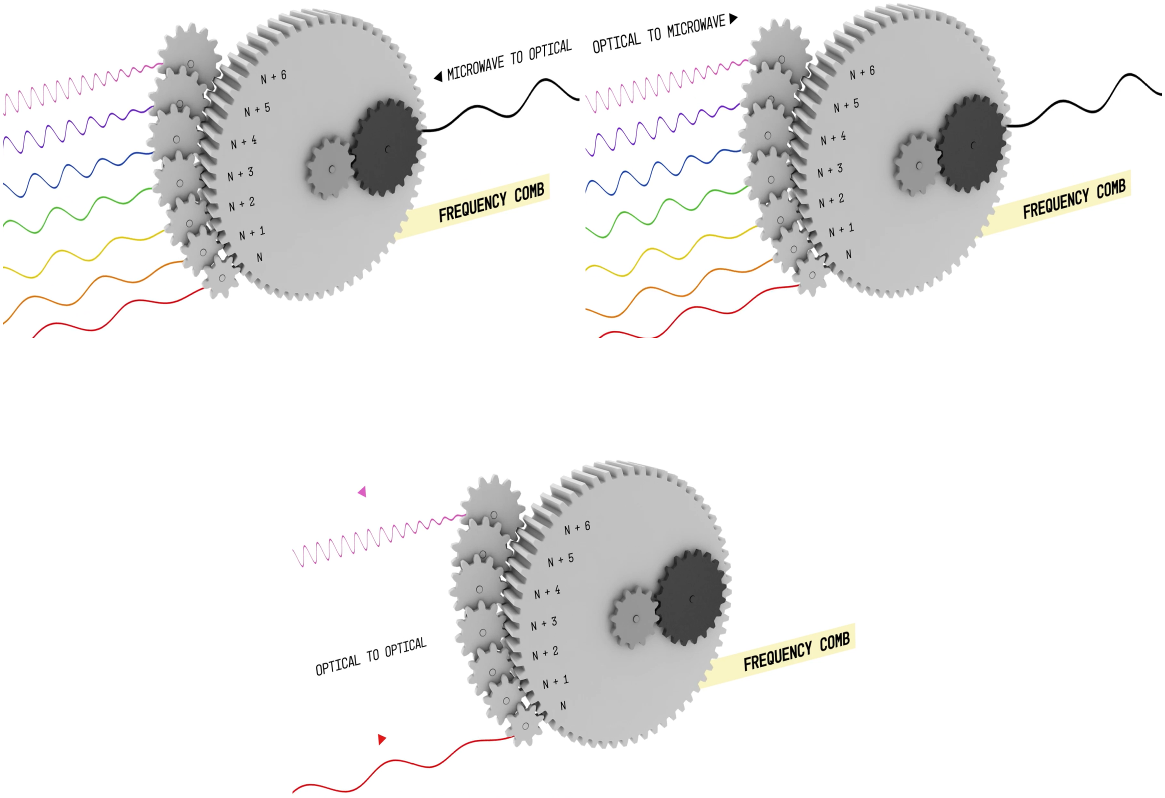

Fig 5: Optical frequency combs can act like gears in a clock[1].

With these 'gears' carrying accurate signals between electronics, microwave-based tools and optical atomic clocks, scientists can use these powerful new clocks for faster, more accurate timekeeping systems.[1]

The continued development of OFC technology holds promise for even more precise timekeeping and new applications in fields such as navigation, and space exploration.

Global Positioning System (GPS) relies on precise timing information to determine the distance between a user's receiver and the GPS satellites, and this timing information is provided by atomic clocks on board the satellites.This improved accuracy translates into better GPS performance, including more precise positioning and timing information.

4.2 Ranging and precise distance measurement

OFCs can be used in lidar systems to improve the accuracy and precision of distance measurements. Lidar is a remote sensing technology that uses laser light to measure distances and create high-resolution maps of objects and surfaces. Lidar works by emitting short pulses of laser light and measuring the time it takes for the light to reflect back from the target object or surface. By measuring the time delay between the emitted and received laser pulses, lidar can determine the distance to the target object or surface with high precision.

In addition to lidar, OFCs have other applications in metrology, where precise distance measurements are required. For example, OFCs can be used in interferometry, a technique used to measure small distances with extremely high precision. Interferometry works by measuring the interference pattern created by two beams of light that have traveled different distances.

5 Concepts applied from lectures

5.1 Mode-locking

Mode-locked lasers are a type of laser that generates extremely short pulses of light with durations in the femtosecond (10^{-15} seconds) to picosecond (10^{-12} seconds) range. These short pulses of light have very high peak powers, making mode-locked lasers useful for a wide range of applications.

Mode locking is achieved by combining in phase a number distinct longitudnal modes of laser, all having slightly different frequencies. When modes of electromagnetic waves of different frequencies but random phase is added, they produce randomly distributed average output of both the electric field and the intensity in time domain.

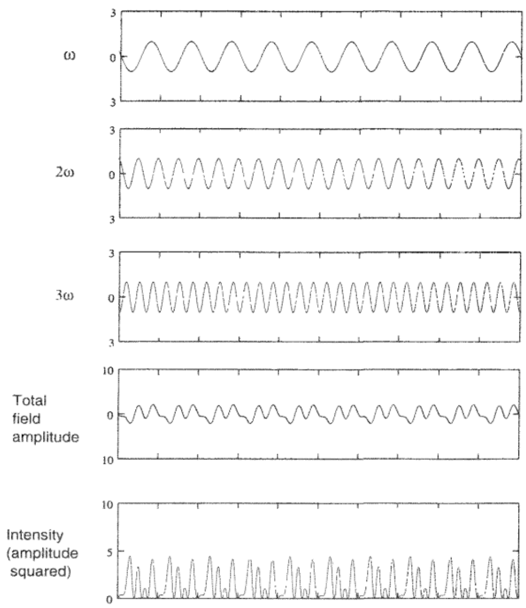

Fig 6: Amplitude and intensity of sum of three out of phase wave added together.

Longitudinal modes in laser cavity oscillates independently of other modes. Amplitude of nth mode is described as :

$$E'(t) = E_0 e^{i(\omega_n t + \phi_n)} \tag{4}$$

where ω_n is the frequency and ϕ_n is phase of that mode. Assuming there are N modes of equal amplitude oscillating simultaneously in cavity, then combined amplitude of all modes can be expressed as

$$E(t) = \sum_{n=0}^{N-1} E'(t) = \sum_{n=0}^{N-1} E_0 e^{i(\omega_n t + \phi_n)} \tag{5}$$

difference in angular frequency between modes is $\omega_{n+1} = \omega_n + n\Delta\omega$.
The total intensity is given by absolute square of the total amplitude :

$$I(t) = |E(t)|^2 = |E_0|^2 \sum_{n=0}^{N-1} e^{i(\omega_n t + \phi_n)} e^{-i(\omega_n t + \phi_n)} = N E_0^2 \tag{6}$$

The combined field amplitude is given by

$$E(t) = E_0 \sum_{n=0}^{N-1} e^{i(\omega_n t + \phi_0)} = E_0 e^{i\phi_0} \sum_{n=0}^{N-1} e^{i\omega_n t} \tag{7}$$

defined $\omega_n = \omega_{N-1} - n\Delta\omega$ which implies

$$\begin{aligned} E(t) &= E_0 e^{i\phi_0} \sum_{n=0}^{N-1} e^{i(\omega_{N-1} - n\Delta\omega)t} \\ &= E_0 e^{i(\phi_0 + \omega_{N-1} t)} \left(\frac{1 - e^{iN\Delta\omega t}}{1 - e^{i\Delta\omega t}}\right) \end{aligned} \tag{8}$$

Therefore, total intensity

$$I(t) = E_0^2 \left(\frac{1 - e^{iN\Delta\omega t}}{1 - e^{i\Delta\omega t}}\right)^2 = E_0^2 \frac{sin^2(N\Delta\omega t/2)}{sin^2(\Delta\omega t/2)} \tag{9}$$

5.2 Self Phase Modulation

Self-phase modulation (SPM) is a nonlinear optical effect that occurs when a high-intensity laser pulse propagates through a medium. In SPM, the intensity of the laser pulse causes a change in the refractive index of the medium, which leads to a change in the phase velocity of the pulse. As a result, different frequency components of the pulse experience different phase shifts, leading to broadening of the pulse spectrum and distortion of the pulse shape.

The phenomenon of SPM is a result of the Kerr effect, which describes the nonlinear response of a medium to an intense electromagnetic field. In the Kerr effect, the refractive index of a medium changes in proportion to the square of the electric field intensity of the light. When a high-intensity laser pulse propagates through a medium, the Kerr effect causes the refractive index to vary in space and time, leading to changes in the phase and amplitude of the pulse.

SPM can be used to compress long pulses of light into much shorter, higher-intensity pulses, which are useful for a range of applications including ultrafast spectroscopy, micromachining, and laser surgery. It can also be used for wavelength conversion, where the broadened spectrum of the pulse can be used to generate new frequencies via processes such as four-wave mixing. However, It can also have negative effects in some applications, such as in fiber optic communications, where it can cause pulse distortion and degradation of signal quality.

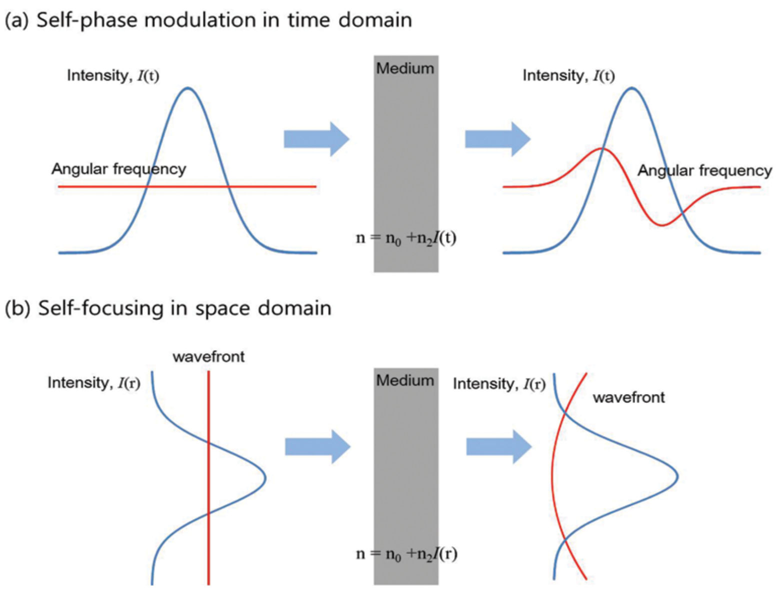

Fig 7: Self-phase modulation in time induces the time-dependent phase variation and Self-phase modulation in space makes the wavefront quadratically curved.

5.3 Ultrashort Pulse

Chirped pulse amplification (CPA) is another technique for generating ultrashort laser pulses. In CPA, a laser pulse with a long duration is first stretched out in time using a dispersive element such as a grating. The stretched pulse is then amplified to a high energy using a laser amplifier. Finally, the amplified pulse is compressed back to its original duration using a second dispersive element. This technique allows for the generation of extremely short pulses with durations on the order of a few femtoseconds.

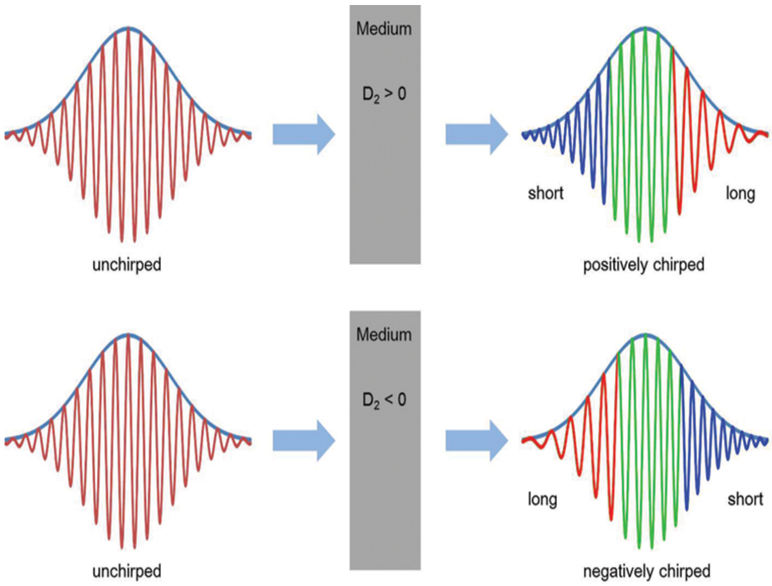

Fig 8: Frequency chirping in the laser pulse.

Nonlinear pulse compression is also a technique for generating ultrashort laser pulses. This technique makes use of the fact that the spectral bandwidth of a laser pulse is inversely proportional to its duration. By using a nonlinear crystal or fiber to broaden the spectral bandwidth of a laser pulse, it is possible to compress the pulse duration to a few femtoseconds or less.

6 Conclusion

Optical Frequency Combs (OFCs) has revolutionized metrology by providing an accurate and stable reference for measuring frequencies in the electromagnetic spectrum. OFCs are generated using mode-locked lasers and rely on the principles of nonlinear optics to produce a comb-like spectrum of equidistant frequencies.

OFCs have found many applications in timekeeping, spectroscopy, ranging, and high-precision measurements. The self-referencing technique of OFCs has enabled the measurement of absolute optical frequencies with unprecedented accuracy, making them a valuable tool in various scientific and industrial applications.

This project has provided an overview of the principles and applications of OFCs in metrology. We have discussed the generation of OFCs using mode-locked lasers and the underlying physics of the nonlinear processes involved in their production. We have also explored the various applications of OFCs in high-precision measurements, including timekeeping and ranging.

Therfore, OFCs are an important tool in metrology and have opened up new possibilities for the precise measurement and manipulation of electromagnetic radiation. Ongoing research in this field is likely to lead to further improvements in the accuracy and stability of OFCs, which will continue to have a significant impact on scientific and industrial applications in the years to come.

References

[1] Optical frequency comb NSIT, (https://www.nist.gov/topics/physics/optical-frequency-combs)

[2] H. Zhang et al. Journal of Electronic Science and Technology 20 100157 2022