Prototype for the Model-Based Assets and Liabilities Management Support System

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Abstract— **This paper presents the prototype of the instrument that supports the control of assets and liabilities by a bank using the mathematical computations. The specifics of the approach lies in the application of the recent advances in the control theory to provide the strategic analysis for various planning horizons and scenarios of external and internal factors.**

I. INTRODUCTION

The main instrument of the bank's management of assets and liabilities is the control of the prices of its products, additionally taking into account the reaction of the market. In this activity, the bank should predict the possible outcomes for its balance and efficiency of its actions.

The control, realized as a result of thorough consideration of possible outcomes, should be efficient. There is a demand for information systems that enable efficient management of the complex and diverse banking activities. The problem of the IT system is to use the available data and the knowledge of the inner governance and proceedings of the organization, to forecast and manage the dynamics of the assets and liabilities.

II. TECHNOLOGY FOR ASSETS AND LIABILITIES MANAGEMENT

A. Motivation

1) ALM by banks: Assets and liabilities management of banks is an important study area in optimal control. Firstly, we assume that banking systems should be based on the solution of the optimal control problem since commercial bank is the economic agent with profit-maximizing behavior. Secondly, main practices in ALM can be rendered in the form of optimal control problem. As the result this can promote better understanding of monetary and prudential policies of national banks. Banks are the main economic agent that policy of monetary authorities are aimed at. Response in bank's behavior on the same policy measures can be different depending on bank's balance sheet composition.

The impact of government policies on the behavior of the banking sector is mainly explored in the context of econometric papers aimed at studying the transmission of monetary policy to the real sector of the economy [1-4]. Papers devoted to modeling the behavior of the banking sector are less common [5-6]. In both types of papers, the banking sector is often considered as an intermediate stage for monetary policy signals to the real sector of the economy. In these papers banking sector is usually described schematically or from the point of view that is important only for real sector of the economy, and assets and liabilities does not correspond to the real composition of the bank balance sheet. This limits the usefulness of the approaches proposed in such works for applied ALM in an actual bank.

2) IT solutions for ALM by banks: Entities of banking and financial domains are in persistent need for the information systems to bolster their processes and their compliance with regulations. The IT solutions which may support the ALM on a basic level are quite common. However the services and functionality provided by such information systems really differ starting from being just beneficial and supportive up to a 'game changing' solution which grants robust efficacy and wast possibilities. IT solutions for the ALM may be divided into two main categories:

1) solutions for commercial banks that help banks to comply with the regulatory demands,
2) solutions for regulators that aid them in controlling the execution of the policies, rules and regulations.

While IT solutions for regulators are required as well as the IT solutions for commercial banks, the automated information system for assets and liabilities management within commercial banks can become increasingly vital in overcoming crises. These solutions allow staying afloat and operational and help maintaining financial stability.

Henceforth the efficiency of the information system for ALM is greatly dependent on the following:

1) the mathematical models which drive the forecasting,
2) goals set,
3) tight cohesion with Risk Management,
4) trustworthiness of parameters and data sources.

For the Technology the goal will be in creating an ALM framework basically focused on:

- long-term profitability,
- short-term profitability,
- forecasting,
- financial institution stability.

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Such framework will be a recommendation information system based on ALM as a collaborative process allowing work with financial institution's complete balance sheet which provides tools for manipulation over mathematical models through their parameters to do the following:

1) manage credit quality,
2) manage liquidity requirements,
3) create enough operating capital.

Forecasting functionality of such information system should allow to create a number of hypothesis with different sets of parameters including different risks' scores, to acquire the predicted assets and liabilities changes. This should allow to elaborate different strategies for financial institution and probe them before managing solutions go live.

The ALM strategic recommendation system should provide a range of tools to fulfil the following:

- assets are to be invested the most optimal way,
- liabilities are to be moderated over the very long term,
- mitigation of broad range of risks i.e. interest rate risk and liquidity risk,
- strategy hypothesis based on mathematical function methods, including for example penalty optimization strategy, etc.,
- functionality for model's verification.

3) Mechanisms of practical management of assets and liabilities by banks: Practical implementation of ALM in Russia involves, among other things, analyzing two types of risk: interest rate risk and liquidity risk.

Liquidity risk or insolvency risk is the risk that a bank may not have enough liquid assets for settlements with its customers. The realization of such a risk leads to an immediate shutdown of the activity of the bank. Therefore, along with interest rate risk, it is one of the most important risks. Measures aimed at mitigation of liquidity risk typically include establishing a liquidity buffer and diversifying sources of bank's funding.

Analyzing the bank's exposure to interest rate risk implies finding out how sustainable the bank's business model is. One of the main functions of a bank is the maturity transformation. The maturity of the bank's liabilities is in most cases shorter than the maturity of assets. As a result, percentage expenses adapt to general interest rate level faster than percentage income. The realization of interest rate risk is usually associated with an unanticipated change in the monetary policy rate, which may lead to a situation when a bank's interest expenses are higher than its interest income. As part of interest rate risk analysis, different metrics are constructed that reflect the bank's income generation over different time horizons under current and scenario compositions of the bank balance sheet. The ultimate goal of changing the structure of the bank balance sheet is to generate sustainable positive earnings over the entire planning horizon under various scenarios of changes in external conditions.

Our approach is based on general principles of constructing the banking model, see [1]-[2]. Recently, many practical issues were considered in the analysis on the ALM problems [3]. For example, in [4], and [5] risk constraints on liquidity were studied.

We propose our approach since it is based on mathematical analysis and is adapted to specifics of concentrated markets, which presently are most common in many countries.

B. The Problem of ALM by Financial Organizations

We have chosen the balance sheet of the model bank in such a way that it corresponds to the real trends observed in the statistics. Russia belongs to the group of developing countries characterized by high interest rates and a large gap between the cost of raising and placing funds. As a result, the largest share of the banking sector's revenues is generated through "standard" banking operations: issuing loans and raising deposits. In this regard, we consider a model balance sheet that consists of loans granted to and deposits raised from the main sectors of an economy: households and firms. We also consider liquid assets in total and debt to central bank since it is main instruments of liquidity management for banks in Russia.

There are different types of liquid assets in real world. For example: cash, liquidity held in national bank, high quality bonds and other securities. In our work we consider all liquid assets together as a single type of asset, because in Russia, with the assistance of monetary authorities, such types of assets can be converted into each other within a day. For example, high quality bonds can be converted into liquidity held in central bank through standing facility repurchase agreements and then this liquidity can be converted into cash.

In order to capture the liquidity risks faced by banks, we introduced two constraints in the form of inequalities. The first constraint is imposed on the amount of required reserves that banks must hold within the central bank. It reflects regulatory liquidity risks. The second constraint sets a minimum bound on the liquid assets that a bank maintains due to its own precautionary motive.

We also impose an additional constraint in the form of an inequality on bank's equity. This constraint reflects the capital adequacy ratio stipulated by international banking regulations. It limits the path of bank equity capital from below to a fixed share of the bank's risk-weighted assets. The purpose of this norm is to make the banking sector resilient to stress situations in which large losses are possible.

Risk-weighted assets are subject to regulation by the authorities. By changing the risk weights on individual assets, the government can make the issuance of certain types of loans more or less profitable for the bank.

Right now types of assets and liabilities established in the model are limited to the main banking operations, i.e. different types of loans and deposits and also debt and reserves held in Central Bank. Tables describing notation of expanded list of assets and liabilities, that we are planning to add into our framework, are placed above.

The dynamic management of assets and liabilities should rely on, firstly, the evolution of the bank's balance as a result of the processes of withdrawal and replenishment and, secondly, on the response of the bank's indicators to the

TABLE I

CAPITAL AND LIABILITIES VARIABLES

Notation	Description
$M_h(t)$	Transferable accounts of households
$S_h(t)$	Deposits of households
$M_f(t)$	Transferable accounts of firms
$S_a(t)$	Deposits of firms
$S_c(t)$	Short-run deposits of Central Bank
$L_{cb}^L(t)$	Long-run debt to Central Bank
$D_g(t)$	Debt to government
$L_o(t)$	Other liabilities
$O_b(t)$	Bank's capital

TABLE II

ASSETS VARIABLES

Notation	Description
$L_m(t)$	Mortgage
$L_h(t)$	Consumer credits
$B_g(t)$	Bonds issued by government sector
$B_p(t)$	Bonds issued by private sector
$L_a(t)$	Loans to firms
$R_c(t)$	Reserves in Central Bank
$A(t)$	Liquid assets
$A_o(t)$	Other assets

decisions by the bank's management. The latter might have not only immediate effect but also the long-term effect that might create both risks and opportunities in the future. That is why we propose to use the optimal control as the appropriate tool for the support of decisions with possible long-term effect on performance.

III. Applications of the Model-Based ALM Support System

Possible applications might be classified, but not limited to, the following groups of managerial problems

- Analysis of the strategy. Forecast of the possible short and long-term outcomes.
- Comparison of strategies. Multilateral analysis of different aspects of benefits and drawbacks of two or more strategies or actions by management that might have impact on the bank's assets and liabilities.
- Design of the strategy based on the target indicators.
- Analysis of risks associated with the external normative constraints on indicators of financial institutions.

The unique feature of the current solution is the possibility to include the analysis of the overall economic context. The model we propose to use is based on the framework which is widely used as a block in commutable general equilibrium models. Therefore, the macro processes might be studied in connection with the description of dynamics of a bank's indicators.

IV. The Framework for ALM

A. Main components and models

The formulation of the model begins with a description of blocks of individual agents, which may include:

- controlled variables, the trajectories of which are determined by the agent himself,
- information variables, which trajectories are determined outside the agent's block, but the agent considers them known,
- parameters of the agent's goal, specifying, for example, the specification of preferences, production set and other characteristics of the agent's activity.

For each block of the model, its own index is set, which will later be used to index the controlled variables and task parameters of this agent. At the same time, at the stage of formulating and solving the problem, indexing is not used in order not to complicate the equations.

The content of the model, in fact, depends on the problem proposed for study. A possible configuration of the framework to compute the assets and liabilities' changes as a result of managerial decisions is a mathematical model in the form of an optimal control problem with phase constraints, where the interest rates stay for the control parameters.

1) Bank Problem: The bank maximizes the utility from dividend income of its owners (owners' welfare)

$$\int_0^T \frac{1}{1-\rho}\left(\frac{Z(t)}{p_\pi(t)}\right)^{1-\rho} e^{-\delta t}\, dt \tag{1}$$

by choosing the volume of dividends $Z(t)$ divided by the price index $p_\pi(t)$ to account for inflation. The parameter $\rho \in (0,1)$ is a characteristic of the utility function.

2) Bank balance: A bank's balance sheet equalizes assets and liabilities. We consider the following types of debt to the bank to be assets: consumer loans to households, mortgage loans to households, loans to companies, bank reserves in the Central Bank. Liabilities are the bank's debt to depositors (households and companies) and the Central Bank's deposits in the bank.

3) Bank assets:

Consumer loans to the population (households). The flow of newly issued loans $K_h(t) \geq 0$ and debt repayment at a rate of $\beta_{lh}(t)$ determines the dynamics of the current debt $L_h(t)$ of households to the bank

$$\frac{d}{dt}L_h(t) = K_h(t) - \beta_{lh}(t)L_h(t). \tag{2}$$

Mortgage loans to the population. The flow of newly issued mortgage loans $K_m(t) \geq 0$ and the repayment of debt at a rate of $\beta_{lm}(t)$ determines the dynamics of the current debt $L_m(t)$ of households to the bank

$$\frac{d}{dt}L_m(t) = K_m(t) - \beta_{lm}(t)L_m(t). \tag{3}$$

Loans to companies. The flow of newly issued loans to firms $K_a(t) \geq 0$ and the repayment of debt at a rate of $\beta_{la}(t)$ determines the dynamics of the current debt $L_a(t)$ of firms to the bank

$$\frac{d}{dt}L_a(t) = K_a(t) - \beta_{la}(t)L_a(t). \tag{4}$$

Bank reserves in the Central Bank. Banks deposit money into the Central Bank at interest. The flow of new reserves

$K_c(t)$ determines the dynamics of the current volume of bank reserves in the Central Bank $R_c(t)$

$$\frac{d}{dt}R_c(t) = K_c(t). \tag{5}$$

Liquid assets of the bank. The variable $A(t)$ denotes the bank's liquid assets, the meaning of which will be described below.

4) Bank liabilities:

Household deposits. The flow of newly created deposits $V_h(t) \geq 0$ and withdrawals from deposits at a rate of $\beta_{sh}(t)$ determines the dynamics of the bank's current debt $S_h(t)$ to households (deposits)

$$\frac{d}{dt}S_h(t) = V_h(t) - \beta_{sh}(t)S_h(t). \tag{6}$$

Deposits for companies. The flow of newly created deposits by firms $V_a(t) \geq 0$ and the withdrawal of deposits at a rate of $\beta_{sa}(t)$ determines the dynamics of the bank's current debt $S_a(t)$ to firms (deposit)

$$\frac{d}{dt}S_a(t) = V_a(t) - \beta_{sa}(t)S_a(t). \tag{7}$$

Central Bank deposits in the bank. Banks accept money from the Central Bank at interest. Replenishment of the Central Bank's deposit in the bank $V_c(t)$ determines the dynamics of the current volume of the bank's debt to the Central Bank $S_c(t)$

$$\frac{d}{dt}S_c(t) = V_c(t). \tag{8}$$

Bank's own capital. This indicator, due to the bank's balance sheet, is the difference between the bank's assets and liabilities.

$$O_b(t) = \sum_{i=h,m,a} L_i(t) + R_c(t) + A(t) - \sum_{i=h,a,c} S_i(t). \tag{9}$$

5) Dynamics of liquid assets of the bank: let us introduce interest variables on loans and deposits with indices corresponding to the indexation of indicators above. For example, $r_{la}(t)$ is the interest on loans to firms, and $r_{sh}(t)$ is the interest on household deposits. These percentages are determined in the market description, so for the bank all percentages serve as external information variables. The same as the $\beta_i(t)$ indicators above. New variable: $OC_o(t)$ - operating and other expenses of the bank.

$$\begin{gathered}\frac{d}{dt}A(t) = -K_h(t) + (r_{lh}(t) + \beta_{lh}(t))\, L_h(t) - K_m(t) + \\ + (r_{lm}(t) + \beta_{lm}(t))\, L_m(t) - K_a(t) + (r_{la}(t) + \\ + \beta_{la}(t))L_a(t) - K_c(t) + V_h(t) - (r_{sh}(t) + \beta_{sh}(t))\, S_h(t) + \\ + V_a(t) - (r_{sa}(t) + \beta_{sa}(t))\, S_a(t) + \\ + V_c(t) - r_{sa}(t)S_c(t) - OC_o(t) - Z(t). \end{gathered} \tag{10}$$

6) Restrictions on variables:

Reservation of deposits. A portion $n_s(t)$ of bank customer deposits must be reserved. The coefficient $n_s(t)$ changes very slowly over time only when the Central Bank revises the standards. So it is approximately considered a constant. We divide liquid assets that help bank to ensure its solvency and reserves in Central Banks since these types of assets are divided in the banking practise. Total buffer of bank's liquid assets include but are not limited to reserves.

$$R_c(t) = n_s(t)\left(S_h(t) + S_a(t)\right). \tag{11}$$

Limitation of bank liquidity. In case of urgent need, part of the bank's funds should be available to the bank for operations. The share of available funds is determined by the parameters τ:

$$A(t) \geq \sum_{i=h,a} \tau_{si}S_i(t) + \sum_{j=h,m,a} \tau_{lj}L_j(t). \tag{12}$$

Capital adequacy ratio determines the proportion between the bank's capital and assets, weighted with weights w determined by the risks of specific assets and the the coefficient k_N is determined by the Central bank and differs for types of banks:

$$\begin{gathered} L_h(t) + L_m(t) + L_a(t) + L_c(t) + A(t) + R_c(t) - \\ -S_h(t) - S_a(t) - S_c(t) \geq k_N * (A(t) + w_{lh}L_h(t) + \\ + w_{lm}L_m(t) + w_{la}L_a(t)). \end{gathered} \tag{13}$$

7) Restrictions on controls: If we accept the assumption that the functions describing the flow of newly issued loans $K(t)$ and newly accepted deposits $V(t)$ are determined by the demand for loans and deposits, respectively, then the control parameters in our model are interest rates on loans and deposits.

Under the law, the maximum daily loan rate is limited, which leads to the control constrain: $r_l(t) \leq R_l$, where $R_l = const > 0$. On the other hand, interest rate restrictions are tied to the key rate of the Central Bank $r_{key}(t)$, so

$$r_{key} - 0.01 \leq r_l(t) \leq R_l, \tag{14}$$

$$0 \leq r_s(t) \leq r_{key} + 0.01. \tag{15}$$

Presence of upper and lower boundaries for $r_l(t)$ and $r_s(t)$ respectively is driven by the fact that Bank of Russia conducts monetary policy in the regime of percent corridor. It means that there is always a possibility for bank to store funds at $r_{key} - 0.01$ or lend them at $r_{key} + 0.01$ to Central Bank. These measures are mainly aimed at stabilising interbank interest rates, but they also ensure that banks do not lend money to other sectors at interest rates that are lower than $r_{key} - 0.01$ and do not accumulate funds at interest rates that are higher than $r_{key} + 0.01$ thus providing additional subchannel of monetary policy to the economy.

To make the set of feasible controls compact, we introduce restrictions on the amount of dividends paid:

$$0 \leq Z(t) \leq M. \tag{16}$$

Reasoning in this way, we can develop a mathematical model for the bank's asset and liability management system based on the optimal control problem with phase constraints. The state variables are deposits $S(t)$, loans $L(t)$ and liquid assets $A(t)$ and the control variables deposit rates $r_s(t)$, loan rates $r_l(t)$ and the amount of dividends paid $Z(t)$.

Our experience showed that introducing the variable $W(t) = A(t) + L(t) - (1 - n_s)S(t)$ instead of $A(t)$ can simplify the system. And $W(t)$ means bank's own funds.

B. Computer realizations and numerical methods

The bank's asset and liability management model led us to the optimal control problem with a constraint on phase trajectories. This class of problems is characterized by mathematical complexity and a small number of completed problems. In order to simplify the model and be able to carry out the first test calculations, we accepted two assumptions:

1. some general form of a bank offering one aggregate type of credit and one type of deposit is taken into account
2. we assume linear dependence of the flows of newly issued loans and newly issued deposits on the corresponding interest rates

The first, obvious way to approach solving such problems is the penalty function method. In this case, phase restrictions pass into the functional as terms and their violation leads to a decrease in the functional. So the problem becomes an ordinary optimal control problem, to the solution of which Pontryagin's maximum principle can be applied. An example of using this method is given in [12].

The next possible approach is based on the works [13] - [16] and uses the Pontryagin's maximum principle for the state constrained problems, and in this case numerical calculations are reduced to the shooting method and going through the initial values of conjugate functions and that takes too much time. In the work [17] a way to speed up the shooting method is suggested using the optimization methods. It is a computational algorithm based on stochastic gradient descent method. But still calculating the case of a trajectory hitting the restrictions causes a lot of difficulties.

Another one approach: using substitutions, a simplified model can be reduced to a model with a linear system and linear constraints, which allows the Frank-Wolf optimization algorithm to be applied. This is the proven working approach to solving this problem. Examples of calculations are given in the work [18].

V. DISCUSSION AND CONCLUSION

A model-based recommendation system with detailed analysis of future outcomes of decisions gives an opportunity to realize strategies in the form of gradual changes.

Another feature of such dynamic description lies in the opportunity to consider strategic interactions between financial institutions (competition, cooperation, mergers and acquisitions etc.). The limiting case where the banks with substantial market power serve as the guide (or indicators) to smaller banks (price-takers) is modelled by the mean-field type models [19], [20], [21], but the part of individual bank's control models might be based on our approach.

Future directions of developments might involve

- identification of parameters of the description of the bank's clients' behavior on particular market,
- connection to the model of the macroeconomic context,
- analysis of the crisis conditions with description of uncertain factors,
- for the IT-sphere the development of the mathematical foundations of the ALM, and improvement of precision of computation. The nonlinear dynamics requires development of special optimization methods,
- scalable and extendable solutions for the analysis of the banking system for the planing of macroprudential policy by central banks.

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