CSE306 - Computer Graphics

Voronoi Diagram - 2D FluidSolver

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1 Introduction

In this report, I present the second assignment of the course CSE306 - Computer Graphics. This second assignment we study Voronoi Diagrams, Power Diagrams for a given set of points and perform some experiments using these structures. The experiments here presented are better described in the lecture notes of the course. In this project I performed the following experiments.

- Voronoi Diagram production
- Centroidal Voronoi Tesselation production
- Power Diagram production
- Optimal Transport simulation
- Fluid Simulation

They will be presented in this order in this report. All experiments and simulations hereby presented were performed in 2D although it can be generalised in 3D. Moreover, the space is considered to be a 1000×1000 lattice.

2 Voronoi Diagram

Let's denote by *seeds* a set of points in a plane. A *Voronoi Diagram* is a partition of the plane into regions, where each one of these regions is the portion of the plane that is the closest to one of the seeds. Following the algorithm shown during lecture we can successfully produce this diagram for a given set of seeds.

In lecture, it was presented the naive algorithm where we perform a clipping algorithm for each one of the bisectors, even if it does not intersect the polygon. This algorithm has complexity $\mathcal{O}(n^2)$. As mentioned in Lecture it is necessary only to check a new bisector m between the current seed P and a new point Q only if $\|P - Q\|$ is smaller than twice the biggest distance between P and a vertex of the current computed polygon. Therefore, in order to improve the algorithm one can implement the following optimization: when computing the voronoi cell of a seed P one can 1) sort all the seeds with respect to their distance to P, 2) while doing the clipping algorithm keep track of the biggest distance between P and the vertices 3) when looking at the next seed check the if the condition is met and if it is not stop the algorithm. As sorting can be done in $\mathcal{O}(n \log n)$, we have an improvement. The results of this experiment are shown in Figure 1

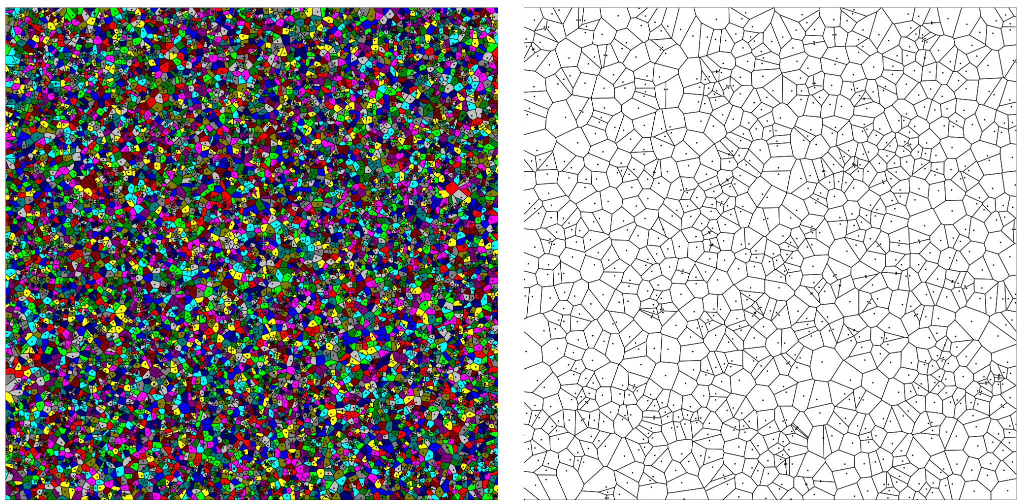

Figure 1: In both pictures the seeds were chosen uniformly random. (Left) Voronoi diagram produced using 10k seeds, it took 10s to be rendered using the naive approach and 3s using the optimization described above. (Right) Voronoi diagram produced using 1k seeds, it took 140ms to be rendered using the naive approach and 45ms using the optimization described above.

3 Centroidal Voronoi Tesselation

In the lecture notes we learn that using Lloyd iterations we can obtain a more uniform partition of the plane. The algorithm in this experiment consists in performing several times the following: 1) compute the Voronoi Diagram 2) compute the baricenter of each cell and take them as new seeds. The results are shown in Figure 2

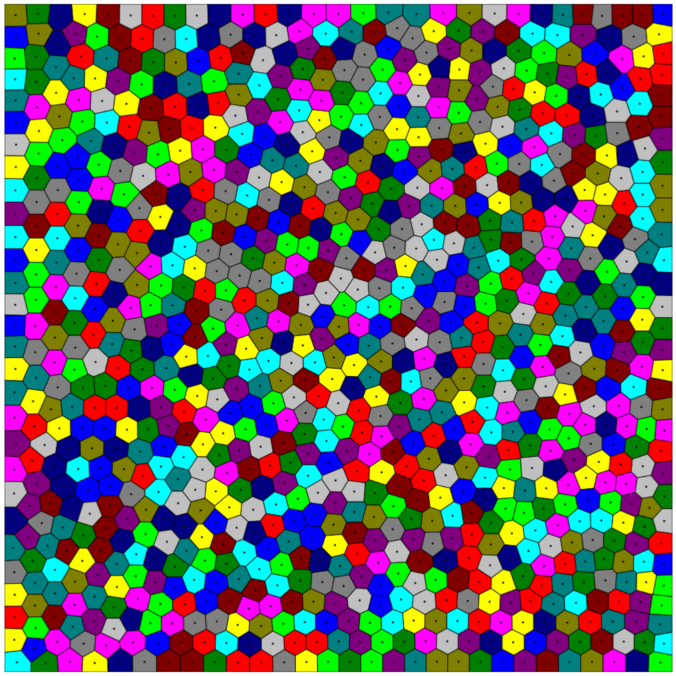

Figure 2: Image of a Centroidal Voronoi Tessselation using initially 1k random seeds and 50 iterations.

4 Power Diagram

Assume we have n seeds and we associate to each one of them a real value r. Thus, a *Power Diagram* is a partition of the plane into cells such that a point P belongs to a cell C_i if $\|P - S_i\| - r_i^2$ is minimal among all seeds S_i.

The approach to build such Diagram is similar to the one to produce Voronoi Diagram, but instead of bisectors of two seeds S_i, S_j we will use the Radical axes between the circumferences $C(S_i, r_i)$ and $C(S_j, r_j)$. It is valid to observe that radical axes share with bisectors some similar properties. For example, given 3 circumferences where the centers are not aligned, their pairwise radical axes are concurrent (About radical axes). The results of such experiment are pictured in Figure 3

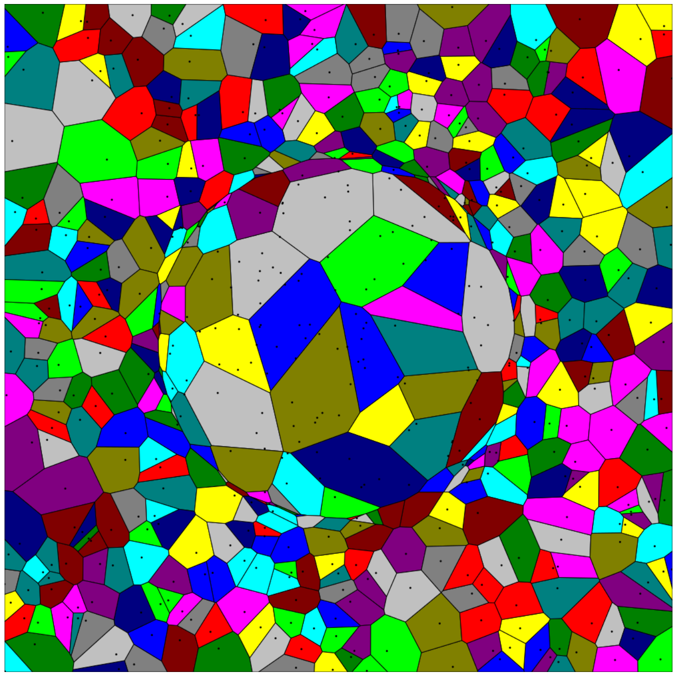

Figure 3: Image of a Power Diagram for 500 seeds and for each seed x_i its weight being $\exp\big(-\frac{\|x_i - C\|^2}{0.02}\big)/20(C = (500, 500))$.

5 Optimal Transport Simulation

As seen in lecture the *optimal transport* problem is the problem of matching a probability distribution with another probability distribution at minimal cost. Moreover, in some cases the optimal transport plan can be represented by a power diagram. Following the analogy made in the lecture notes in Figure 4 we have a representation of 2000 bakeries in a city. Assuming also that bakeries closer to the center can produce more bread, we have that they attract more people, thus a larger area since the population is assumed to be uniformly distributed.

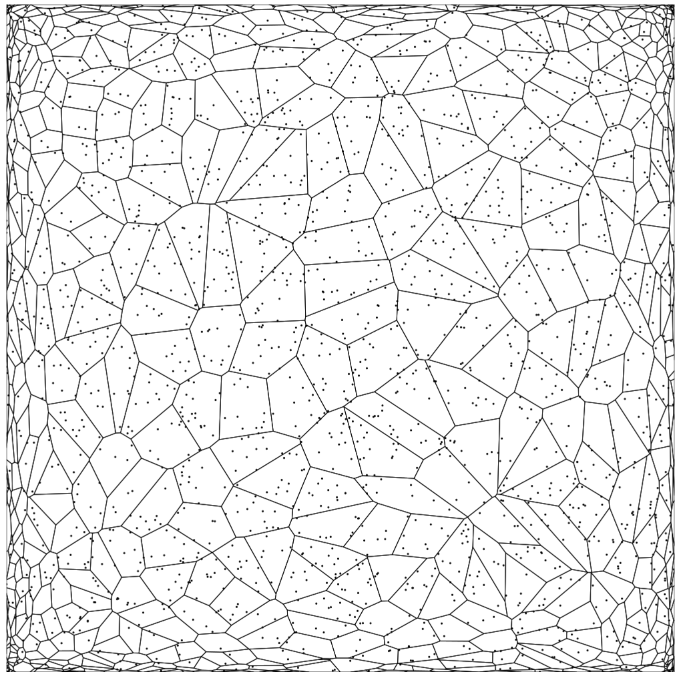

Figure 4: Use of semi-discrete optimal transport so that the cell associated to one of the 2000 seeds S_i has an area proportional to $exp(\|S_iC\|^2/0.02)$ where $C = (500, 500)$.

6 Fluid Simulation

As the last part of the project we simulated a drop of some fluid falling. The algorithm is described precisely in the lecture notes, the idea is to add gravity with a spring force pushing each cell towards its centroid, and use optimal transport to make the volume of each particle constant. In Figure 5, we have 4 frames of the simulation, with XX particles of water and XX particles of air. A small video can be found on git repository.

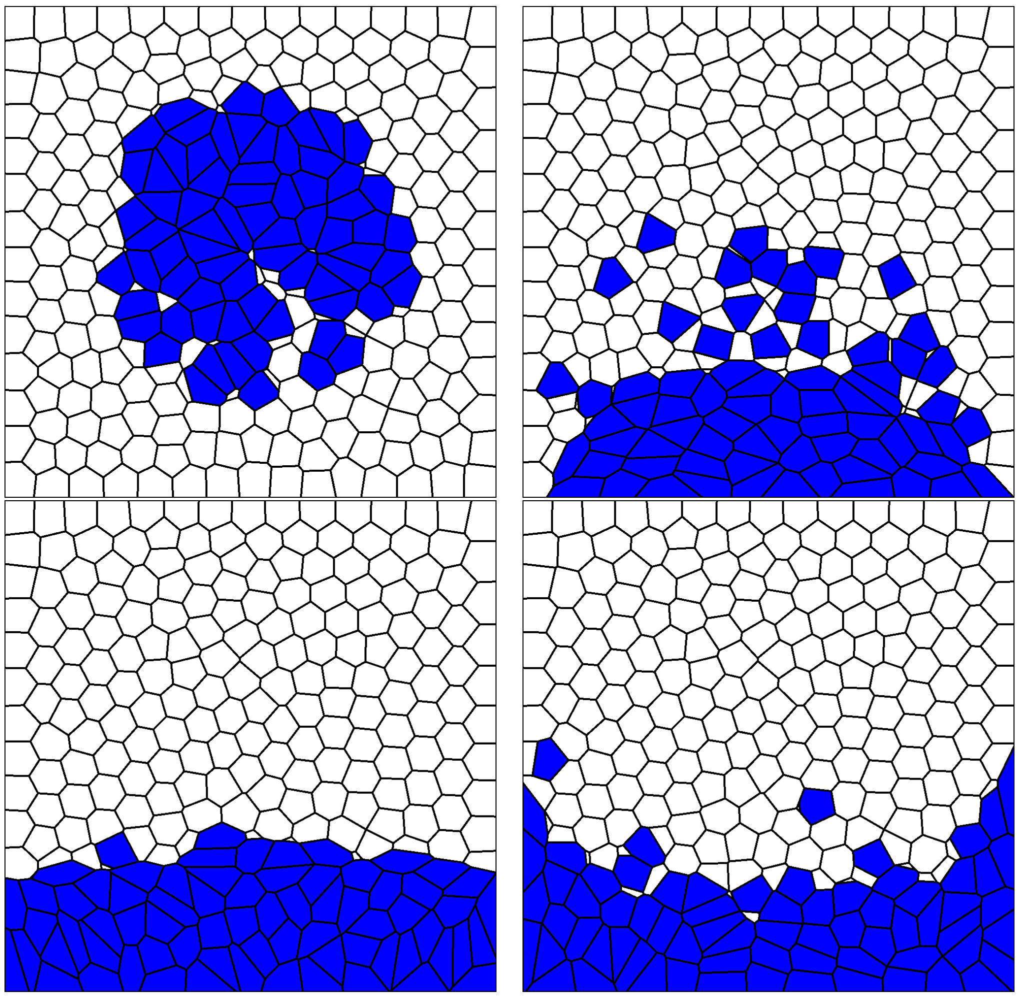

Figure 5: Four frames of a simulation of a fluid drop. These frames were produced usingXX particles of water, XX particles of air, $\epsilon = 0.004$, $dt = 0.002$ and each particle with a mass of 200.