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Sheet no. (1)

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- 1- Given the vectors  $\mathbf{A} = -6\mathbf{a}_x + 2\mathbf{a}_y - 4\mathbf{a}_z$  and  $\mathbf{B} = 4\mathbf{a}_x + 3\mathbf{a}_y - 2\mathbf{a}_z$  Find (a) a unit vector in the direction of  $\mathbf{A}+2\mathbf{B}$ ; (b) the magnitude of  $\mathbf{A}+2\mathbf{B}$ ; (c) the vector  $\mathbf{C}$  such that  $\mathbf{A}+\mathbf{B}+\mathbf{C}=\mathbf{0}$ .
- 2- The vectors  $\mathbf{A} = 4\mathbf{a}_x + 5\mathbf{a}_y - 2\mathbf{a}_z$  and  $\mathbf{B} = 2\mathbf{a}_x + 8\mathbf{a}_y + 3\mathbf{a}_z$  are represented by directed line segments that extend outward from the origin of a cartesian coordinate system. (a) What is the separation of their tips? (b) Find a unit vector in the direction of  $\mathbf{A}$ . (c) Find a vector  $\mathbf{C}$  that is parallel to  $\mathbf{A}$  and has the length of  $\mathbf{B}$ .
- 3- A certain vector field is specified by:
$$\mathbf{A} = \frac{100(x-y+z)}{x^2+y^2+z^2}\mathbf{a}_x - xyz\mathbf{a}_y + 8(x+y-z)\mathbf{a}_z$$
For the interval  $0 \leq x \leq 10$  along the line  $y = 1, z = 2$ , sketch the variation of (a)  $\mathbf{A}_x$  versus  $x$ ; (b)  $|\mathbf{A}|$  versus  $x$ .
- 4- At the point C (2,  $30^\circ$ , 5), a vector  $\mathbf{A}$  is expressed in cylindrical coordinates as  $20\mathbf{a}_\rho - 30\mathbf{a}_\phi + 10\mathbf{a}_z$ . (a) Find  $|\mathbf{A}|$  at C; (b) Calculate the angle between  $\mathbf{A}$  at C and the surface  $\rho = 2$ .
- 5- The surfaces of a volume are defined by  $\rho = 5$  and  $12$ ,  $\phi = 0.1\pi$  and  $0.4\pi$  and  $z = -1$  and  $3$ . (a) Find the length of the straight line connecting diametrically opposite corners of the volume. (b) Find the area of the six faces. (c) Find the volume enclosed.
- 6- Express the vector field  $\mathbf{W} = (x^2 - y^2)\mathbf{a}_y + xz\mathbf{a}_z$  in: (a) cylindrical coordinates at P ( $\rho = 6$ ,  $\phi = 60^\circ$ ,  $z = -4$ ); (b) spherical coordinates at Q ( $r = 4$ ,  $\theta = 30^\circ$ ,  $\phi = 120^\circ$ ).
- 7- At point B ( $r = 5$ ,  $\theta = 120^\circ$ ,  $\phi = 75^\circ$ ) a vector field has the value  $\mathbf{A} = -12\mathbf{a}_r - 5\mathbf{a}_\theta + 15\mathbf{a}_\phi$ . Find the vector component of  $\mathbf{A}$  that is: (a) normal to the surface  $r = 5$ , (b) tangent to the surface  $r = 5$ , (c) tangent to the cone  $\theta = 120^\circ$ ; (d) Find a unit vector that is perpendicular to  $\mathbf{A}$  and tangent to the cone  $\theta = 120^\circ$ .
- 8- A field is given in spherical coordinates as  $\mathbf{F} = [(\cos\theta) / r^2]\mathbf{a}_r + [(\sin\theta / r)]\mathbf{a}_\theta$ . (a) Express  $\mathbf{F}$  in terms of  $x, y, z, \mathbf{a}_x, \mathbf{a}_y$  and  $\mathbf{a}_z$ . (b) Evaluate  $\mathbf{F}$  at (1, 2, 3).
- 9- Two vectors are defined at point P as  $\mathbf{F} = 10\mathbf{a}_r - 3\mathbf{a}_\theta + 5\mathbf{a}_\phi$  and  $\mathbf{G} = 2\mathbf{a}_r + 5\mathbf{a}_\theta + 3\mathbf{a}_\phi$ . Determine (a)  $\mathbf{F} \cdot \mathbf{G}$ ; (b) the scalar component of  $\mathbf{G}$  in the  $\mathbf{F}$  direction at P; (c) the vector component of  $\mathbf{G}$  in the  $\mathbf{F}$  direction at P; (d)  $\mathbf{G} \times \mathbf{F}$ ; (e) a unit vector perpendicular to both  $\mathbf{F}$  and  $\mathbf{G}$  at P.