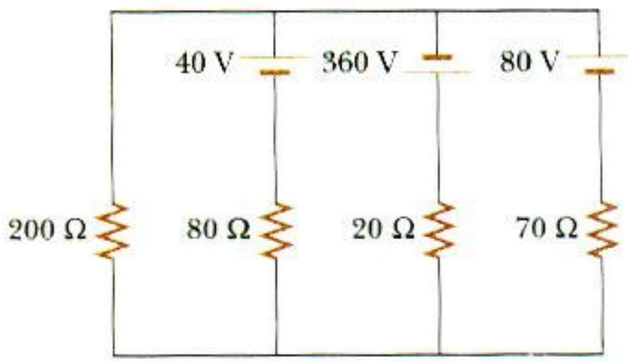
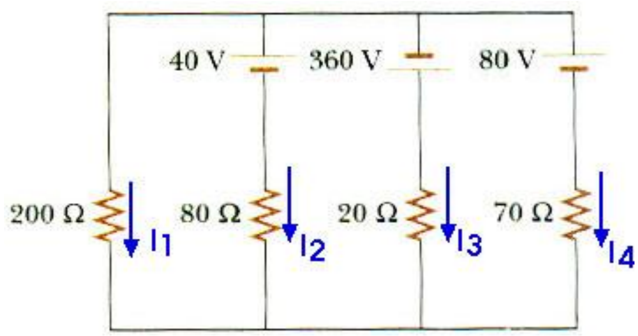
Homework #8

In the circuit of Figure below, determine the current in each resistor and the voltage across the 200-ohm resistor.



This requires the application of **Kirchoff's Rules**. Begin by assigning a current through each of the resistors.

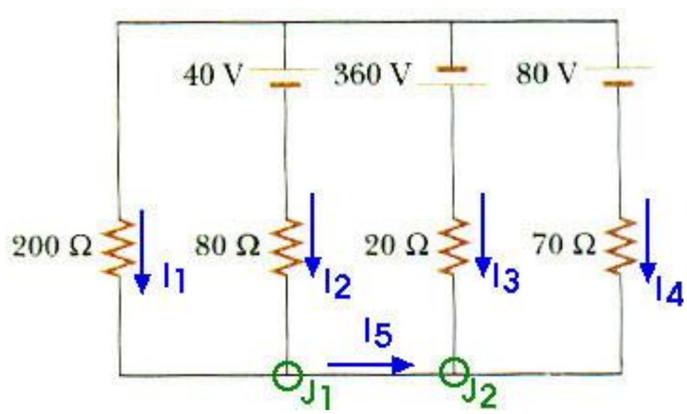


Apply Kirchoff's **Junction Rule** at the two junctions.

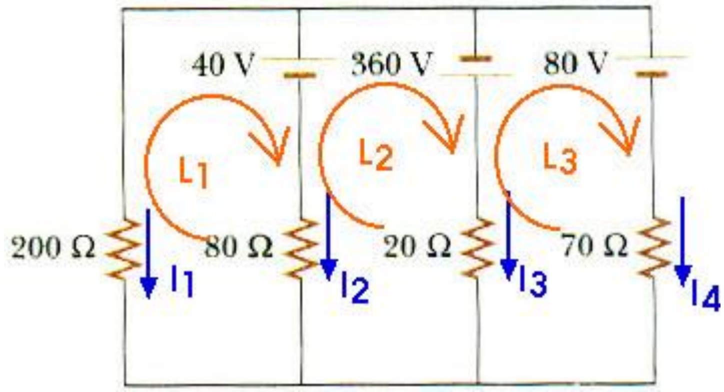
J1: $I_1 + I_2 = I_5$

J2: $I_5 + I_3 + I_4 = 0$

$I_1 + I_2 + I_3 + I_4 = 0$



Certainly one or more of our currents will turn out to be **negative**. (That's **okay!**) We have **four** unknowns and only one equation. We will have to get **three** more equations from Kirchoff's **Loop Rule.**



L1: - 40 V = I_2 (80 Ω) - I_1 (200 Ω)

L2: 40 V - 360 V = I_3 (20 Ω) - I_2 (80 Ω)

L3: - 360 V - 80 V = I_4 (70 Ω) - I_3 (20 Ω)

We are finished with the "Physics" of this problem. It's "just" math from here on. We have **four unknowns** so now we need only **four equations**,

$I_1 + I_2 + I_3 + I_4 = 0$

- 40 V = I_2 (80 Ω) - I_1 (200 Ω)

40 V - 360 V = I_3 (20 Ω) - I_2 (80 Ω)

- 360 V - 80 V = I_4 (70 Ω) - I_3 (20 Ω)

80 V = I_4 (70 Ω) - I_1 (200 Ω)

$$
\begin{array}{rrrrl}
I_1 & +I_2 & +I_3 & +I_4 & = 0 \\
-200I_1 & +80I_2 & & & = -40A \\
 & -80I_2 & +20I_3 & & = -320A \\
 & & -20I_3 & +70I_4 & = -440A
\end{array}
$$

$$
\begin{array}{rrrrl}
I_1 & +I_2 & +I_3 & +I_4 & = 0 \\
-20I_1 & +8I_2 & & & = -4 \\
 & -8I_2 & +2I_3 & & = -32 \\
 & & -2I_3 & +7I_4 & = -44
\end{array}
$$

$$
\begin{array}{r}
20I_1 + 20I_2 + 20I_3 + 20I_4 = 0 \\
-20I_1 + 8I_2 \qquad\qquad\qquad = -4 \\
\hline
28I_2 + 20I_3 + 20I_4 = -4
\end{array}
$$

$$
\begin{array}{r}
14I_2 + 10I_3 + 10I_4 = -2 \\
-8I_2 + 2I_3 \qquad\quad = -32 \\
-2I_3 + 7I_4 = -44
\end{array}
$$

$$
\begin{array}{r}
112I_2 + 80I_3 + 80I_4 = -16 \\
-112I_2 + 28I_3 \qquad\quad = -448 \\
\hline
108I_3 + 80I_4 = -464
\end{array}
$$

$$27I_3 + 20I_4 = -116$$

$$
\begin{array}{r}
27I_3 + 20I_4 = -116 \\
-2I_3 + 7I_4 = -44
\end{array}
$$

$$
\begin{array}{r}
54I_3 + 40I_4 = -232 \\
-54I_3 + 189I_4 = -1188 \\
\hline
229I_4 = -1420
\end{array}
$$

$$I_4 = -\frac{1420}{229}$$

$$I_4 = -6.201A$$

$$
\begin{aligned}
-2I_3 + 7A &= -44 \\
-2I_3 + 7(-6.201) &= -44 \\
-2I_3 - 43.406 &= -44 \\
-2I_3 &= -44 + 43.406 \\
-2I_3 &= -0.594 \\
I_3 &= \frac{0.594}{2} \\
I_3 &= +0.297
\end{aligned}
$$

$$-8I_2 + 2I_3 = -4$$

$$-8I_2 + 2(+0.297) = -4$$

$$-8I_2 + 0.594 = -4$$

$$-8I_2 = -4 - 0.594$$

$$-8I_2 = -4.594$$

$$I_2 = \frac{4.594}{8}$$

$$I_2 = 0.574A$$

$$-200I_1 + 80I_2 = -40$$

$$-200I_1 + 80(0.574) = -40$$

$$-200I_1 + 45.939 = -40$$

$$-200I_1 = -40 - 45.939$$

$$-200I_1 = -85.939$$

$$I_1 = \frac{85.939}{200}$$

$$I_1 = 0.430A$$

$$I_2 = 0.574A$$

$$I_3 = 0.297A$$

$$I_4 = -6.201A$$

Deciding how to choose a "loop" sometimes seems difficult. **Any** continuous loop is a candidate. We might have used a loop around the **outside** of the entire circuit diagram.