Data Structures & Algorithms 2
Tutorial 1
Introduction

OBJECTIVES

- Proof by Induction vs. Counter-example vs. Contradiction
- Recursion

Exercise 1

Prove the following formulas:

a) $\log(A^B) = B \log A$

b) $\log X < X$ for all $X > 0$

Exercise 2

Using Proof by induction, prove the following formulas:

a) $\sum_{i=1}^{N} (2i - 1) = N^2$

b) $\sum_{i=1}^{N} i^3 = \left(\sum_{i=1}^{N} i\right)^2$

c) $n^3 - n$ is divisible by 6 for $n >= 0$

You can make use the formula: $\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$

Exercise 3

- Prove by Contradiction the following :

a) For all integers n, if n^3+5 is odd then n is even.

b) If $a^2 + b^2 = c^2$ for integers a,b,c , then at least one of a or b is even.

- Prove the following statement by proving the contrapositive.

If $2^n - 1$ prime, then n is prime.

Exercise 4

Evaluate the following sums:

- $\sum_{i=0}^{\infty} \frac{1}{4^i}$
- $\sum_{i=0}^{\infty} \frac{i}{4^i}$
- $\sum_{i=0}^{\infty} \frac{i^2}{4^i}$

Exercise 5

Write a recursive algorithm that returns the number of 1s in the binary representation of *N*. Use the fact that this is equal to the number of 1 in the representation of *N/2* when N is even. If N is odd, there is an additional 1.

Exercise 6

Given a decimal number (For instance : 34892), write a recursive function to flip the number from right to left (→29843) (Please, don't use String functions)

Additional exercises

Exercise 7

Let F_i be the Fibonacci numbers defined as $F_{k+1}=F_k+F_{k-1}$ such that $F_0=1$and $F_1=1$
Prove the following:

- $\sum_{i=1}^{N-2} F_i = F_N - 2$
- $F_N < \phi^N$ such that $\phi = (1 + \sqrt{5})/2$ (ϕ is known as the Golden Ratio)
- $F_N = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ for any n>=1

Exercise 8

For a vending machine to return the change to the customer, it must return a minimal number of coins. Write a recursive function to determine the number of each type of coin to give back to the customer as part of the change.

(For example : 290 → 1x200DA + 1x50DA + 2x20DA)