The National Higher School of Artificial Intelligence	**2023-2024**	**Probability Semestre 3**

Midterm Exam (1h15)

Exercise 1 *Determine the distribution of the random variable $T = n - S$, where S is a real random variable that follows the $\mathcal{B}(n, p)$ distribution.*

Exercise 2 *Let $X \rightsquigarrow Geo(p), 0 < p < 1$. Find $\mathbb{E}\left[\frac{1}{2^X}\right]$.*

Exercise 3 *At the entrance to a restaurant, n people give their hats to the cloakroom. After the meal, they find their hats completely mixed up, and each person takes a hat at random. Let $X_k\,(k = 1, \cdots, n)$ be the random variable that takes the value 1 if the k^{th} person picks up his hat, and 0 otherwise. Let $S_n = X_1 + \cdots + X_n$ be the number of people who retrieved their hats.*

1. *Construct a probabilistic space describing this experiment.*
2. *Calculate $\mathbb{E}[S_n]$ and $Var(S_n)$.*

Exercise 4 *We consider a function f defined by*

$$f(x) = \frac{1}{2\theta} \log\left(\frac{\theta}{|x|}\right) \mathbb{I}_{[-\theta,\theta]}(x), \text{ where } \theta > 0.$$

1. *Show that the function f is a density of probability.*
2. *Let X be a random variable admitting the density f.*

   a. *Determine F_X the cumulative distribution function of X.*

   b. *Calcualte $\mathbb{E}\left[X^k\right]$ and deduce $Var(X)$.*