Worksheet N 1
Extrema of multivariable functions

Exercise 1. Find all local maxima and minima of :

$$f(x,y) = x^2 + xy + y^2 - 3x, \quad f_1(x,y) = x^2 + y^2$$
$$g(x,y) = xy - x^3 - y^2, \qquad g_1(x,y) = 4x^2 - 4xy + 2y^2 + 10x - 6y$$

Exercise 2: To treat a bacterial infection, the combined use of two chemical compounds is employed. Studies have shown that in the laboratory, the duration of the infection can be modelled by

$$D(x,y) = x^2 + 2y^2 - 18x - 24y + 2xy + 120,$$

where x is the dosage in mg of the first compound and y is the dosage in mg of the second. How can the duration of the infection be minimized?

Exercise 3: For a rectangular solid of volume $1000\,\mathrm{m}^3$, find the dimensions that will minimize the surface area. (Hint: Use the volume condition to write the surface area as a function of just two variables.)

Exercise 4: For a rectangle whose perimeter is $20\,\mathrm{m}$, use the Lagrange multiplier method to find the dimensions that will maximize the area.

Exercise 5: Find the points on the circle $x^2 + y^2 = 80$ which are closest to and farthest from the point $(1,2)$.

Exercise 6: Maximize (and minimize):

$$\begin{cases} f(x,y,z) = x + z, \\ \text{Under the constraint:} \\ g(x,y,z) = x^2 + y^2 + z^2 = 1. \end{cases}$$

Exercise 7: Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$