Theory of Computing 2023/2024

Midterm Exam - 90 mins

2ND YEAR - ENSIA

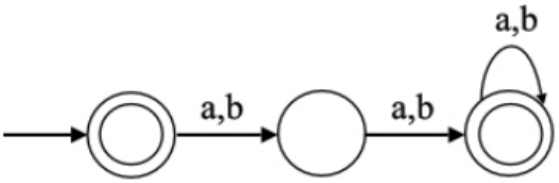
IMPORTANT NOTES

- Closed-book Exam : No phones on tables, PCs, nor Books.
- Bring your own stuff (Pen, Corrector, Pencil, Ruler, …)
- Strictly, No talking to each other (Can I have a pen, ruler, charger, cable … **NO**)
- Lecturers will **not answer nor discuss** any query during the exam.
- **Be brief** when answering the questions. Long paragraphs will be ignored.
- Bad or non-clear writing, will not be appreciated and you **will** be penalized.

EXERCISES : Answer all questions.

Exercise 1 (8 points) [DFA & NFA] :

1. [1 point] Describe informally the language for the following machine :

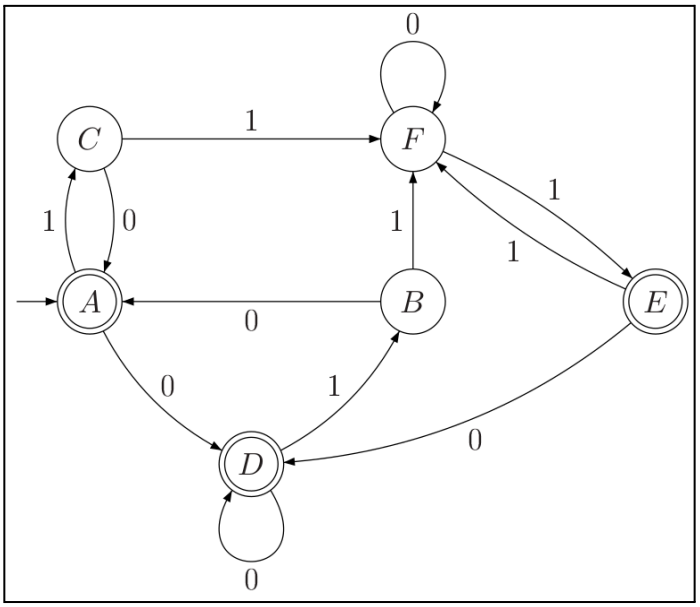


2. [2 points] Draw the DFAs for the following language over the alphabet {0,1} :
   a. Alternating 0 and 1 (Examples for accepted strings 101, 10,01 1010 , … , not accepted strings 0, 00, 01110)
   b. The language where the word contains exactly two blocks of 1's and both these blocks have odd length. For example, 1010 and 01001110000 should be accepted, but not 00110001 or 10101.

3. [2 points] Draw the NFAs for the following language :
   a. $\{wc^{3n+2} \mid w \in \{a,b\}^*, |w| \bmod 3 = 1 \text{ and } n >= 1\}$(Length of w modulus 3 is 1), The alphabet is {a,b,c}
   b. $\{b^m w \mid m>=2, w \in \{0,1\}^* \text{ and } |w| + m \text{ is even }\}$. The alphabet is {b,0,1}

4. [2 points] For the alphabet {0,9}, construct the DFA for the language where the sum of the digits is a multiple of 3. (Accepted word 3012, 3+0+1+2=6 which is divisible by 3)

5. [1 point] Minimize the following DFA, no need to show steps :



Exercise 2 (4 points) [Regular Expressions] :

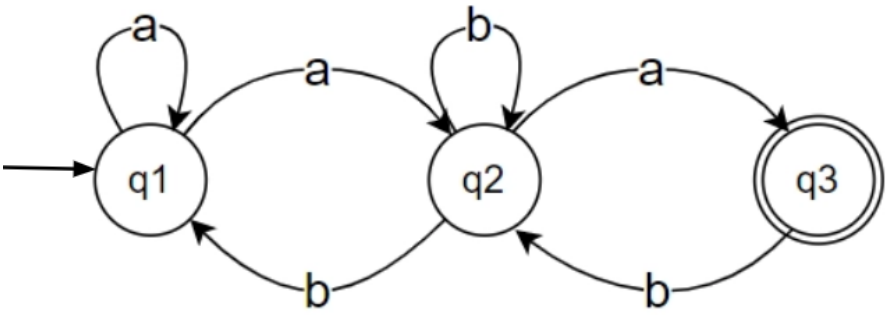
1. [3 points] Write regular expressions for the following regular languages, the alphabet is {0,1}

   a. [1 point] All words which start and end with the same letter

   b. [1 point] Words must start and end with 1. Any 0 in the word must be immediately preceded and followed by 1. The length of the word must be at least 4.

   c. [1 point] The language of all strings not containing the substring 000.

2. [1 points] Convert the following NFA to regular expressions showing the steps



Exercise 3 (4 points) [Regular Languages] :

Prove that the following languages are **not regular**:

1. [1 points] The number of 0 is not the same as the number of 1.

2. [1 points] Let Σ = {0, 1, +, =} and ADD = {x = y + z | x, y, z are binary integers, and x is the sum of y and z}.

3. [2 points] L ={0^k | k is a prime number }. Provided that the set of prime numbers is infinite.(no need to prove this).

Exercise 4 (4 points) [CFG] :

Generate the Context Free grammar for the following languages:

1. [2 point] The number of ‘a’ is not the same as the number of ‘b’. (Alphabet is {a,b})

2. [2 point] The complement of the language a^nb^n such that n>0 (The big set of {a,b}* , Alphabet is {a,b})