Complexity Theory 2022/2023

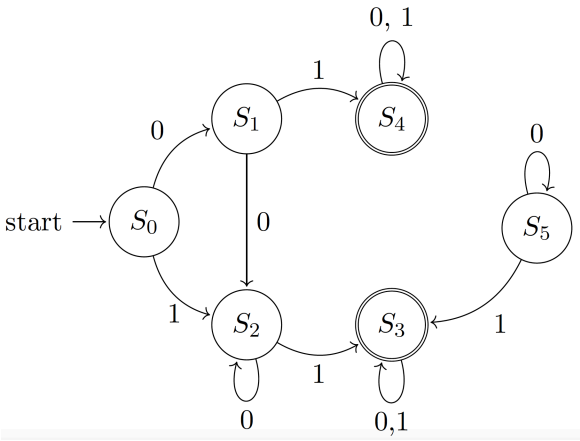
Midterm Exam - 90 mins

2ND YEAR - ENSIA

EXERCISES : Answer all questions.

Exercise 1 (12 points) :

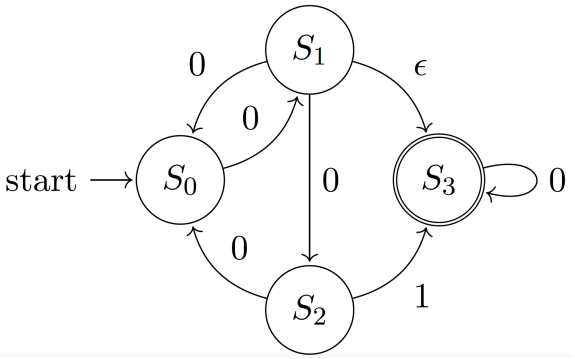
1. [4 points] Find a DFA (Deterministic Finite Automaton) for the following languages. Here, the set of alphabet is ∑={0, 1}:
   a. L = {w | w is a binary string that is multiple of 3}.
   b. L = {w | w contains an even number of zeroes **and** an odd number of ones}.
2. [4 points] Find an NFA (Non-deterministic Finite automaton) for the following languages.
   a. L = {w Ė Σ^* | w contains two 0s or exactly two 1s, and or is exclusive}.
   b. L = {w | w are of the form $0^*1^*0^+$} (note: $0^+=0^*0=00^*$).
3. [4 points] Minimize the following DFA. Here, the set of alphabet is ∑={a, b}:



Exercise 2 (8 points) :

1. [5 points] Write regular expressions for the following regular languages:
   a. [2 pts] {w | w contains an even number of 0s, or w contains exactly two 1s} (here or is logical or).
   b. [3 pts] Give a regular expression for the language L containing all strings in a* b* whose length is a multiple of three. E.g., L contains aaaabb but does not contain ababab or aaabb.

2. [3 points] Convert the following NFA into a regular expression:



January 19, 2023

Exercise 3 (13 points) :

1. [1 point] Demonstrate that the following context-free grammar is ambiguous:

   S → aXY

   X → bYb

   Y → X | ε

2. [3 points] Provide the CNF (Chomsky-Normal Form) for the following production rules:

   S → aXbX

   X → aY | bY | ε

   Y → X | c

3. [6 points] Provide context-free grammar's production rules for the following languages:
   a. L = $\{0^n1^n2^m3^m \mid n\geq 0 \text{ and } m\geq 0\}$.
   b. L = $\{0^n1^m2^m3^n \mid n\geq 0 \text{ and } m\geq 0\}$.
   c. L = $\{a^i\, b^j\, c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$.

4. [3 points] An elevator is controlled by two commands: "↑" to raise the elevator one floor and "↓" to lower the elevator one floor. Suppose the number of floors in the building is infinite and the elevator starts at the ground floor. Find the CFG that generates valid command sequences verifying the following conditions:

   a. The elevator never descends below the ground floor.

   b. The elevator must always return to the ground floor at the end of the control sequence.

   c. The empty control sequence is valid.

Exercise 4 (7 points) :

1. [4 points] Apply the pumping lemma to prove that the following languages are non-regular:
   a. L = $\{0^{x^n} \mid n\geq 0\}$ (0 is repeated x^n times).

   b. L = $\{0^m1^n \mid m<\sqrt{n} \text{ or } n<\sqrt{m}\}$ (Hint: $\sqrt{k^2+1} > k$, $\forall k\geq 0$).

2. [2 points] Draw a pushdown automaton for the following language L = $\{a^ib^jc^k \mid j+k\geq i\}$. Only one symbol is allowed to be pushed/popped to/from the stack at a time.

3. [1 point] Convert the following context-free grammar into a pushdown automaton:

   S → aXc | ab

   X → SX | ε