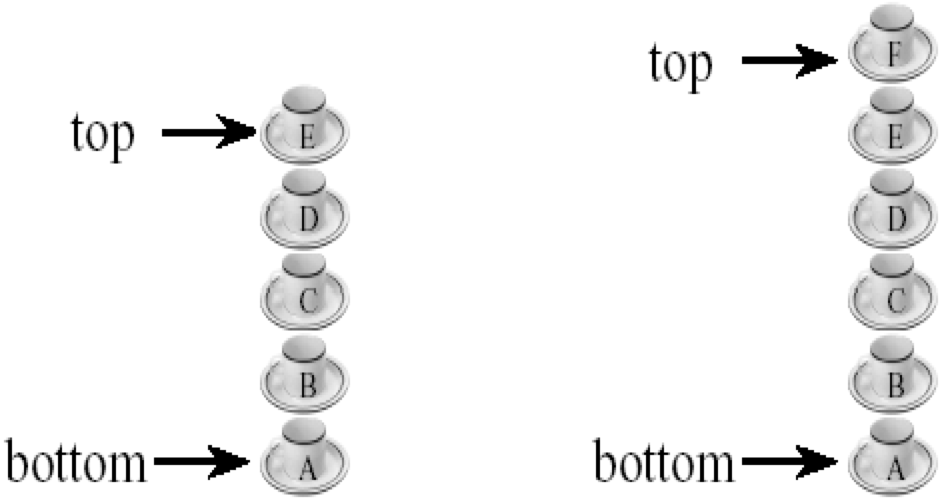
Stacks

CSE, POSTECH

Stacks

- Linear list
- One end is called top.
- Other end is called bottom.
- Additions to and removals from the top end only.

Stack of Cups



- Add a cup on the stack.
- Remove a cup from the stack.
- A stack is a LIFO (Last-In, First-Out) list.
- Read Example 8.1

Observations on Stack & Linear List

- Stack is a restricted version of linear list
  - All the stack operations can be performed as linear list operations
- If we designate the left end of the list as the stack bottom and the right as the stack top
  - Stack add (push) operation is equivalent to inserting at the right end of a linear list
  - Stack delete (pop) operation is equivalent to deleting from the right end of a linear list

Stack ADT

AbstractDataType stack {

instances

linear list of elements; one end is the bottom; the other is the top.

operations

empty() : Return true if stack is empty, return false otherwise;

size() : Return the number of elements in the stack;

top() : Return top element of stack;

pop() : Remove the top element from the stack;

push(x) : Add element x at the top of the stack;

}

- See the C++ abstract class stack definition in Program 8.1

Derived Classes and Inheritance

- Stack can be defined as a special type of linear list
- Class B may inherit members of class A in one of three basic modes: public, protected and private

```
class B: public A
```

  - protected members of A become protected members of B
  - public members of A become public members of B
  - cannot access private members of A

```
class B: public A, private C
```

  - public and protected members of C become private members of B

Array-based Representation of Stack

- Since stack is a restricted version of linear list, we can use an array-based representation of linear list (given in Section 5.3.3) to represent stack
- Top of stack → `element[length-1]`
- Bottom of stack → `element[0]`
- The class derivedArrayStack defined in Program 8.2 is a derived class of arrayList<T> (Program 5.3)

Derived Array-based class Stack

```
template<class T>       // program 8.2
class derivedArrayStack : private arrayList<T>, public stack<T>
{
   public:
      derivedArrayStack(int initialCapacity = 10)
        : arrayList<T> (initialCapacity) {}
      bool empty() const
           {return arrayList<T>::empty();}
      int size() const
          {return arrayList<T>::size();}
      T& top()
         {
            if (arrayList<T>::empty())
               throw stackEmpty();
            return get(arrayList<T>::size() - 1);
         }
      void pop()
           {
              if (arrayList<T>::empty())
                 throw stackEmpty();
              erase(arrayList<T>::size() - 1);
           }
      void push(const T& theElement)
           {insert(arrayList<T>::size(), theElement);}
};
```

Base Class arrayStack

- We learned that a stack class can be defined by extending an arrayList class as in Program 8.2
- However, this is not a very efficient implementation
- A faster implementation is to develop a base class that uses an array stack to hold the stack elements
- Program 8.4 is a faster implementation of an array stack
- Read Section 8.3.2

Efficiency of Array-based Representation

- Array-based representation of a stack can waste space when multiple stacks are to coexist
- An exception is when only two stacks are to coexist
- How do we implement two stacks in an array?
  - Fix the bottom of one stack at position 0
  - Fix the bottom of the other at position MaxSize-1
  - The two stacks grow toward the middle of the array
- See Figure 8.4

Linked Representation of Stack

- Multiple stacks can be represented efficiently using a chain for each stack
- Which end of chain should be the stack top?
  - If we use the right end of the chain as the stack top, then stack operations push and pop are implemented using chain operations *insert(n,x)* and *erase(n-1)* where n is the number of nodes in the chain → **$\Theta(n)$** time
  - If we use the left end of the chain as the stack top, then *insert(0,x)* and *erase(0)* → **$\Theta(1)$** time
- What should the answer be?
  - **Left end!**
- See Program 8.5 for linked stack definition

Application: Parenthesis Matching

- Problem: match the left and right parentheses in a character string
- (a*(b+c)+d)
  - Left parentheses: position 0 and 3
  - Right parentheses: position 7 and 10
  - Left at position 0 matches with right at position 10
- (a+b))*((c+d)
  - (0,4)
  - Right parenthesis at 5 has no matching left parenthesis
  - (8,12)
  - Left parenthesis at 7 has no matching right parenthesis

Parenthesis Matching

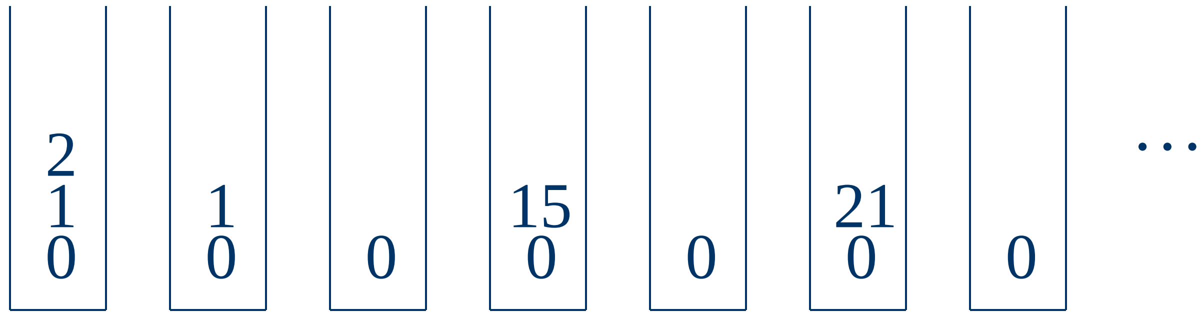
- (((a+b)*c+d-e)/(f+g)-(h+j)*(k-1))/(m-n)
  - Output pairs (u,v) such that the left parenthesis at position u is matched with the right parenthesis at v.

    (2,6) (1,13) (15,19) (21,25) (27,31) (0,32) (34,38)
- **How do we implement this using a stack?**
  1. Scan expression from left to right
  2. When a left parenthesis is encountered, add its position to the stack
  3. When a right parenthesis is encountered, remove matching position from the stack

Example of Parenthesis Matching

(((a+b)*c+d-e)/(f+g)-(h+j)*(k-1))/(m-n)

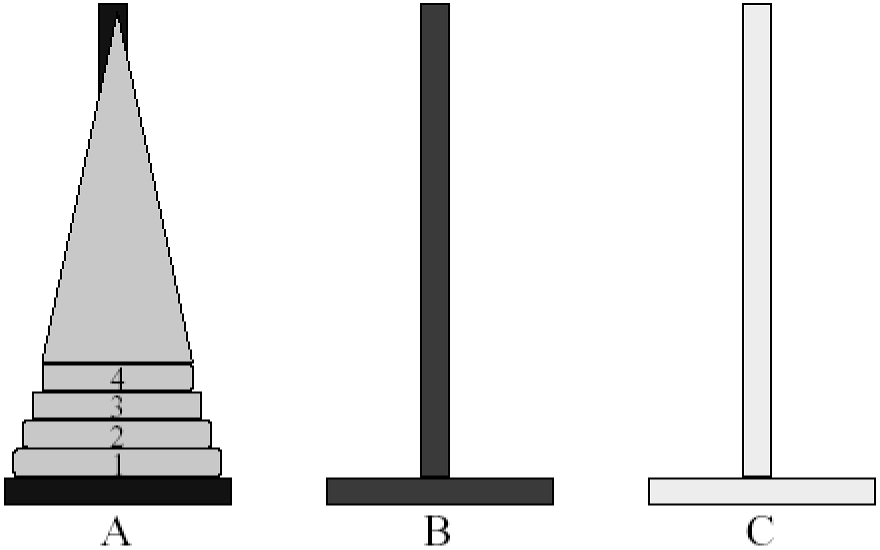
stack



output (2,6) (1,13) (15,19) (21,25)…

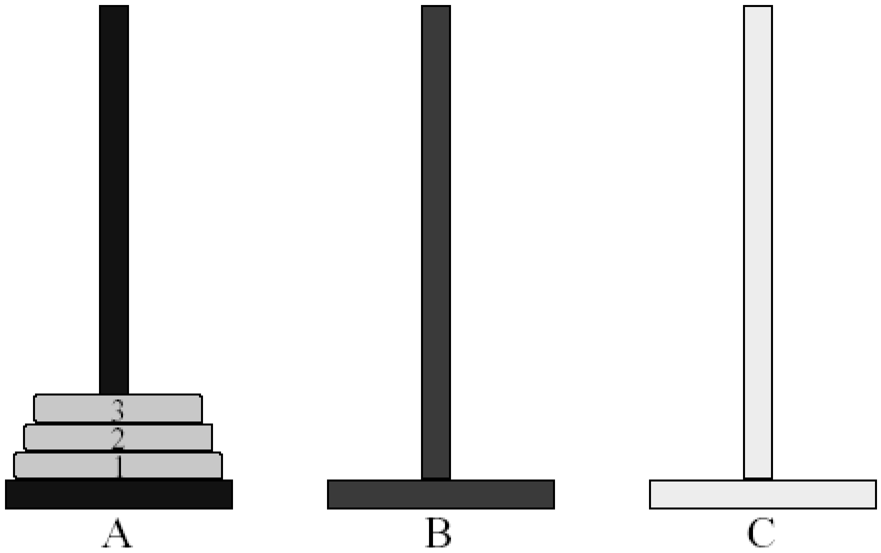
– See Program 8.6
– Do the same for (a-b)*(c+d/(e-f))/(g+h)

Application: Towers of Hanoi

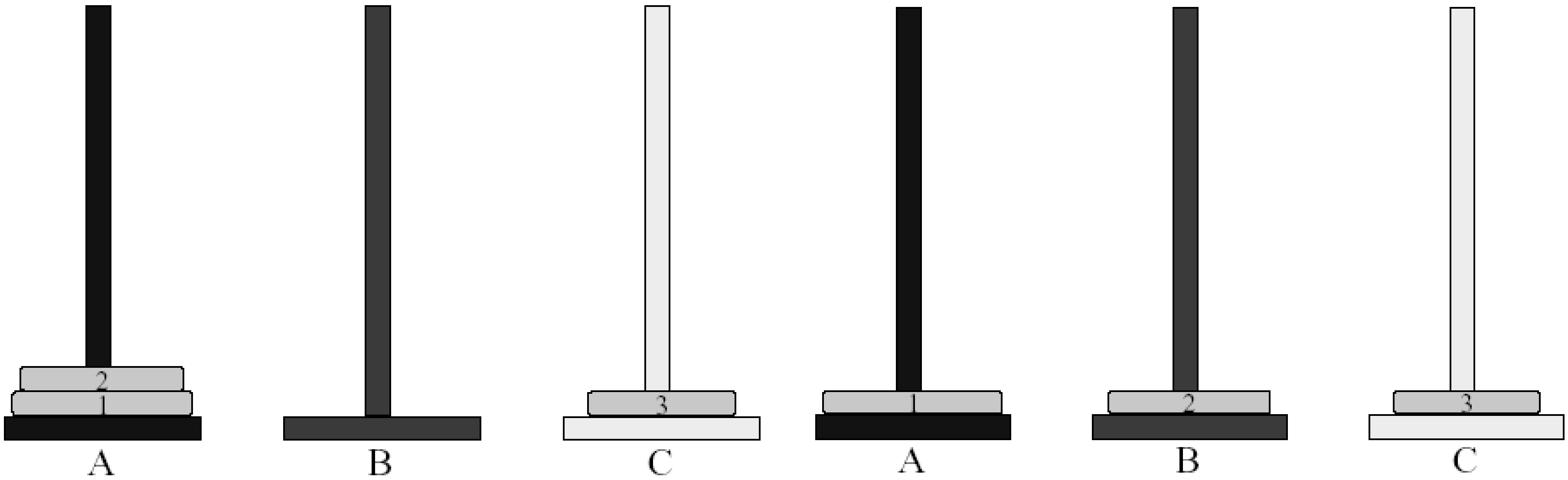


- Read the ancient Tower of Brahma ritual (p. 285)
- ***n*** disks to be moved from tower **A** to tower **C** with the following restrictions:
  - Move 1 disk at a time
  - Cannot place larger disk on top of a smaller one

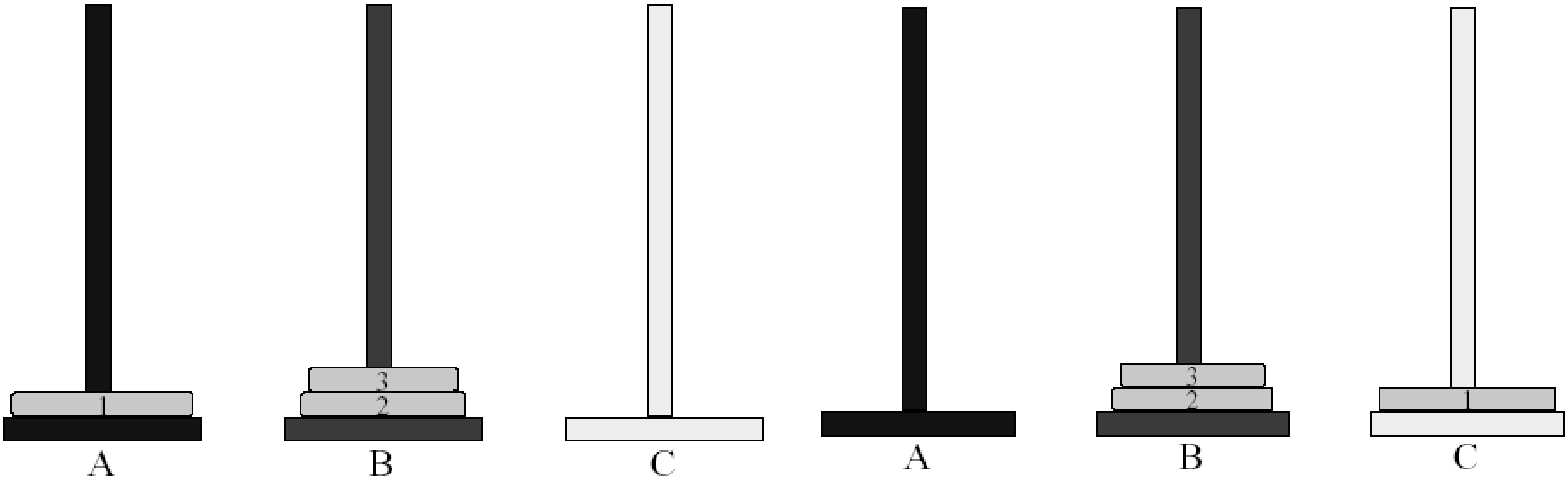
Let’s solve the problem for 3 disks



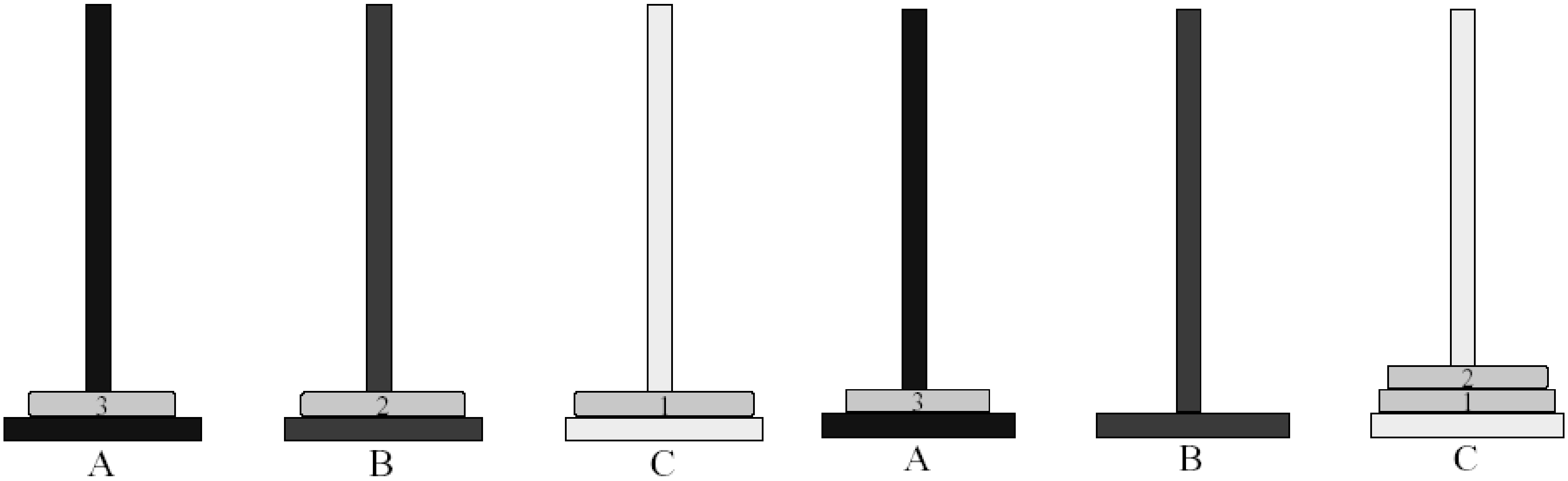
Towers of Hanoi (1, 2)



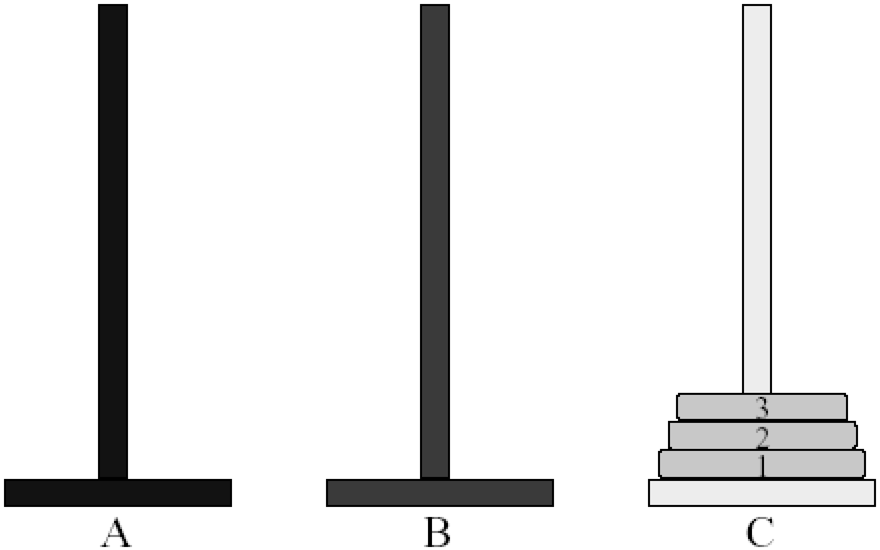
Towers of Hanoi (3, 4)



Towers of Hanoi (5, 6)



Towers of Hanoi (7)



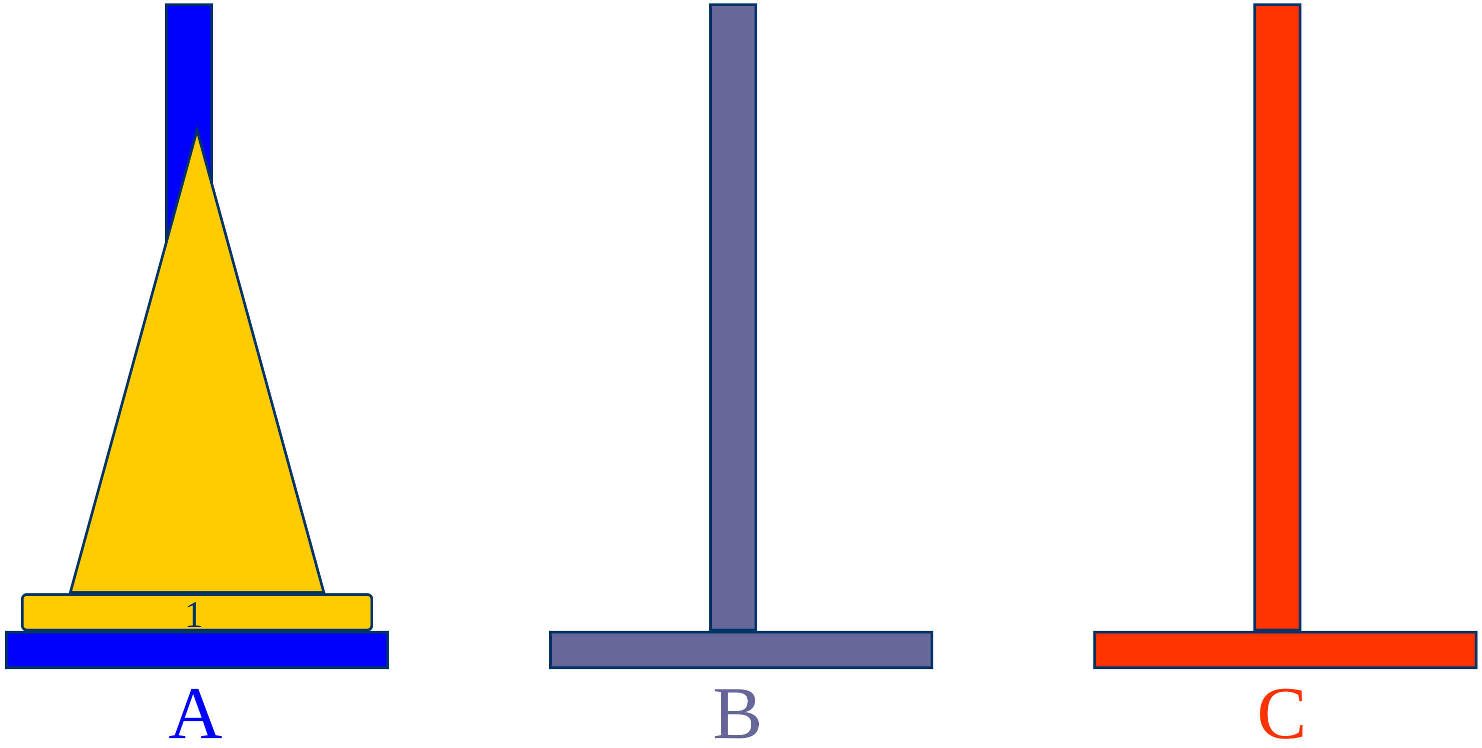
- So, how many moves are needed for solving 3-disk Towers of Hanoi problem?

  ➔ 7

Time complexity for Towers of Hanoi

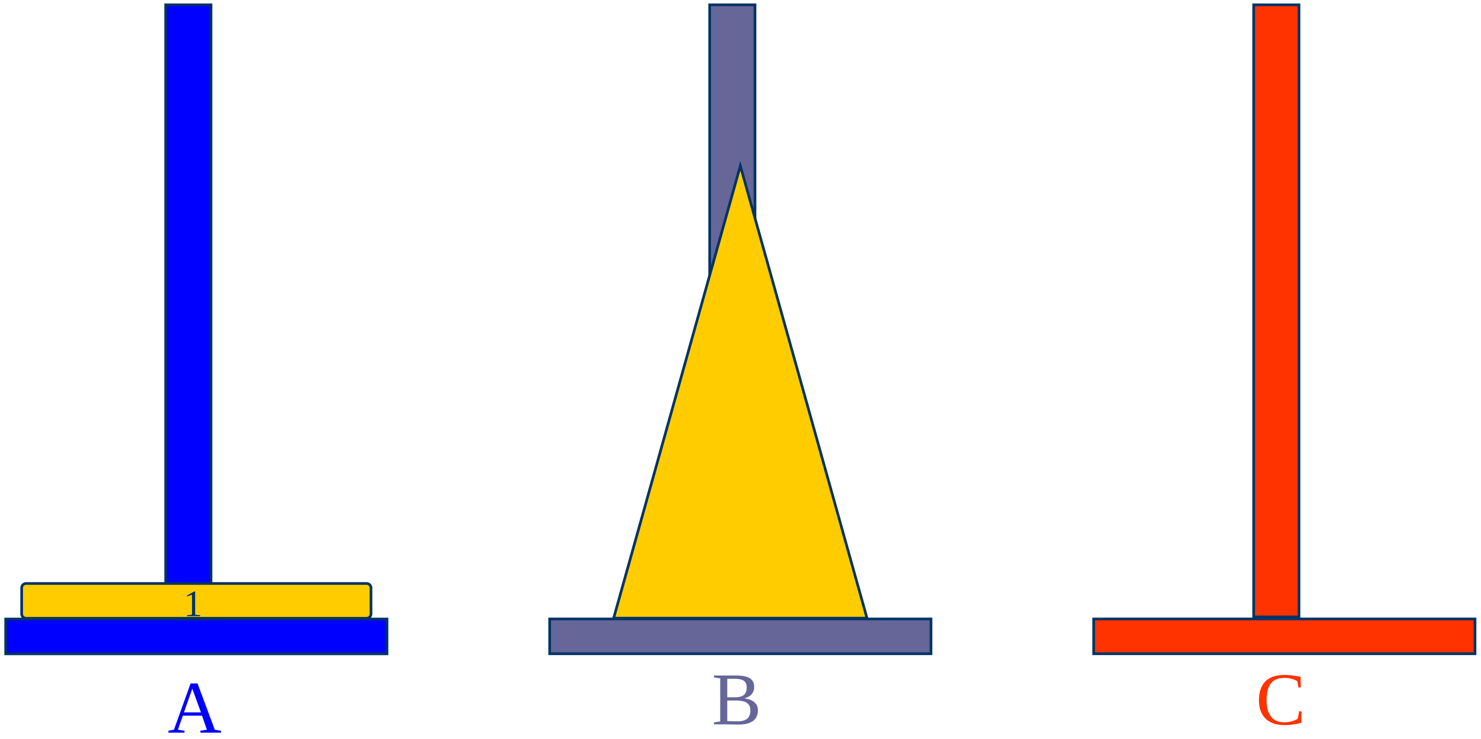
- A very elegant solution is to use **recursion.** See Program 8.7 for Towers of Hanoi recursive function
- The minimum number of moves required is $\boldsymbol{2^n-1}$
- Time complexity for Towers of Hanoi is $\Theta(2^n)$, which is exponential!
- Since disks are removed from each tower in a LIFO manner, each tower can be represented as a stack
- See Program 8.8 for Towers of Hanoi using stacks

Recursive Solution



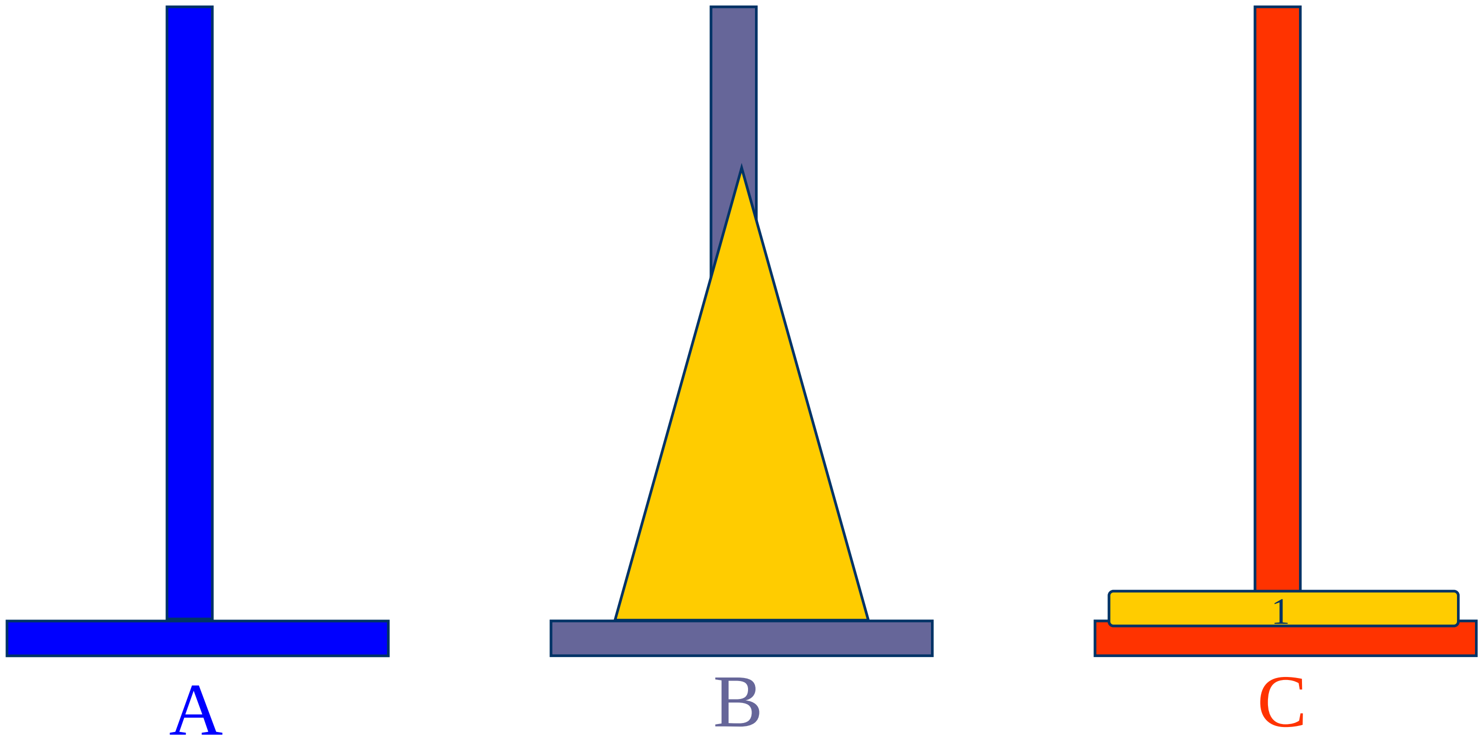
- **n > 0** gold disks to be moved from A to C using B
- move top **n-1** disks from A to B using C

Recursive Solution



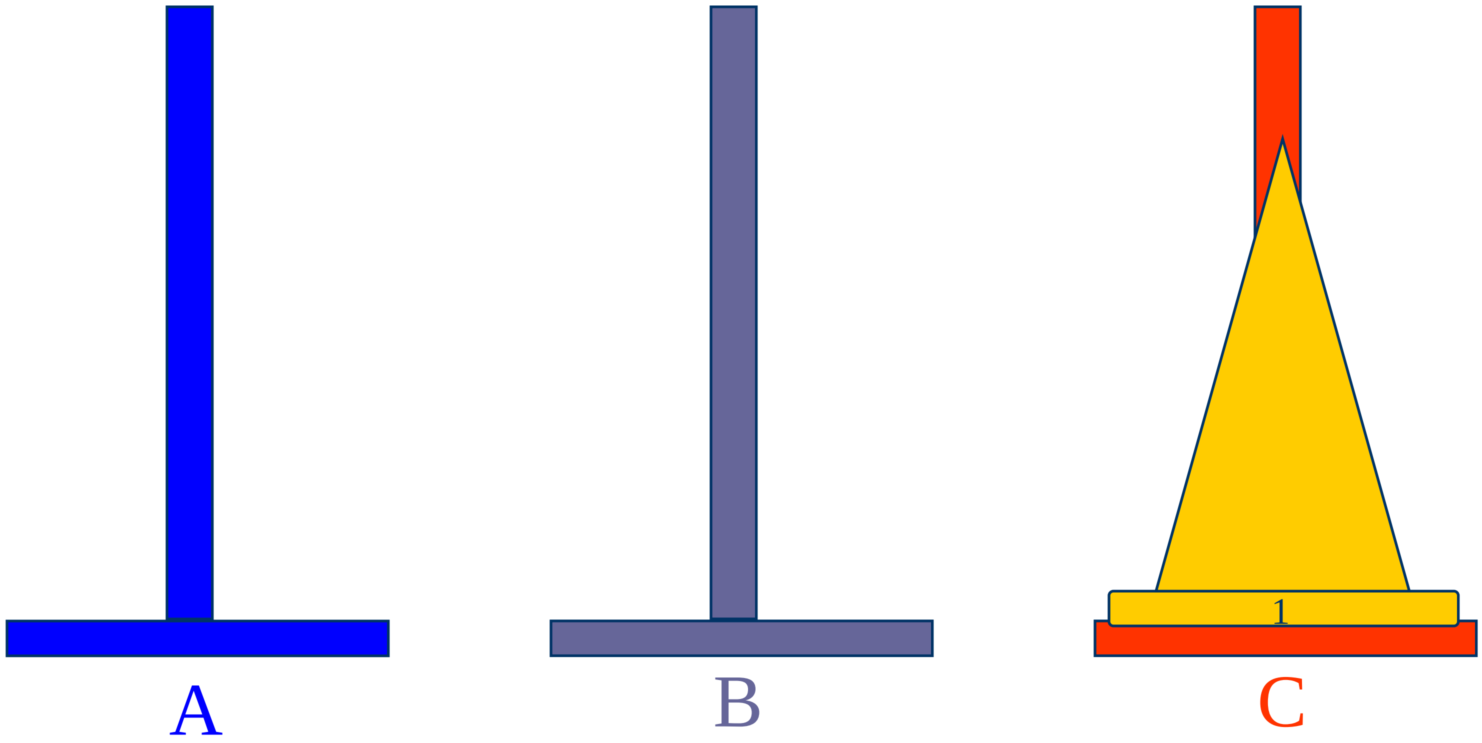
- move top disk from A to C

Recursive Solution



- move top n-1 disks from B to C using A

Recursive Solution



- moves(n) = 0 when n = 0
- moves(n) = 2*moves(n-1) + 1 = 2^n-1 when n > 0

64-disk Towers of Hanoi Problem

- How many moves is required to move 64 disks?

  ➔ moves(64) = $2^{64}-1 = 1.8 * 10^{19}$ (approximately)
- How long would it take to move 64 gold disks by the Brahma priests?

  ➔ At 1 disk move/min, the priests will take about **$3.4 * 10^{13}$ years**.

  ➔ "According to legend, the world will come to an end when the priests have competed their task"
- How long would it take to move 64 disks by Pentium 5?

  ➔ Performing 10^9 moves/second, a Pentium 5 would take about **570 years** to complete.

  - try running hanoiRecursive.cpp on your computer using n=3, 4, 10, 20, 30 and find out how long it takes to run

Application: Rearranging Railroad Cars

- Read the problem on p. 289
- Rearrange the cars at a shunting yard that has an input track, and output track, and *k* holding tracks between the input and output tracks
- See Figure 8.6 for a three-track example
- See Figure 8.7 for track states
- See Program 8.9, 8.10 and 8.11

READING

- Read 8.5.4 Switch Box Routing Problem
- Read 8.5.5 Offline Equivalence Problem
- Read 8.5.6 Rat in a Maze
- Read all of Chapter 8