Using Agent Based Modeling To Explore The Role Of Spatial Competition On The Behaviour Of Rival Firms, Selling Homogeneous Products, In Two-Dimensional Markets With Non-Uniform Population Distributions.

Tarek Thomas Bou Nassif

MInf Project (Part 2) Report
Master of Informatics
School of Informatics
University of Edinburgh

2023

Abstract

In order to investigate the conditions and behaviours that drive rival firms selling homogeneous products to settle in close proximity to one another, a comprehensive ABM package was implemented. Building on previous works in market competition (Harold Hotelling [8]) and ABM as a means to explore spatial competition (Bernard [2]), an ABM was developed supplementing previous works with the inclusion of non-uniform population density, random firm price initialisation and the modeling of population centers. As a result of repeated simulations, emergent behaviour in regards to the evolution of firm price, position, market share and overall market equilibrium have been derived. This has provided deeper insight into the dynamics of market competition while also further qualifying ABM as an effective methodology in experimental economics.

Table of Contents

Chapter 1

Introduction

1.1 Coffee, Gas And Fast Food - A Phenomena

Any individual that has filled their vehicle with gas, purchased a coffee in a busy population centre, or eaten in a fast-food chain will likely have noticed a key similarity shared amongst each of these entirely separate and disconnected experiences. Within each example, one seldom fails to notice the congestion of competing firms, each setting up operation in incredibly close proximity to rivals, while selling similar, almost substitute, products.

Upon initial observation, one finds themselves questioning the logic driving the decisions taken to carry out business so geographically close to a direct competitor. After all, proximity to a rival firm selling a similar product to the same customer pool, would lead to an assumed increase in cost of customer acquisition. As a result, the common observer of this phenomenon, remains in possession of a pressing unanswered question.

> *Why don't competing firms spread out, to each individually capture the uncontested demand of customers?*

One can better rephrase the question to be more specific about the observations and behaviours that have triggered the initial curiosity.

> *Why do rival firms congregate in close geographic proximity, especially when each firm produces and distributes near substitute products or services?*

It is easy to rationalise these observations as results purely derived from inherent characteristics of the geographical location in question. Population centres, tourist destinations, places with a dense populace of customers inclined towards the consumption of a specific product or service, are bound to draw in the business of rival firms. After all, sharing a dense pool of customer demand may often lead to larger profits, as opposed to individually claiming uncontested, but more shallow pools of demand.

However, a locations inclination to support economic activity, is not the only factor to be considered, since the congregation of competing businesses is a phenomenon that exists in a wide variety of geographic and economic contexts.

Yes, one will find a franchise of every notable coffee brand near the bustling crowds of Leister Square, but one will also find clusters of identical competing gas stations clustered around locations as remote from economic activity as they are barren of customer populations.

This piece of research seeks to explore this observation and answer these fundamental questions by leveraging agent based modeling (ABM) to simulate and study the fundamental idea of spatial competition.

1.2 Types Of Market Competition

Because this study is motivated by the observation of behaviours enacted by competing firms, one must first examine the general idea of economic competition, and its forms taken in a free market.

To begin, the most recognized form of competition is price competition. This entails the leveraging of price changes to manipulate the interest and demand of consumers in a market. Rival firms will set their price to simultaneously maximise profit, but also undercut competition. This type of competition appeals to the idea that customers are driven by a fundamental desire to minimize their cost.

Rival firms will also compete regarding non-price factors. These factors include various characteristics of a good or service, such as quality, marketing, packaging, and customer experience. This type of competition leverages variance in customer needs and takes advantage of the fact that human economic decision making is not exclusively driven by a core desire to minimise cost. For example, humans develop brand loyalty and are willing to pay more for a product perceived as more luxurious. This general idea is supported by the article ”The New Science of Customer Emotions” written by Scott Magids, Alan Zorfas, and Daniel Leemon [9]. This article studied the relationship between a customers emotional perception of their experience with a firm and their propensity to purchase from that firm. The study demonstrated that emotions had a strong correlation to a customers consumption from a firm and as a result showed that price alone was not the sole contributor.

Competition need not be tied specifically to aspects of a good or service. Competition can also entail the behaviour of firms. This can be seen in monopolistic competition, where a market is populated with several smaller rival firms each selling similar, yet clearly differentiated products. Unlike price and non-price competition, customer demand is regulated by the abundant availability of choice. Price is regulated by the fact that each firm has some market share, but due to an abundance of choice, cannot dramatically increase price. In this case, choice induces a market-wide elasticity of demand, thus ensuring lower fluctuations in price.

In other cases, the dynamics of competition are instead parameterized by co-operation. This is evident in an oligopoly. In this case the market is dominated by a small coalition of firms that leverage co-operative behaviour such as collusion and price fixing to maintain collective market share. In the extreme case the market may be controlled by a Monopoly, where a single firm controls the entire market.

A deeper dive into what category of market competition the initial observations belong to will be carried out in the literature review (Chapter 2), however it must be ultimately understood that market competition is an incredibly complex and versatile dynamic to study and model. At any given moment price, quality, geographic location, consumer preference, firm strategy, and government intervention, could all simultaneously influence the current state of a competing market. It is this complexity that motivates the more specific study of spatial competition as a means to explain the tendency of firms to settle close to one another.

1.3 Spacial Competition & Harold Hotelling

Spatial competition is a form of market competition where rival firms leverage geographic location within a market to acquire market share. Instead of using strategic co-operation between firms, or variations in the fundamental properties of a good or service, firms seek to position themselves within a market to use proximity to consumers to attract their business.

Spatial competition was explored and popularised by Harold Hotelling, an established mathematician, statistician, and economist. Hotelling explored this type of market competition, through his model of spatial competition, described in his pioneering paper "Stability in Competition" [8].

His theory will be explored in depth in the literature review. However, to sum up his model of spatial competition considered the case two rival firms competing against each other for market share. The market was represented by a straight line with consumer population being uniformly distributed along each unit of length, with each firm selling products considered to be perfect substitutes. Firms would only be able to leverage their location along the line to attract consumer business. Hotelling showed, that given this experimental setup, the firms would reach an equilibrium at the centre of the line, equally dividing the market.

Ultimately Hotelling was able to prove the convergence of his model to a central equilibrium using an analytical approach subject to the defined conditions he laid out. This was a pioneering exploration regarding why rival firms settle close to each other, however his approach was not void of critique. His research remains a key study in the investigation of spatial competition on the market equilibrium. However given the simplicity of the system defined in his approach, this study needs to be supplemented in order to derive conclusions more applicable to real world markets.

To better explore the dynamics of behaviours shared by competing firms, one must consider a methodology that can capture various forms of market competition, as well as geographical market landscapes with varied population densities. Doing so will permit a more fruitful analysis and understanding of the initial observation and will yield greater ability to generalise to the real world. For this reason, agent-based modelling will be the central approach of this exploration.

1.4 Building an Agent Based Model

Hotelling's analytical approach was sufficient for the case of a two-firm system, in a one-dimensional market with uniform population density. This, combined with the fact that firms could only leverage location, lead to a direct analytical solution by solving the system of linear constraints that defined his system. This approach does not lead to such clear solutions in the case of more complex systems. More complex systems lead to an explosion in constraints and as a result are difficult to use for the study of deeper and more realistic experiments.

As a result, ABM was chosen as the optimal methodology for this exploration. ABM was first popularised in the book "Artificial Societies: Social Science from the Bottom Up" [5] and has progressively evolved as a computational modelling technique with applications in fields such as financial, epidemic and traffic modelling.

ABM is a modelling technique that leverages the simulation of the repeated interactions between complex agents constrained by a defined set of rules and functions that inform decision making. ABM carries out simulations between agents in the context of an environment. This environment may change over the course of the simulation and can play a role in the interactions carried out. As a result of repeated simulations, the trends in agent behaviour can be used to extrapolate and study the dynamics of the system the model was designed to emulate.

1.5 Objectives

Overall, the initial observations that inspired this study, the original work of Harold Hotelling [8], and the methodology to be taken have been established and discussed. Before engaging in a more thorough discussion regarding previous works, this section will highlight the overall objectives of this exploration. The objectives will correspond to two main subject areas: The creation of an ABM package and testing suite and the study of emergent behaviours.

To begin, the ultimate goal is to build a modular and agile ABM package that is capable of carrying out repeated simulations to extract and study emergent behaviours displayed by rival firms competing in a market. The simulations produced by this package will have the following key characteristics:

- Simulations are deterministic. Given a set of input configurations and a random seed, the same output simulation will be produced. This allows for the integration of randomness into a simulation, but also ensures that a simulation can be repeated and studied more confidently.
- Simulations will allow for the customisation of meaningful environments that best replicate real world markets. This includes variations to population density, as well as overall market size.
- Simulations will track relevant metrics relating to their execution, and will produce meaningful visuals to guide the study of execution results.

- Agent behaviour will be modular. The ABM package will allow for the adjustment of firm and consumer behaviours in order to ensure coverage of several potential hypotheses. This will include the inclusion or exclusion of price as a decision making factor, as well as the ability to set custom thresholds for decision making.

Given that an effective ABM suite has been created, it is essential to use the results of its simulations to make meaningful extrapolations to real world markets. Overall, the following factors will be studied in regards to the behaviour or rival firms selling homogeneous products in a common market:

- The influence of spatial competition on the geographic location rival firms choose to settle at in a competing market.
- The effects of population density, both uniform and non-uniform, on the location rival firms choose to settle at.
- The effects of price changes on the location rival firms choose to settle at as well as the evolution of their market share over time.

Overall this list in not conclusive. Attempts to predict agent behaviour can be made using previous works combined with an understanding of economic laws and theories, however there is no certainty that agents will behave as anticipated. Ultimately the goal is to build a reliable and informative model, that will allow for the study of both anticipated and non-anticipated emergent behaviours.

Chapter 2

Literature Review

This chapter will entail the exploration of inspiring works in two main subject areas: Previous studies and works with an emphasis on the study of competing firms, duopolies and spatial competition, and ABM as a means to explore the behaviour of competing firms.

2.1 Harold Hotelling & Competing Duopolies

This section will primarily focus on the work of Harold Hotelling, as well as more general work in the field of market competition.

A deep dive into the mathematical proof established by Hotelling [8] will not be carried out as the proof itself does not generalise sufficiently to be relevant to an ABM. Instead, his overall methodology and subsequent conclusions will be discussed.

2.1.1 Strategic Competition & Partial Equilibrium Analysis

Hotelling's model of spatial competition is built on the foundations of two main economic concepts. The first being the study of participants in an economy from a game theoretic standpoint. This was first instantiated by Augustin Cournot, a pioneering economist that first introduced the concept of strategic interdependence in a market. The second concept, popularised by Alfred Marshall [10], concerns the study of a subsection of economic activity, assuming all other factors remain stable. This is also referred to as "Partial Equilibrium Analysis".

Cournot's [4] work laid out the foundations for modern game theory, as their initial exploration sought to model the behaviour of competing firms in a market where firms would only be able to alter the quantity they produce. This was clearly an inspiration to Hotelling as his work emulated this model, the difference being Hotelling focused on geographical location, price and resulting market share instead.

Cournot's work is important in regard to both Hotelling, and this exploration, since his research showed that competing firms will not always independently compete relative to market conditions. Instead, equilibrium price and quantity will be determined by the strategic interactions between the firms in the market. This was a pioneering step in the study of microeconomics, as it was one of the first academic papers to represent firms as rival entities with separate self serving strategies. The market equilibrium was no longer the result of an overall study of the market conditions regarding the supply and demand of the a good or service, but instead the equilibrium was the result of the competing strategies of rival firms.

Partial equilibrium analysis is also essential to Harold Hotelling and ABM as a chosen methodology. This type of analysis will study a part of an economy, such as the market for a certain product or service, and will assume that the rest of the economy remains constant. This concept set the precedent that one could study and derive conclusions about the behaviours in an overall global economy, by the study of a smaller subset of economic activity. Generally, this made economic research more accessible, as fewer factors needed to be studied and controlled in order to validate conclusions resulting from said research. Partial equilibrium analysis is also a core concept of ABM, since in an agent based model, one does not need to model the entire universe in order to derive meaningful conclusions about the models subject. Instead a small subset of a subject area is modeled, and as agents interact within that environment, conclusions can be drawn and applied to a wider subject area.

This concept of partial equilibrium can also be referred to as ”Ceteris Paribus”. This term, and research paper, published by Edward Chamberlin [3], further reaffirmed the idea that in the context of economic study, it is essential to assume external factors to be constant in order to derive meaningful conclusions.

Overall both of these concepts represent the pillars behind Hotelling’s work in spacial competition, and to this day present themselves as essential topics in both ABM and game theoretic study.

2.1.2 Critiques Of Hotelling

It is important to note however that Hotelling, in the context of this research, is a source of inspiration regarding the desire to explore spatial competition and its influence on firm behaviour. Hotelling’s work was pioneering for its time but presents itself as a rather reductionist approach to the study of how firms interact. This is evident in the book ”The Theory of Industrial Organization” by Jean Tirole [13]. Tirole claims that Hotelling’s deductions are based on a simplistic view of the economy as they do not factor in product differentiation, government regulation, and other more complex market factors.

His work may also be difficult to expand to more complex systems, such as the case of more than two firms, due to his methodology chosen to qualify his claim: An analytical proof using the linear constraints that define his market. This stance was taken by Gaëtan Fournier and Marco Scarsin in their research paper ”Location games on networks: existence and efficiency of equilibria” [6]. Their paper claimed that for the case of more than two firms, there can exist multiple equilibria. In fact, as the number of firms increase they showed that the number of equilibria can increase exponentially.

Together these critiques provide valid justification for ABM as a chosen methodology.

To begin ABM allows for the implementation of complex reactive systems that address the issues with the simplistic assumptions taken by Hotelling. Secondly, ABM does not revolve around the need to *solve* the system. This makes ABM far more resistant to changes in system complexity.

2.1.3 Economic Stability & Duopolies

In Hotelling's paper, Stability in Competition [8], he sought to explore and analytically prove that a duopoly would converge to a natural market equilibrium in regards to both price and physical location within the market. Overall, the idea of a naturally occurring stable equilibrium in a duopoly is a concept that is both supported and rejected by various pieces of literature.

For example, looking at the paper "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look." [7]. This paper claims that duopolies can exist in a state of equilibrium if they both adopt a lean strategy where each firm attempts to slightly undercut the price of its competitor. However, if one firm deviates from this strategy and seeks to raise prices to acquire greater profit margins, then a price war may result as the rival will react by dramatically undercutting its price point. This study does describe the possibility of a market equilibrium, however by no means affirms it as a guaranteed outcome.

On the other hand, one work that does support Harold Hotelling's implication that duopolies converge to a market equilibrium is the book, "Sunk Costs and Market Structure" by John Sutton [12]. Here, Sutton claims that duopolies are inherently stable as a result of steep costs of market entry. Acquiring and maintaining market share is incredibly difficult and costly, and so duopolies are said to be less inclined to engage in price wars. This idea is useful to consider, since studies relating to the game theoretic analysis of economic behaviour seldom consider these sunk costs in regard to actions taken. Changes to quantity, price and other factors are typically represented as instantaneous cost free decisions.

A different take on this stability is demonstrated in the paper "A Study of Cartel Stability: The Joint Executive Committee" by Robert H. Porter [11]. This study investigates historic cases of cartels and duopolies in order to derive conclusions regarding the convergence to a market equilibrium. The study demonstrates that throughout history duopolies have been able to maintain equilibrium's as a result of coercion between the members of the duopoly. This could take the form of punishments for deviating from agreed commitments to price and quantity supplied, or monitoring systems to ensure firms have close to perfect information regarding the other firms market share, price and quantity.

Overall the notion of duopolies maintaining a stable equilibrium is not a concept unique to Harold Hotelling. There does remain a key distinction however. Hotelling claimed, and subsequently attempted to prove, that duopolies will converge to a market equilibrium as a result of fundamental economic factors; price, proximity and market share. Studies such as [11] make the claim that such equilibrium's are instead a result of careful collusion and social engineering.

Ultimately the previously mentioned works all attempt to explain why firms tend to converge to equilibrium prices, locations and strategies. However, the studies either used analytical and mathematical approaches or the analysis of real world examples to defend their positions. In order to obtain a more versatile view of the subject as a whole, one must now consider experimentation as a means of studying rival firms. This approach is evident in previous works that have used ABM as a means of investigating spatial competition.

2.2 ABM & Spatial Competition

Having now established the previous works regarding the study of competing firms in a market, one must also understand previous works regarding ABM as a means to study spatial competition. A core work that served as an inspiration for this investigation was the paper ”Spatial Competition with Interacting Agents” by Bertrand Ottino-Loffler [2]. This study uses ABM to model the behaviour of competing firms selling a homogeneous product in a two-dimensional market. The work used repeated simulations of a market in order to study how the relationship between consumers and competing firms would influence the distribution of firms in a market.

This study took a large step forwards from Hotelling's initial work. These main differences will all be highlighted in the following section (2.3), however, this was one of the first studies to take an experimental approach towards spatial competition, incorporating a market of two dimensions, and considering the case of multiple rival firms. This paper studied the correlation between types of market interactions and the prices and positions chosen by firms. The study considered localised interaction, the case when consumers only interact with the closest firm, and global interaction, the case when consumers interact with the entire market according to a probability distribution.

Overall this study was a key inspiration to this work as it was able to show that markets focused on localised interactions led to firms settling around the center of the market, and global interactions led to firms being more evenly distributed. This work was essential to this exploration as it validated ABM as a sufficient methodology for the study and derivation of emergent behaviours in competing firms.

2.3 Differentiating Factors

To finalise the context of this exploration it is important to highlight the key differentiating factors this research will leverage to discoverer and explore different conclusions from original works.

2.3.1 The Two Firm System

One main critique of previous works is their focus on duopolies as their target market structure. To begin Harold Hoteling's paper focused exclusively on a two firm system. This focus on duopolies does not negate the work and research carried out on the subject, but it does lead to difficulty when generalising to more complex systems. This leads to two problems, the first being that duopolies are rare in the real world, and the second being that the initial observations that drove this study did not directly indicate an observation of duopolies.

For this reason, in alignment with the ABM model developed by [2], this research will involve multiple firms that collectively operate in a market. This is because the real world observation of rival firms settling near each-other concerns a market that more closely resembles monopolistic competition. Following previous works this exploration will assume that each firms sells an identical product. This initially contradicts the definition of monopolistic competition were rival firms leverage slight variations in products and services to acquire market share.

One could argue that the initial observations of fast food chains, coffee shops and gas stations better resembles an oligopoly. This may be the case for larger institutions and the distributions of their franchise locations, however this definition does not explain the observation that rival firms in close proximity do not all share traits regarding scale. Large conglomerates may all establish franchise locations near each other, but so do smaller locally owned businesses. Ultimately so long as the products are sufficiently similar, rival firms appear to settle close to one another, regardless of firm size.

Overall the intention of this section is not to over-invest into categorising the observed market competition. Instead one must understand that the motivation behind this experimentation is not to derive conclusions specifically about duopolies. For this reason this study will use the case of multiple competing firms.

2.3.2 Population Density

Previous works, both experimental [2] and analytical [8] make the assumption that consumer population is uniform across the market. This does not represent the population distribution of the real world and as a result may lead to difficulties when extrapolating experimental observations. Population density is a core characteristic of markets, and to assume uniformity leaves conclusions susceptible to doubt and critique. For example looking at the article ”The Inequality of Urban Density” by Shlomo Angel and Patrick Lamson-Hall [1], one sees that not only is population density non-uniform globally, but even within dense population centers such as cities, population density is not distributed evenly.

For this reason this study will implement a model that considers non uniform population distributions in order to best emulate real world populations. This will also allow for the potential to derive conclusions regarding the behaviour of competing firms in relation to population density.

2.3.3 Price Changes

Previous works have either not considered price at all, for example Hotelling's [8] original paper only discussed the use of a firms location as a means to acquire market share, or have initialised price equally amongst all firms, this being evident in Bertrand's ABM [2]. In the case of this exploration, it is important to note that price will be included as a means for firms to acquire market share, effectively supplementing the base model described by Hotelling. However, in order to ensure that results are not correlated too heavily to the initial price chosen, this exploration will also consider the case of randomised price initialisations for firms in the market. This will supplement Bertrand's ABM, as this study will not only consider the case of a globally unique starting price.

Chapter 3

Methodology

This chapter will describe the methodology and approach taken to achieve the goals set out by the study. The chapter will progressively explore the technical implementation of the ABM, making sure to highlight assumptions made and high level decisions taken.

The ABM package built for the purpose of this study provides value in three key areas:

- Allows for repeatable deterministic simulations.
- Implements a modular system of logic for agent decision making.
- Builds insightful visuals that drive the study of emergent behaviours.

Overall this package acts as the technical implementation of a market where rival firms selling a homogeneous product will compete over market share. The ABM will involve both uniform and non-uniform population distributions. The package will capture the work done by Harold Hotelling and Bertrand Ottino-Loffler [2], but will also supplement their ideas with alterations to agent decision making and environment design.

It should also be noted that the language used to implement this model was python with version 3.9. All supporting packages and imports were managed with a virtual environment. This environment will not be included in the files associated to this paper, however all relevant libraries required to run this model are present in the Appendix. A requirements file has also been included in the package, this will allow for simple set-up and testing.

3.1 Class Design & Architecture

Before deep diving the characteristics of the environment, the agents and the algorithms that drive the simulation, the overall structure of the code package will be explored. This information will be stored in the following table.

Class Name	*Class Description*
env_generator	This class generates environments for simulations. It owns logic related to building one and two-dimensional environments, and plotting both uniform and non-uniform population distributions.
firm	This class represents the firm. It serves to track a firms position, revenue, and price as well as the progressive evolution of each of these values.
model	This class generates a model. A model owns an environment object as well as a dictionary of firms. This class contains all logic related to operations on an environment such as the assignment of market share to firms.
simulator	This class represents the final ABM simulation. This class owns a model object, and contains all logic related to the iteration by iteration execution of a simulation.
region_plotter	This class plots all visuals relating to the evolution of firm positions and market share.
stats_plotter	This class plots all visuals relating to the evolution of firm prices, market share, revenue and the distance to closest firms.
env_plotter	This class plots all visuals relating to the depiction of the current environment.
gaussian_dist	This is a custom class that is used to generate values in accordance to a normal distribution. This class is used when non-uniform population density is selected.
config	This class stores the parameters of an experiment.

At a high level, each experiment is conducted within a simulation object. Each simulation owns a model, which respectively owns an environment and a set of firms. Every iteration, the simulation uses the logic owned by its model to propagate changes to the models environment and agents. Finally, the plotter classes are used to produce insightful visuals and analytics regarding the simulation. Given that the context of the package is established, the constituents of the ABM will now be discussed.

3.2 The Consumer Class & Our Environment

This section will deep dive the representation of the consumer and the environment, although in the chosen implementation, both represent the same thing.

The agent-based model that will be used in this experiment will not include a separated agent class encapsulating the behaviours and characteristics of the consumer in the procedurally generated market. The reason for this, is that given the context of the problem, the consumer does not possess sufficient autonomy to justify the implementation of a standalone class. When studying Hotelling's model [8] as well as other works [2] the consumer agent acts as a metric through which firms evaluate their potential set of decisions. The consumer seeks only to select the most favourable firm given the current firm positions and prices offered.

This allows for consumers to effectively represent binary variables. In fact, consumers are ultimately the primary heuristic firms use to make informed decisions. As a result, this study will capture the consumer as an element of the environment. Each location in the procedurally generated environment will contain a finite number of consumers represented by an integer. To ensure consumers play an adequate role in the decisions carried out by the firms, the value of acquiring a location in a firms market share will be weighted by the number of consumers that populate that location.

3.3 Procedurally Generated Markets

Before looking into the logic that drives consumer assignment to a given firm, the generation and structure of the environment will be explored. The environmental setup that this study implements allows for key differentiation from previous works in the field.

For the rest of this document let it be understood that environment, market and grid may be used interchangeably.

First, the case of one dimensional examples will be taken. Honoring Hotelling's initial exploration [8], it was important to ensure that the package developed had the functionality to emulate the original inspired work. One dimensional environments are represented by a grid containing a single row, and K columns. Each point along this line represents a position that a firm or consumer may situate themselves upon. Conversely, in the case of two dimensional environments, the market will be represented by a $K \times K$ grid.

The value at each position represents the population of consumers that occupy that location. This is best represented by equation 3.3. The number of consumers that occupy a given position will vary based on if the simulation enables non-uniform population densities. In the case of uniform population densities, the number of consumers occupying each square is 1 at every position.

In the case of non-uniform population densities, the consumers occupying a given square is calculated based on a more complex process. To begin, the grid will be initialised with a random consumer count per position in the range $[0,5]$. Then, based on the configuration specified, C cluster centers will be defined. A cluster center represents a population center, and will effectively act as the peak of a normal distribution. All positions around a cluster center will have progressively lower population densities. Overall, the population at a center will be the highest, and following a normal distribution, positions around this center will have progressively lower consumer counts.

Figure 3.1a demonstrates the case of 4 randomly initialised firms along a one dimensional environment with 100 possible positions. This market has 4 population centers, and as a result depicts a more realistic consumer population distribution. Figure 3.1b demonstrates the case of 4 randomly initialised firms along a two dimensional environment with 10000 possible positions. Both cases capture the non-uniform distribution that is present in the real world.

It is important to note that a normal distribution is assumed as the statistical distribution to model real world human population. Although real world population distribution does not necessarily follow this trend, this is still a considerable improvement when compared to the flat uniform distribution taken by previous works.

One dimensional examples will not be depicted, as they emulate the same setup at the markets in Figures 3.1a and 3.1b, with the only difference being a constant value of 1 for population density. Is is important to note that consumers will follow the same trend as described by localised competition defined in Bertrand's study [2]. This means that firms will interact exclusively with the closest firm to their current position. If the simulation also permits for firms to use price changes to collect market share, then consumers will consider firms that have the most favourable combination of both price and location. A deeper study into the exact dynamics of this assignment of demand to a firm as well as overall agent behaviour will be carried out in following sections.

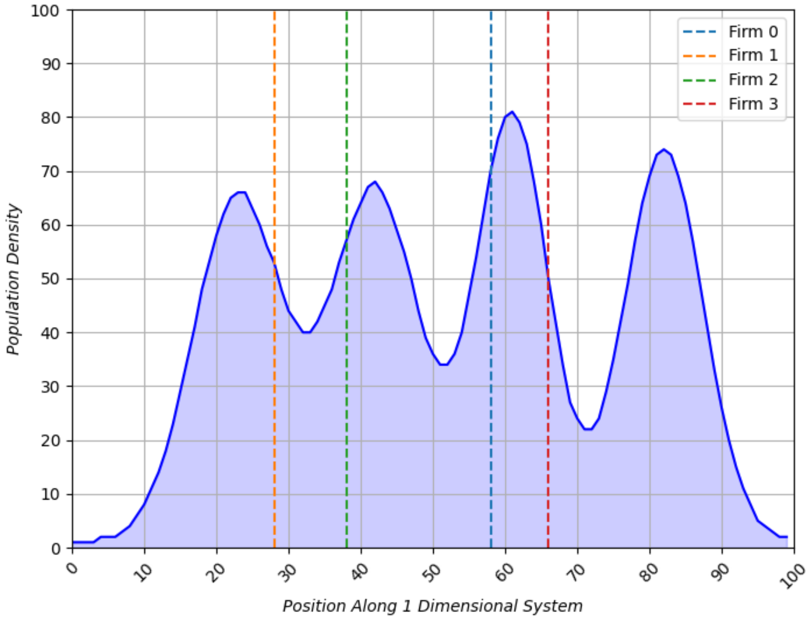
3.4 The Firm Class

The firm class serves to capture and track the state and actions taken by a given firm during each iteration of the simulation. The firm class tracks the following key variables:

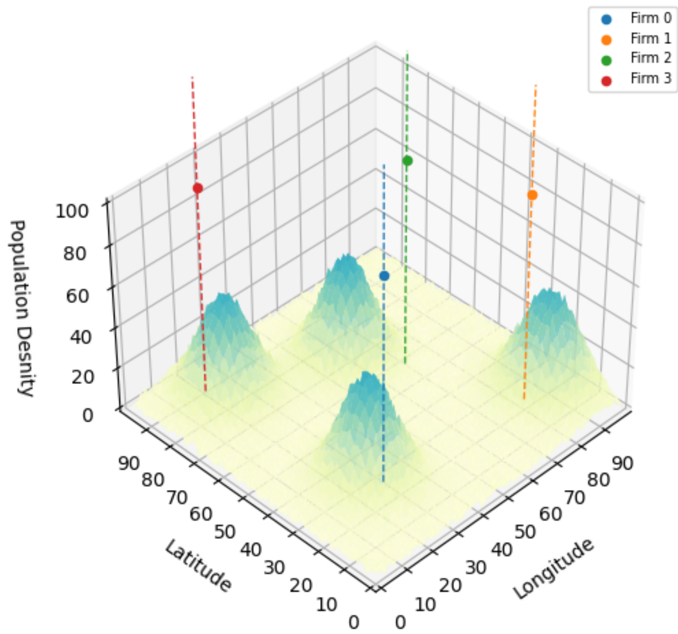
Variable	***Description***
Position	The firms current position in the market.
Position History	This variable tracks the sequence of positions taken by a firm over the course of a simulation.
Price	The firms current price.
Price History	This variable tracks the evolution of a firms price. This is only relevant when firms are able to leverage price changes.
Revenue	The firms current revenue given its position, price, and market share.
Revenue History	This variable tracks the evolution of a firms revenue per iteration of the simulation.

Firms will interact with the market every iteration. They check the potential revenue at a new position and new price, and will alter their price and position to maximise profits. Firms are initialised with a random position in the market and are able to move to any adjacent unit to their current position, including diagonals. Firms may not share a position in the market.

Each firm will have the opportunity to move once per iteration. Firms will do so sequentially with an order that is randomised each iteration. This is a key difference from the ABM developed by Bertrand [2], as their model involved firms all moving at once at the start of each iteration. This is a key distinction as sequential decision making better relates to the overall goal to study the interaction of rival firms in relation to *each other*. When firms interact all at once, each iteration, they are only able to make decisions based on the previous state of the market. However, in this study firms are able to consider the current actions taken by rival firms within the same iteration.



(a) 1-D Market With 4 Population Centers & 4 Firms



(b) 2-D Market With 4 Population Centers & 4 Firms

Figure 3.1: Sample Environment Initialisation

3.5 Market Share & Customer Decision Making

Market share will be the primary driving factor behind a firm's decision to move its location and change its price. The notion of market share will remain constant over all configurations of a simulation: Market share denotes the size of the consumer population that chooses a given firm. However, this calculation differs when firms are able to change their price.

The following equations establish the baseline for how market share is defined and calculated:

$$P = \{P_0...P_k\} \leftarrow \text{Set Of All Positions In An Environment} \quad (3.1)$$

$$F = \{F_0...F_n\} \leftarrow \text{Set Of All Firms} \quad (3.2)$$

$$Consumers(P_k) = \text{Number Of Consumers At Position } P_k \quad (3.3)$$

$$OptimalFirmNoPrice(P_k) = F_n \in F \mid F_n = MIN(DIST(P_k, F)) \quad (3.4)$$

$$MarketShare(F_n) = \sum_{0}^{k}(\ Consumers(P_k) \mid F_n = OptimalFirmNoPrice(P_k)\ ) \quad (3.5)$$

- Equation 3.1 denotes the set of all positions within an environment.
- Equation 3.2 denotes the set of all firms actively participating in the simulated market.
- Equation 3.3 indicates the number of customers situated at a position in the grid. In the case of uniform population density simulations this function will always return 1.
- Equation 3.4 describes how consumers chose their best firm. In the case that firms are not able to change their prices, this function will simply return the closest firm based on euclidean distance.
- Equation 3.5 describes how a firms market share is calculated. The market share for a firm is the sum of all consumers that choose that firm as their best.

In the case of firms being able to leverage price changes, equations 3.4 and 3.5 would instead take the following forms:

$$OptimalFirmWithPrice(P_k) = F_n \in F \mid F_n = MIN(DIST(P_k, F)\ +\ Price(F_n)) \quad (3.6)$$

$$MarketShare(F_n) = \sum_{0}^{k}(\ Consumers(P_k) \mid F_n = OptimalFirmWithPrice(P_k)\ ) \quad (3.7)$$

- Equation 3.6 will return the firm that minimises the linear sum of the distance from a firm to a given position and the price offered by that firm.

Market share is then used to determine a firms revenue. This is depicted in equation 3.8. Depending on if the simulation involves price changes, either 3.5 or 3.7 will be used in the calculation of revenue.

$$Revenue(F_n) = MarketShare(F_n) \times Price(F_n) \tag{3.8}$$

3.5.1 Market Assumptions

Consumers will directly assign 100% of their demand to the single closest firm, or to the best firm also considering price. In either case, consumers do not distribute their demand to more than one firm and each will consume a single unit per iteration. It is also important to note that price and distance have equal weight in regards to calculating the optimal firm for a position.

Another assumption that is shared by both Hotelling's original study [8] and Bertrand's experimental ABM model [2] is that firms pay no cost in regard to moving their position and changing their price. Not only so, but consumers are said to be perfectly rational relative to the absolute cost of a product. Consumers will always assign their demand to the firm that yields the lowest cost. Consumers do not develop loyalty to firms, and their demand is said to be perfectly elastic in regards to consuming from one firm or another. This means that the demand assigned to a given firm is incredibly sensitive to any change in price, or in this case, the total cost passed on to the consumer. Because all firms in this experiment are theoretically producing perfect substitute products, any slight deviation in cost that makes a firm more favourable, will assign it the entire market, and any change that makes the firm more costly, will lose it the entire market.

Having elaborated the setup of the environment, the agent classes, as well as the means through which both firms and consumers make decisions, the simulation and core algorithms will now be discussed.

3.6 Algorithms

The algorithms that drive the simulation are contained within the simulation class. The algorithms differ only with respect to a firms ability to change its price in order to acquire market share.

Both Algorithms follow the same overall approach. Each iteration begins with the randomisation of the firms order in a list. Because this study involves firms taking turns moving each iteration, it is important to ensure no firm is consistently at an advantage regarding its ability to change its position. This allows for the encoding of a more natural system where rival firms iteratively alter their position sequentially each iteration.

Now, for each firm in the randomised order, all possible moves are explored. In the case that price changes are enabled, every possible price change is also explored for each potential valid move. It is important to note here that when a firm *"explores"* a potential position, this means the model will simulate the market share as if the firm had moved to that new price and position, assuming all other firms maintain the position and price they owned at the end of the previous iteration.

Algorithm 1 ABM Simulation Algorithm - Prices Changes Disabled

$F = \{F_0 ... F_n\} \leftarrow$ The Set Of All Firms
while $Iterations < MaxIterations$ **do**
  Randomize the order of F
  for $F_i \in F$ **do**
    Track Current Position & Revenue
    Let $V = \left\{V_0 ... V_y\right\} \leftarrow$ The Set Of Valid Moves For F_i
    for $V_j \in V$ **do**
      $OldRevenue \leftarrow Revenue(F_i)$
      $NewRevenue \leftarrow Revenue(F_i)$ If F_i Moves To V_j
      if New Revenue $>$ Old Revenue **then**
        Update Trackers For Position & Revenue
      end if
    end for
    Update F_i Position Based On Highest Revenue For Best V_j
  end for
  $Iterations \leftarrow Iterations + 1$
end while

Algorithm 2 ABM Simulation Algorithm - Prices Changes Enabled

$F = \{F_0 ... F_n\} \leftarrow$ The Set Of All Firms
while $Iterations < MaxIterations$ **do**
  Randomize the order of F
  for $F_i \in F$ **do**
    Track Current Price, Position & Revenue
    Let $V = \left\{V_0 ... V_y\right\} \leftarrow$ The Set Of Valid Moves For F_i
    for $V_j \in V$ **do**
      for Price Change $\in \{-1, 0, 1\}$ **do**
        $NewPrice = Price(F_i) \ + \ PriceChange$
        $OldRevenue \leftarrow Revenue(F_i)$
        $NewRevenue \leftarrow Revenue(F_i)$ If F_i Moves To V_j And Uses $NewPrice$
        if New Revenue $>$ Old Revenue **then**
          Update Trackers For Position, Revenue Price
        end if
      end for
    end for
    Update F_i Position & Price Based On Highest Revenue For Best V_j
  end for
  $Iterations \leftarrow Iterations + 1$
end while

After every potential price and position is explored, the firm will be updated with the price, position and revenue of the best decision at that time step. The firms history parameters will also be updated in order to support visualisation once the simulation has finished.

3.7 Experimental Design

This section will describe the overall experimental setup through which conclusions will be derived. The series of experiments will begin with a one-dimensional, multi firm, non-uniform population example. This initial case will consider a more simple environment to establish context for the behaviours observed in more complex two-dimensional experiments. The following list contains the specifications of the two-dimensional experiments that will be explored in the next chapter:

- Experiment 1 - Non-uniform population density, price changes disabled, $N > 3$ Firms.
- Experiment 2 - Non-uniform population density, price changes enabled, $N > 3$ Firms, price initially set at 25.
- Experiment 3 - Non-uniform population density, price changes enabled, $N > 3$ Firms, price initialised randomly for each firm within the range $[10, 30]$.

In each of the preceding experiments the environment will be set to have the following constant variables:

- A two-dimensional environment represented by a 100×100 grid.
- Non-uniform population density across the entire market.
- $C = 4$ Population centers will be initialised.
- When price changes are enabled, price changes may only increase or decrease by a single unit, or remain the same.
- Each experiment will entail 50 total iterations.

Chapter 4

Results

This chapter will explore and discuss the results of the various experiments and simulations carried out, as well as any resulting observations in relation to emergent behaviours. The chapter will explore observations regarding the following main subject areas:

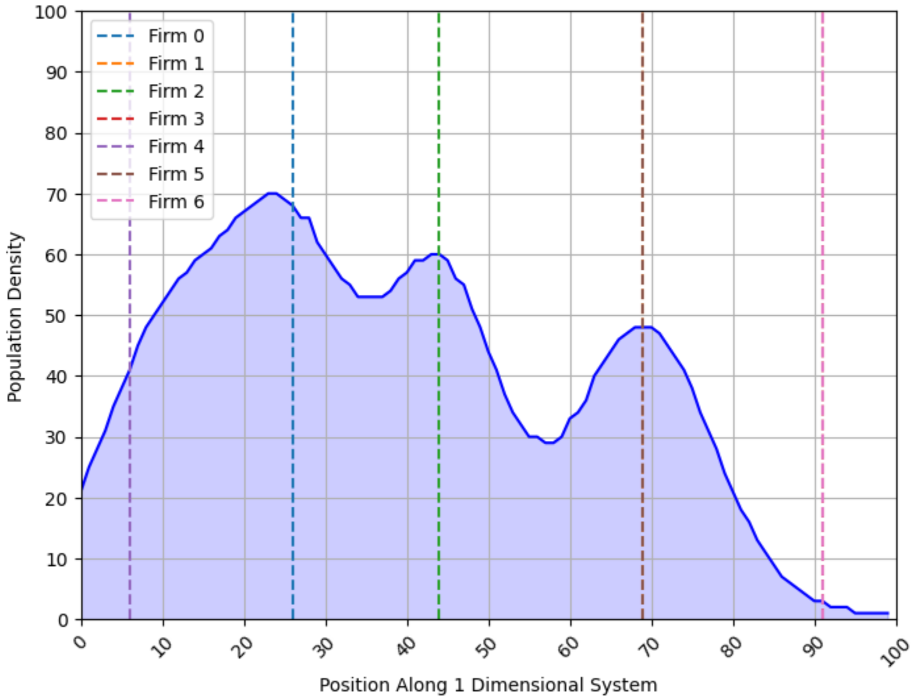
1. The evolution of firm positions per iteration.
2. The evolution of firm price, and the effect of price changes on firm behaviour.
3. The effect of population density on firm position and price changes.

Each of these subject areas will be sequentially explored in order to study the behaviour of agents and to make extrapolations to the real world.

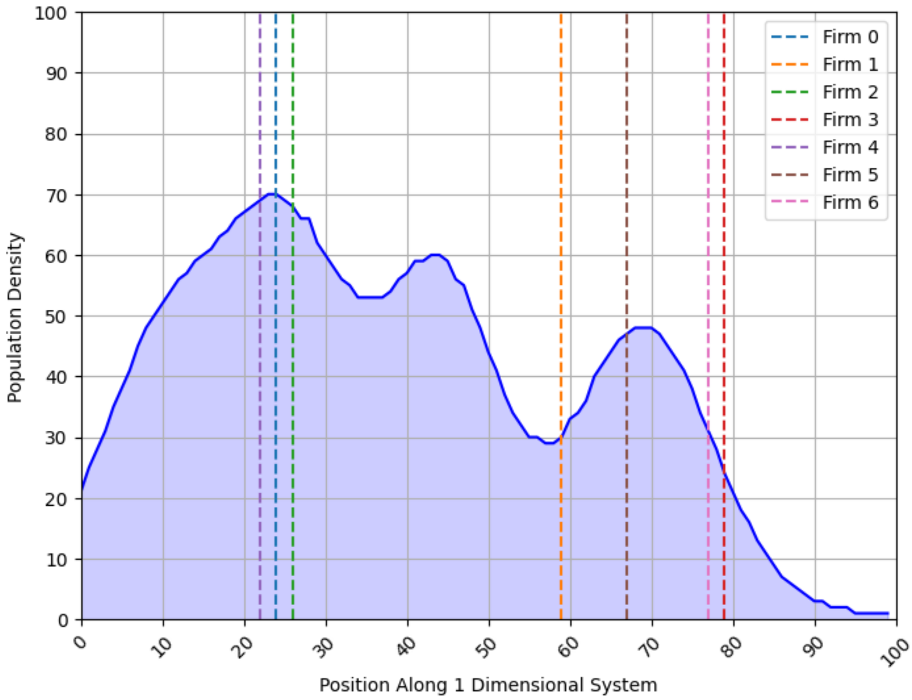
4.1 Experimental Results & Visuals

The goal of this section is to display the results of the experiments described in section 3.7. The following should be noted regarding the following figures and visuals:

- All figures represent experiments with the same random seed chosen for the initialisation of firm positions and the plotting of populations. This seed is 1999111111111, and represents a completely arbitrary number chosen independently of any testing and development carried out on the package. A common seed will ensure equal initial environments for each experiment, allowing for conclusions to be derived based on core aspects of the model, and not factors relating to the randomised initial state of the market.
- All visualisations are produced by code developed for this package, no pre-built visualisation tools were used.

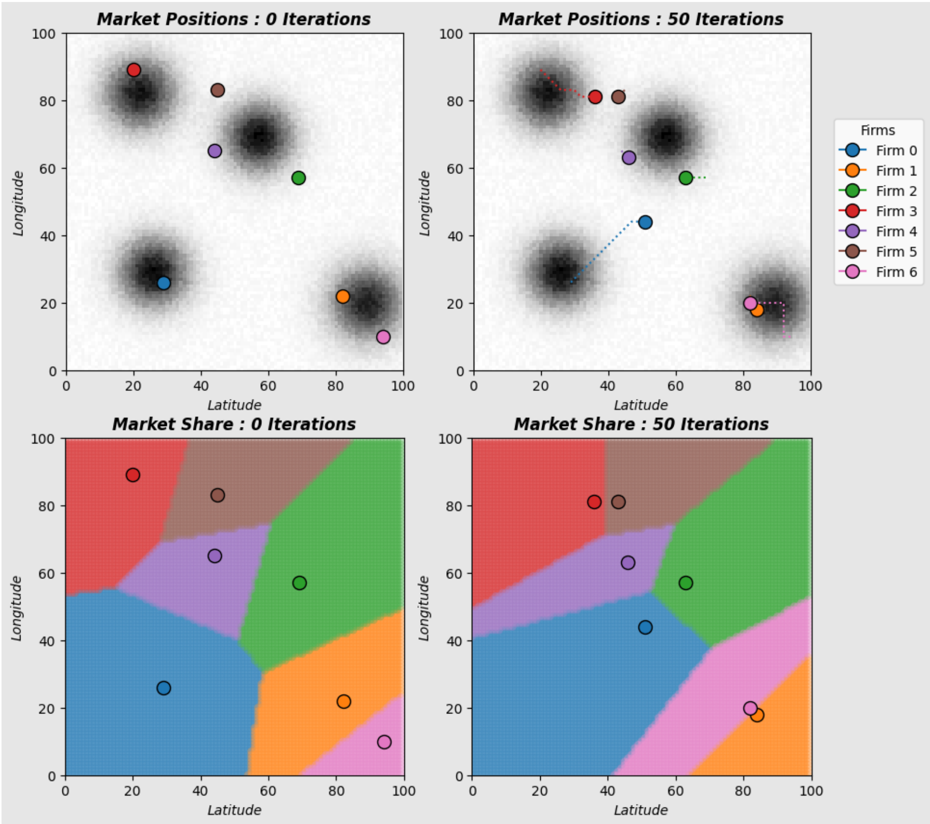


(a) 2-D Experiment - 0 Iterations

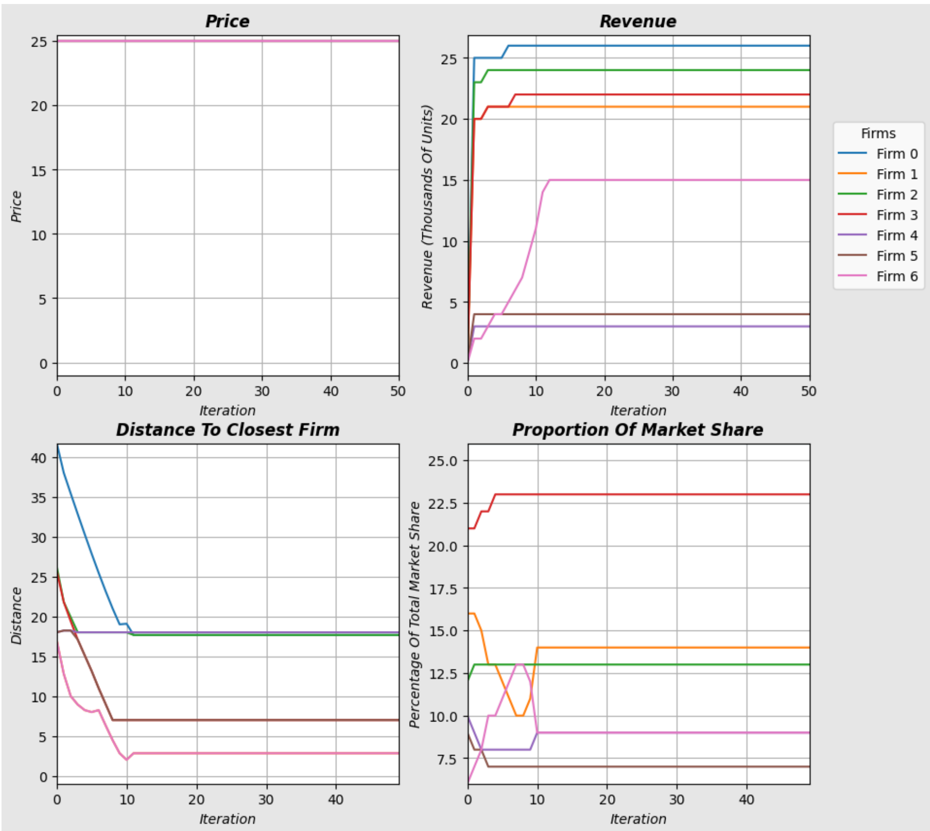


(b) 2-D Experiment - 50 Iterations

Figure 4.1: 2-D Experiment With Price Changes Disabled

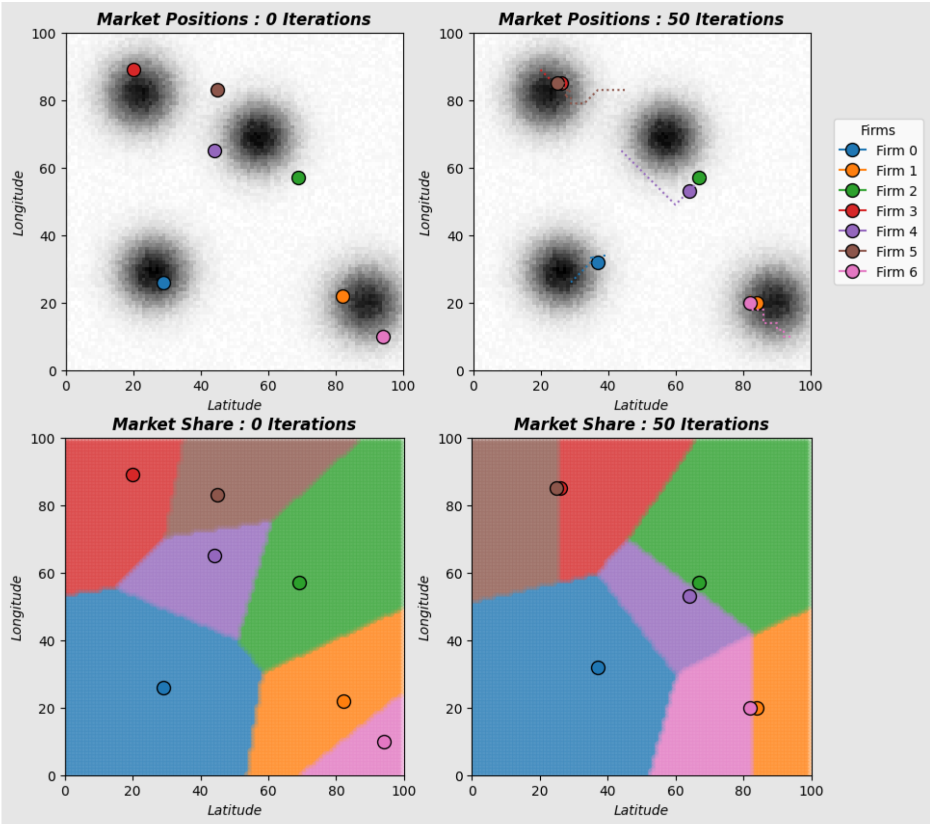


(a) Evolution of Firm Positions & Regional Market Share

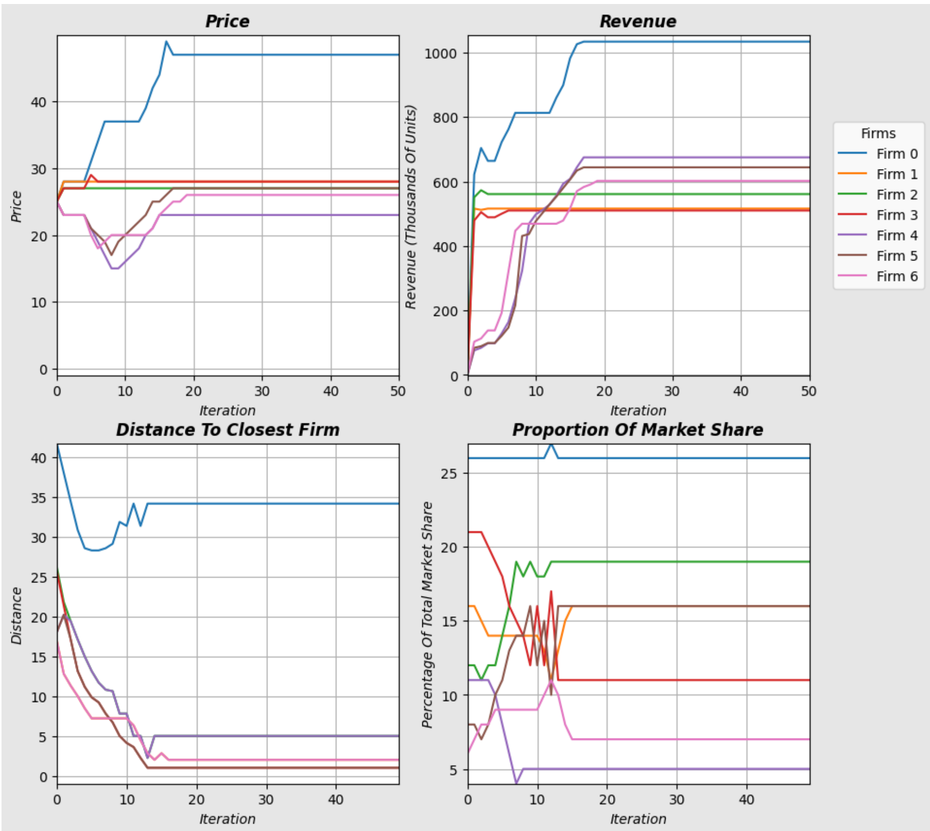


(b) Evolution of Firm Prices, Revenue, Distance to Closest Firm & Proportion of Market Share

Figure 4.2: Experiment 1 - Price Changes Disabled

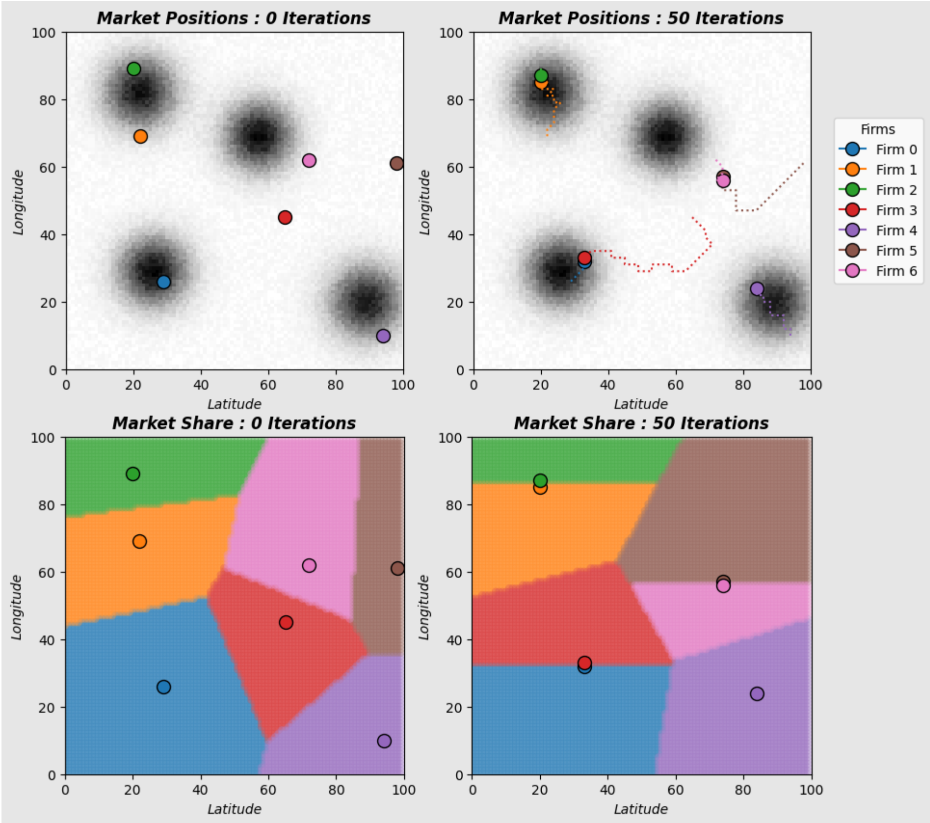


(a) Evolution of Firm Positions & Regional Market Share

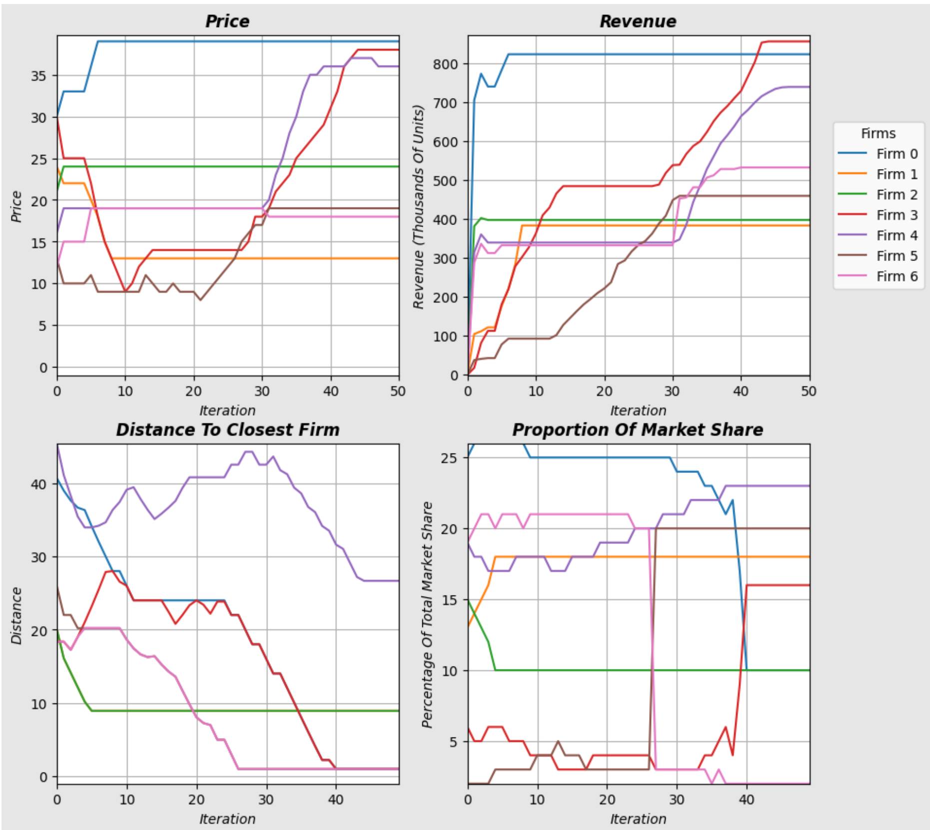


(b) Evolution of Firm Prices, Revenue, Distance to Closest Firm & Proportion of Market Share

Figure 4.3: Experiment 2 - Price Changes Enabled With All Prices Initialised To P=25



(a) Evolution of Firm Positions & Regional Market Share



(b) Evolution of Firm Prices, Revenue, Distance to Closest Firm & Proportion of Market Share

Figure 4.4: Experiment 3 - Price Changes Enabled With Prices Initialised Randomly In Range $[10, 30]$

4.2 Evolution Of Firm Positioning

This section will discuss the general idea of firm positioning as a means to study agent behaviour.

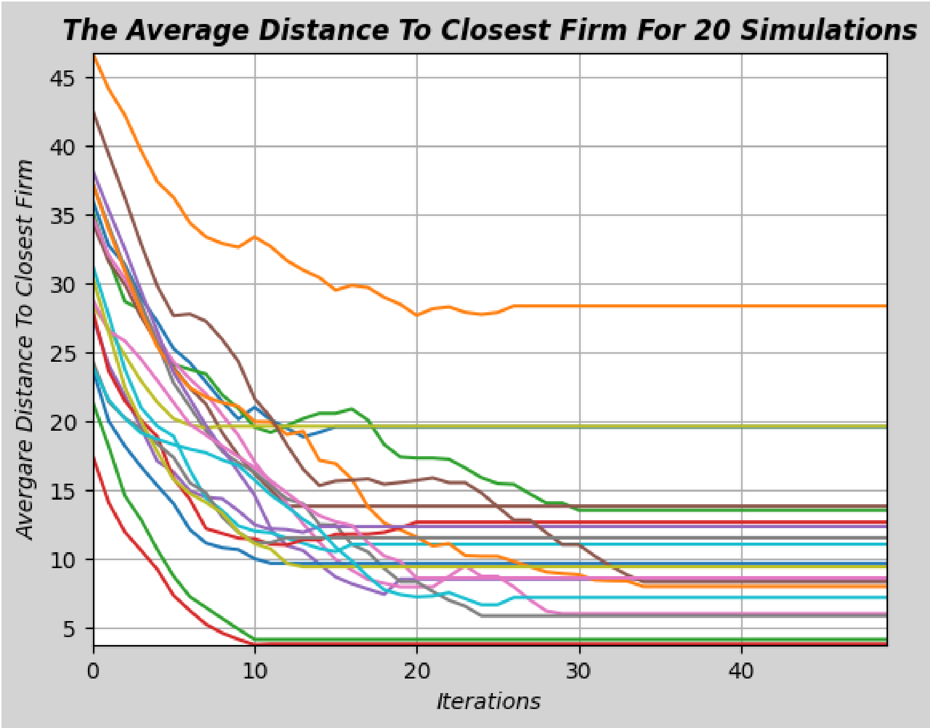
To begin, as shown by both the previous work of Bertrand [2], and the model theorised by Hotelling [8], the experimental results show a clear tendency for firms to approach one another as the simulation progresses through its iterations. Markets appear to be consistently divided into sets of ”Pseudo” duopolies. Simulations produce markets where firms form pairs that each compete compete over smaller subsets of the total market. The reason they cannot be fully considered as duopolies is that they still remain amongst a larger group of 7 firms selling the same product in the same market. However clearly after 50 iterations firms form pairs that each compete over a local subset of the market.

Looking at all experiments there is a clear trend: The distance to the closest firm, for each firm, decreases per iteration. This is clearly evident in Figure 4.5. This figure displays the average distance to the closest firm for 20 randomised simulations. The figure considers both enabled and disabled price changes. It is clear that both the overall trend, and the trend for each simulation, is decreasing. This observation is evident irrespective of price changes and population density. This is an incredibly important finding because it indicates the importance of spatial competition regarding a firms desire to acquire more market share. Regardless of a markets population density, or a firms ability to leverage price to boost its revenue or undercut competitors, firms consistently approach one another in order to both maximise their own market share, and protect their own market share from rivals.

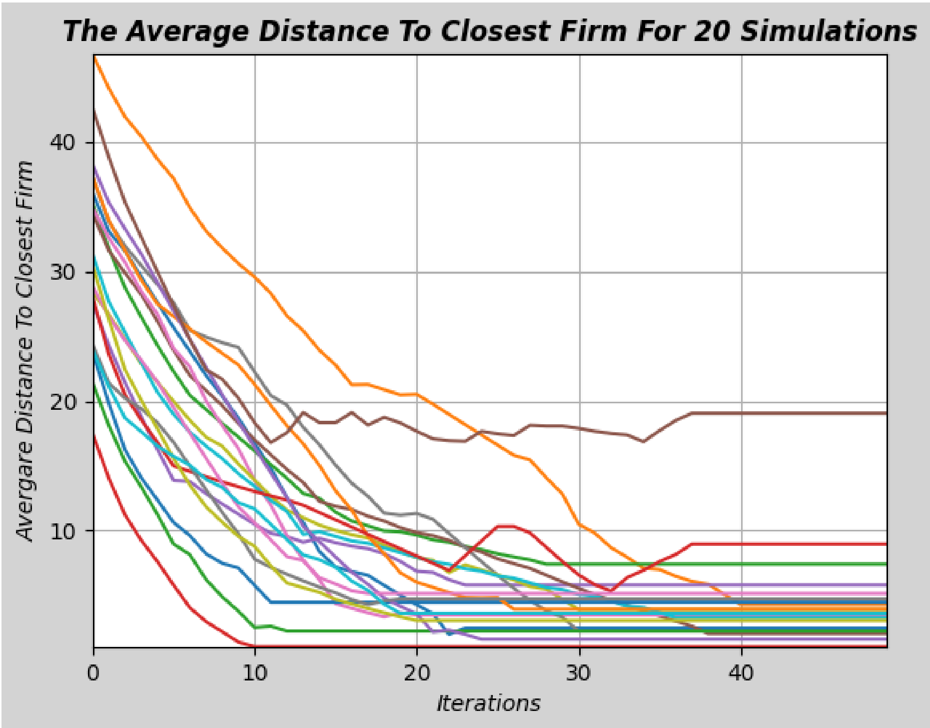
It is also important to note that in figure 4.5a, with price changes enabled, the rate at which average distance decreases is lower than figure 4.5b where price changes are disabled. This shows that when spatial competition is the *only* form of market competition a firm can utilise, firms are more likely to approach each other. When price changes are enabled firms still approach each other, but to a lesser extent, as they instead will use price competition to acquire market share.

The progressive change in position, and the final positions chosen by firms, is directly influenced by the the ability of a firm to change its price. This is clearly evident in the observed distinctions between the evolution of firm positions seen in figures 4.2a and 4.3a. In the absence of price changes, firms are only able to leverage their position to increase market share. When firms can change price they are not always inclined to move their positions. This is clearly evident in the behaviour of firm 0 in Figure 4.3. In the absence of price changes firm 0 would have approached firms 2 and 4 as seen in the behaviour depicted in figure 4.2a. However, when price changes are enabled, firm 0 chooses to remain relatively stable in terms of its position and instead raise its price to increase revenue. This can be validated in the price and revenue evolution graphs seen in figure 4.3b.

This shows that even in the case of identical products in a common market, firms may not always be inclined to only leverage a single form of market competition. Instead, firm decision making is based on a combination of various forms of market competition.



(a) Price Changes Enabled - Approaching Firms After 20 Iterations



(b) Price Changes Disabled - Approaching Firms After 20 Iterations

Figure 4.5: Approaching Firms After 20 Iterations

4.3 Price Changes & Agent Behaviour

Regarding the evolution of price, we see that when price changes are enabled, prices set by firms tend to be correlated to the relative positioning of firms. As previously discussed, firms tend to approach one another and form pairs of duopolies that together control smaller subsets of the overall market. One can clearly notice that firms establish price points that match the price set by the firm they are closest to.

This observation is further supported by the fact that this phenomena appears to be present in both the case when price is initialised to a common global value, and when price is set to a random initial value. This is clearly evident in Figure 4.4. In this experiment the final state of the market yields pairs of firm duopolies. Looking at the following table of information:

Firm Pairs	***Firm Prices***
Firm 0 + Firm 3	Price Firm 0 = 39 and Price Firm 3 = 35
Firm 1 + Firm 2	Price Firm 1 = 22 and Price Firm 2 = 24
Firm 5 + Firm 6	Price Firm 5 = 18 and Price Firm 6 = 17

Clearly we see that despite firms each having their prices initially randomised, after 50 iterations firms will not only converge towards each other in terms of positioning, but will also reach an established price point shared by their closest firm. This is also evident in the case that price is initialised to universal global value. Looking at figure 4.3:

Firm Pairs	***Firm Prices***
Firm 3 + Firm 5	Price Firm 3 = 27 and Price Firm 5 = 26
Firm 2 + Firm 4	Price Firm 2 = 26 and Price Firm 4 = 24
Firm 1 + Firm 6	Price Firm 1 = 27 and Price Firm 6 = 25

The difference is that each duopoly shares the same rough price point when firms all begin at the same price. However, firms that do not converge towards other firms, and instead remain isolated and in control of their own private subset of the market, choose to raise prices to a level that exceeds the price point typically observed by the firm pairs.

To conclude, it can clearly bee seen that when firms are able to use price changes to gain market share, price levels converge to a stable price level. In a given market, price does not continue to increase. This is an important observation because it clearly demonstrates that the presence of competition acts as a limiting factor to the overall price level of a market. In the absence of competition prices would infinitely increase (Assuming that consumers are encoded the same way, and continue to consume a single unit from the closest firm no matter what).

4.4 Population Density & Agent Behaviour

Looking at the effects of population density on agent behaviour one will notice that firms tend to converge to form duopolies in and around population centers. This is clearly evident in Figures 4.2a, 4.3a and 4.4a. Although firms do not form pairs in the one-dimensional example seen in figure 4.1, firms still do settle in and around population peaks. This is important since it provides a valid indication as to how and why rival firms settle near each other, providing insight into the initial observations discussed in the introduction. Firms not only settle in pairs of duopolies competing over a localised market, they also settle in and around densely populated population centers.

The consideration of non-uniform population density was essential in regard to supplementing Hotelling's initial model [8]. Looking at figure 4.1 we see that with non-uniform populations firms do not always converge towards each other, even when price changes are disabled. It is important to note, that this does not negate previous observations concerning approaching firms. The distance from each firm to its closest firm still does decrease per iteration. However, according to Hotelling's model, Firm 5 in figure 4.1b should approach firm 6, and firm 1 should subsequently approach firm 5. However this does not occur. Firms 1 and 5 stabilize at positions further away from other firms, since moving closer to the cluster of firms on either side would lead to the loss of densely populated locations. This observation is important since it shows the the dependence of Hotelling's model on the rigidly defined market specified in his work. It also demonstrates how population density plays a key role on the final positions firms choose to settle at.

4.5 Evolution Of Revenue & Proportion Of Market Share

Overall the evolution of revenue and proportions of market share do not indicate any clear trend that indicates a deeper understanding of agent behaviour. However, two observations still lead to potentially useful understandings about the market as a whole.

To begin, regarding the proportion of market share controlled by each firm, one will notice that the market does not converge to a state where a single firm controls the vast majority of the market. Firms by no means share the market equally, however, no single firm is able to entirely "defeat" all other firms. This is an important observation because it can be used to justify the claim that in the presence of perfect information and equality amongst competitors in terms of costs of production and product characteristics, the natural state of the market does not involve market dominance of a single firm.

This could be used to argue that real world monopolies and oligopolies cannot result in the presence of perfect competition. So long as firms share comparable products, costs of production, market mobility, and possess equal information about consumer preference and the state of the market, a single firm, or group of firms cannot entirely consume the market share. Instead, market structures that involve the dominance of a few firms require those firms to poses some level intrinsic advantage. This could be strategic collusion, greater information and technology, or lower cost.

Secondly, one will notice that just as positions converge to a stable state, so does market share. Clearly this is because a firms position is related directly to its pursuit of market share, so there is an obvious connection between one factor and the other. However, the convergence of both these elements to a stable point implies the existence of a market equilibrium. This is a key observation as it implies that given parity amongst firms, the market will not fluctuate indefinitely, *Ceteris Paribus*. Market fluctuations instead are related to a lack of parity amongst participants in an economy. Markets are victim to changing conditions regarding the in and outflows of an economy, as well as developments in technology that are not equally available to all competitors. In the case of this experiment, with perfect parity amongst firms, and a rigidly defined market with no flows to and from the outside economy, one will see a natural convergence to a stable state.

Chapter 5

Conclusions

The overall objective of this chapter will be to address the initial objectives and success criteria highlighted in the introduction of this paper. Having discussed the methodology and results of several experiments, an overall evaluation of the exploration's ability to address initial inspiring observations as well as supplement previous academic work is also in order. Finally, a closing discussion regarding improvements to the methodology will be carried out.

In order to avoid repetition, detailed observations and references to specific figures and experiments regarding emergent behaviours will not be included as they have been described in detail in the previous chapter. This section will serve as an overall reflection of the study, the quality of its implementation, and the potential improvements.

5.1 A Comprehensive ABM Package & Simulation Suite

The first objective of this work was to build and develop a versatile ABM package that was capable of building reliable and informative simulations of competing firms in a dynamic market. At a high level, one can confidently claim that this was achieved within this piece of research. In order to qualify this claim one must look at the overall narrative taken in this paper:

1. Real world observations regarding the behaviour of competing firms were taken into consideration.
2. A high level discussion regarding market competition was then carried out.
3. Previous works and literature were then considered, taking a progressively more precise approach. The discussion progressed from a high level, such as general works in market competition, to a more refined level such as works specifically relating to spatial competition. Finally the review of previous literature concluded with an exploration of specific works that used ABM to explore spatial competition.
4. A detailed experimental methodology was then presented, making reference to previous literature and the differentiating factors utilised.

5. Finally, results of various experiments were visualised and discussed in order to study observations relating to emergent behaviours of agents and characteristics of market competition.

The reason for highlighting the narrative taken in this paper is to validate that the decisions made in the development of the ABM package came as a result of a comprehensive investigation of previous work, with a clear goal to investigate a set of real world observations and curiosities. The methodology used was built on a solid foundation of research, and each element of the implementation was validated with meaningful evidence. Addressing the specific goals relating to the package, and more importantly the simulations, the following can be said:

- Simulations are deterministic. Given the same input configurations with specific random seeds, the outcome of two simulations will always be equal. Each solution still uses random elements for population density, the order of firm decision making and firm initialisation. This leads to repeatable simulations, and allows for the trust of their outputs. Because simulations produce reliable and repeatable outputs, a greater degree of confidence can be assigned to the observations and extrapolations from their results.
- Simulations allow for customised comprehensive environments. Environments can be configured for both one and and two dimensions and can have customised sizes. The markets specified by the package can also allow for non-uniform population distributions via the plotting of population centers. The number of these population centers can also be specified.
- Simulations keep track of meaningful metrics and provide key visuals to guide the study of agent behaviour. This condition has also been met, as simulations keep adequate track of the market and agents as iterations progress. The package includes comprehensive visualisation classes that enable for clear depictions of simulations in order to allow for the confident study of emergent behaviour.
- Agent behaviour is modular. Agent decision making can be modeled and altered via the addition or supplementation of functions in the model class. Behaviours can be fine-tuned regarding decision making thresholds, and the overall package allows for the testing of various hypothesis.

Overall a comprehensive ABM package with sufficient features for the study of firm behaviour in a competitive market was successfully created.

5.2 Emergent behaviours

The criteria outlined in the introduction also qualified a set of objectives regarding the ability to derive meaningful conclusions on the behaviour of rival firms in a competitive market. This section will tackle each of these criteria one by one.

The first criteria was to effectively study the influence of spatial competition on firm positions. As seen in the results chapter, the study was able to effectively observe and study several emergent behaviours relating to spatial competition and firm positioning.

For one, it was clearly shown that firms will approach each other and form clusters. Typically these clusters will consist of two firms. The study effectively showed how in a competitive market firms divide themselves into smaller groups that compete over subsets of the entire market share. This was observed irrespective of population density, and was a clear indication of how spatial competition drives the observed behaviour of rival firms settling in close proximity to each other.

The second criteria was to study the influence of population density on firm behaviour. This was also successfully carried out. To begin, the addition of population density demonstrated a clear deviation away from the original inspiring work of Hotelling [8]. Although firms still approached each other, they did not necessarily always converge to positions directly adjacent to each other. It was also observed that firms would be more likely to form duopolies in and around dense population centers. This presented itself as a supplementation to the original works as neither Hotelling [8] nor Bertrand [2] implemented such complex environments, but also provided a key insight into the original observations regarding rival firms described in the introduction.

The final criteria was to study the influence of price changes on firm behaviour and the subsequent state of the market. This was also effectively carried out, as clear trends in price were observed. Firms shared price ranges in accordance to the closest firm, or cluster that they belonged to. This was a clear indication that when firms group together to control a smaller subset of the market, they establish their own price point. It was also observed that isolated firms tend to show prices different to the price ranges of the clustered firms, these prices typically being higher. This was an important observation as it demonstrates how in the absence of direct competition firms will raise their prices as their personal market share remains relatively uncontested. This can be extrapolated to real world behaviour where firms raise prices in the absence of competitors.

Overall the study was effective in regards to exploring emergent behaviour. The study was able to explore behaviours relating to the objectives initially described, but was also able to derive more insightful deductions regarding other factors such as revenue and market share trends. Ultimately the study was able to produce meaningful conclusions about firm behaviours and market factors and provided insight as to why rival firms selling homogeneous products tend to settle in close proximity to each other.

5.3 Improvements & Future Works

Despite the informative simulations and the effective study of agent behaviours, the potential for improvement remains.

To begin, the study lacks a deep dive into aggregate firm behaviour. Although conclusions regarding the behaviour of rival firms where logically qualified and backed by empirical observations resulting from several simulations, the experiments lacked a high number of aggregated simulations as a means to study trends in metrics. Clusters of simulations were used to study trends in the average distance to firms, as seen in Figure 4.5, however this was the only metric studied this way. This may lead to doubts regarding derived conclusions, as one may argue that the experimental results described in the previous chapter are a result of chance. Considering the degree of randomness in the initialisation of agent positions and population density, it may be considered problematic to take the results of a select few experimental samples as a means to generalise to the overall behaviours of competing firms in a market.

Another issue would be the handling of randomness. The initial location of firms plays a considerable role in regards to the final equilibrium of a market. The clusters that are formed, as well as the overall observed trends in a simulation are directly correlated to the initial firm positions as well as to the placement of population centers. This is not a serious critique of the study as similar behaviours and trends are observed over a variety of random seeds and initialisations, however it must still be mentioned that not enough effort to normalise results relative to the initial environment state was taken.

An issue more rooted towards the core methodology would be parameter choice. Currently consumers will use the sum of the euclidean distance to a firm as well as that firms price to determine their choice. Initially this does not sound problematic, however one must consider the effect of market dimensions on this function. For the duration of this study a 100×100 market was chosen with a starting price of 25. If the dimensions of the simulation were to change, for example a 50×50 market size, the price would play a far more significant role on consumer decision making. The opposite would be said if the market size was increased. Ultimately the relative difference in size between the market dimensions and the price plays a key role on the decision making of the consumer. In the case of this study this was accounted for by taking a price roughly one fourth of the market dimension, this was also chosen by Bertrand [2]. However, this ratio has not been backed by any further parameter validation, and the consequence of the price choice on larger or smaller markets is ambiguous.

The most pressing issue present in this study is the presence of perfect information amongst both firms and consumers. This study may have built on top of its inspiring works with the addition of non-uniform population distributions, population cluster centers and randomised price initialisation, however it did not include factors relating to the enforcement of imperfect information. For example, Bertrand's [2] ABM included cases when firms could only sample a subset of the total market to influence their decision. This is more realistic, as real world firms do not posses perfect information regarding consumer preference. Although this was not implemented, having firms decide their best move based on a subset of the consumer population would not be a computationally intensive feature to implement.

In regards to future works, a key feature to supplement the current implementation would be the modeling of franchises. Based on the initial observations described in the introduction, many competing firms that settle near one another in the real world correspond to stores that belong to larger conglomerates. An insightful addition to this ABM would be to have firms posses multiple locations in the market. Each firm would have multiple locations spread around the environment, and the simulation would run in the same way, the only differences being that firms belonging to the same parent company would not compete with each other for market share, they would instead only compete with the locations of rival companies. This would allow for an interesting exploration regarding if clusters would form where each cluster only contains a single firm from each parent company, effectively emulating real world franchises.

A more complex model of consumer choice could also be implemented. It would be possible to model consumer preference. For example, consumer choice could take into consideration the number of historic iterations where a given consumer has consumed from a given firm. If a firm has a historic trend of contentiously providing for a customer, this would make a sudden improvement in a rival firm less likely to completely steal the business of that customer. These changes would better model real world consumer behaviour.

5.4 Closing Remarks

Overall, the ABM developed has led to fruitful deductions regarding trends in the behaviour of rival firms competing in a common market. Following in the steps of Bernard [2], it has once again been proven that is it possible to produce meaningful analysis using experimental ABM. This is especially important in a field that has historically been dominated by analytical approaches.

Ultimately analytical approaches may yield air-tight proofs from a mathematical perspective, yet they suffer in two main subject areas. As systems become progressively more complex, the computational power required to analytically prove economic theory explodes exponentially, especially in the case of non linear systems of constraints. Analytical proofs also tend not to be resistant to variations in the definition of the systems they operate on; any changes to the contextual definition of the environment will require the entire proof to be repeated. More importantly however, many analytical methodologies fail to capture the intrinsic stochasticity of economic markets. Actors in economic settings do not always follow rigid laws, and often times, representing their behaviours as non-flexible equations may lead to reductionist outlooks. This is not to say that economic actors behave randomly either, actors clearly follow logical trends that have been established after careful study of market conditions. However, pure analytical approaches will often fail to capture the inherent variance of actor decision making that characterises the very systems one hopes to study in the first place.

ABM has proven to be an incredibly resilient and informative methodology for the exploration of economic behaviour. ABM allows for reactive environments, and can effectively draw conclusions while still maintaining a systems intrinsic entropy. ABM also allows for reactive experimentation, as redefining substituent elements of a model does not require significant effort, and so a wider context can both be considered and studied. Ultimately this has allowed for the meaningful study of spatial competition, and has lead to a deeper understanding of market competition.

Bibliography

[1] Shlomo Angel and Patrick Lamson-Hall. The inequality of urban density. *Journal of the American Planning Association*, 2020.

[2] V. Veetil U. Wilensky B. Ottino-Loffler, F. Stonedahl. Spatial competition with interacting agents. *International Journal of Microsimulation*, 2017.

[3] Edward Hastings Chamberlin. Ceteris paribus. *The Journal of Political Economy*, 1948.

[4] Augustin Cournot. Recherches sur les principes mathématiques de la théorie des richesses. *Hachette*, 1838.

[5] Joshua Epstein and Robert Axtell. *Growing Artificial Societies: Social Science from the Bottom Up*. MIT Press, 1996.

[6] Gaëtan Fournier and Marco Scarsini. Location games on networks: existence and efficiency of equilibria. *Mathematics Of Operations Research*.

[7] Drew Fudenberg and Jean Tirole. The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. *American Economic Review*, 1983.

[8] Harold Hotelling. Stability in competition. *Economics Letters*, 1929.

[9] Scott Magids, Alan Zorfas, and Daniel Leemon. The new science of customer emotions. *Harvard Business Review*, 2015.

[10] Alfred Marshall. *Principles of economics*. Macmillan, 1890.

[11] Robert H. Porter. *A Study of Cartel Stability: The Joint Executive Committee*. Bell Journal Of Economics, 1983.

[12] John Sutton. *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration*. MIT Press Books, 1991.

[13] Jean Tirole. *The Theory of Industrial Organization*. The MIT Press, 1988.

Appendix A

First appendix

A.1 Libraries And Environment Dependencies

```
fonttools==4.39.0
importlib==1.0.4
importlib-metadata==6.1.0
importlib-resources==5.12.0
ipykernel==6.22.0
ipython==8.12.0
jupyter_client==8.1.0
jupyter_core==5.3.0
kiwisolver==1.4.4
matplotlib==3.7.1
matplotlib-inline==0.1.6
numpy==1.24.2
packaging==23.0
parso==0.8.3
pexpect==4.8.0
pickleshare==0.7.5
Pillow==9.4.0
platformdirs==3.2.0
prompt-toolkit==3.0.38
ptyprocess==0.7.0
pure-eval==0.2.2
Pygments==2.14.0
pyparsing==3.0.9
python-dateutil==2.8.2
scipy==1.10.1
six==1.16.0
stack-data==0.6.2
tornado==6.2
traitlets==5.9.0
zipp==3.15.0
```