

Exercise 2

Deadline: 18. November, 2020

Please upload your solutions to to your personal **Submissions** folder in OLAT.

Use the **Latex template** provided in Exercise_0 for the report.

Hand-written results will not be accepted!

Theory

1. Homography Definition

In the lecture a planar homography was introduced as $h : \mathbb{P}^2 \mapsto \mathbb{P}^2$.

- (a) Give the definition for a general homography $h : \mathbb{P}^n \mapsto \mathbb{P}^n$.
- (b) How many degrees of freedom does it have?

2. Line Preservation

Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \in \mathbb{P}^2$ be three points on a line. Show that a homography \mathbf{H} preserves this property. Hint: Use the implicit definition of a line $ax + by + c = 0$, thus $\mathbf{l}^\top \mathbf{x}_i = 0$ with $\mathbf{l} = (a, b, c)^\top$.

3. Camera Center in World Coordinates

We are following the convention that a point \mathbf{X}_W in homogeneous world coordinates is transformed to camera coordinates as follows: $\mathbf{X}_C = [\mathbf{R} | \mathbf{t}] \mathbf{X}_W$.

The camera center in world coordinates \mathbf{C}_W can be expressed in terms of the extrinsic camera parameters by

$$\mathbf{C}_W = -\mathbf{R}^\top \mathbf{t}.$$

- (a) Prove this property. (Hint: Where is the camera center in camera coordinates?)
- (b) In this context, illustrate the meaning of $\mathbf{t} = -\mathbf{R}\mathbf{C}_W$, i.e. from where to where does this vector point?

Practical Part

For the following tasks, you find the required intrinsic parameters of the camera $(\alpha_x, \alpha_y, x_0, y_0, s)$ and the homographies \mathbf{H}_i in the file `data/ex2.mat`.

1. Relative Rotation Estimation from Homography

A homography between two images taken with the same camera can be used to compute the relative rotation \mathbf{R}_{rel} when the camera has undergone a pure rotation, i.e. no translation between the two views. Homography \mathbf{H}_1 (from `ex2.mat`) was obtained from a perfect pure rotation, whereas \mathbf{H}_2 was computed after imperfect (manual) rotation of the camera. Write a function `compute_relative_rotation` that:

- (a) loads a homography from `ex2.mat`
- (b) computes \mathbf{R}_{rel} and prints it to the console
- (c) checks whether \mathbf{R}_{rel} fulfills the properties of a rotation matrix
- (d) if necessary, corrects \mathbf{R}_{rel} and prints the new rotation matrix to the console

Apply `compute_relative_rotation` to \mathbf{H}_1 and \mathbf{H}_2 . Why does the rotation matrix computed from \mathbf{H}_2 need correction?

2. Camera Pose Estimation from Homography

- (a) A homography between a plane in the world coordinate system and a camera image can be used to compute rotation \mathbf{R} and translation \mathbf{t} of the camera. Homography \mathbf{H}_3 was computed from the corners of a (fully visible) checkerboard. The checkerboard lies in the xy -plane of the world coordinate system centered around the origin. Write a function `compute_pose` to determine \mathbf{R} and \mathbf{t} from \mathbf{H}_3 and print them to the console.
- (b) The third element of \mathbf{t} in part (a) above might be negative. What does this mean in this particular case (consider the location of the checkerboard corners)? Why can this happen?

Remark:

1. Make sure your code executes the tasks above sequentially by simply calling `python main.py` (include `ex2.mat` alongside `main.py` in your `.zip` file).

Good Luck!