

# Image Enhancement in the Frequency Domain

Recall : The purpose of image enhancement is to improve the interpretability or precision of information in images for human viewers and providing better input for other image processing and image processing vision techniques.

This is achieved by changing the attributes of images to make them more suitable for a given task.

Image enhancement doesn't increase the amount of data in an image, it just changes the dynamic range of the chosen feature so they can be detected easily.

We already talked about image enhancement in the spatial domain. Today we will be talking about image enhancement in the **frequency domain**. This is mostly done using the **fourier transform**.

There is no "general theory" to determine what a good image is when judged by humans. The rule of thumb is : if it looks good, it is good.

However, when we use image enhancement as preprocessing for other image processing or computer vision techniques, there are quantitative techniques to find out if the image is good. An example of this is histograms to determine if the contrast is suitable or not.

In Frequency domain images, The image is first transferred into the frequency domain. This means we take the fourier transform of the image first, do operations on that, then we perform the **inverse** fourier transform to get the resultant image.

These methods enhance the image by performing a convolution operation on the image with a linear, position invariant operator → this means that the image will have the same signature regardless of the position of the pixels in the image. They modify the distribution of the gray-level values. As a consequence, the values of the pixels in the image change. It is important to know that **It is possible for 2 images to have the same signature in the frequency domain**

The Pixel values of the input image will be changed according to the transformation function applied. For an image F and output G This is given by :

$$s = T(r)$$

where r and s represent the pixels in F and G respectively.

The process is as follows :

$F(x,y) \rightarrow \text{preprocessing(normalization)} \rightarrow \text{Fourier Transform} \rightarrow \text{Filter function} \rightarrow \text{Inverse Fourier Transform} \rightarrow \text{Post-Processing} \rightarrow G(x,y)$

Alternatively, we can do this directly

$$G(u,v) = H(u,v)F(u,v)$$

where H is the transfer function of the image.

Using the frequency domain allows us to capture properties that are not present in the spatial representation of the image, mainly the shape and texture of the image.

We also use the frequency domain because  
It is computationally faster to apply filters in the frequency domain.  
This is particularly useful for larger filters.

## Definition of frequency

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Frequency is formally defined as:

The number of times in a specified period that a phenomenon occurs within a specified interval.

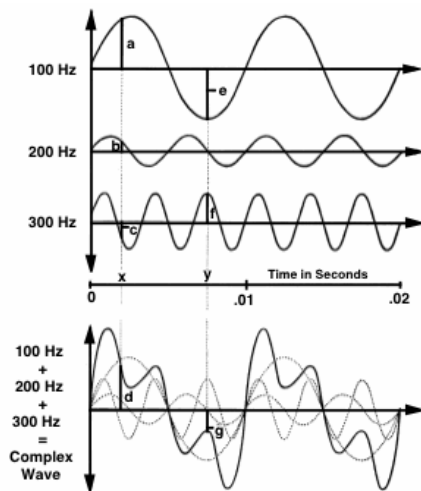
## The Fourier Transformation

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The basic principle is that any equation that repeats itself can be expressed by the sum of sines and cosines in various frequencies.

Essentially, it's the sum of the different periods of a periodic function expressed as multiple sines.

For example, Here is a Fourier transform for a sound wave:



The more terms there are to the Fourier function(series), the closer the approximation of the original function(since we rarely get a perfectly periodic wave).

An image that has had the fourier transform applied to it is represented as the variation of its colors and brightness.

It is important to know that 2 images can have the same fourier transformations. This might happen if they contain the same objects.

Because the fourier transform tells us what is happening in an image, It is often convenient to describe image processing operations to describe what happens to the frequencies in the image. For example, eliminating low frequencies gives us edges. eliminating high frequencies blurs the image.

There are many applications for Fourier Transform Image processing, some of them include :

- Feature Extraction
- Image Filter
- Image Compression

## Discrete fourier transform

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A Discrete Fourier transform is calculated over a sample window of samples. Calculating A DFT assumes that the image is repeated horizontally and vertically since it is a continuous signal.

To counteract this, we can do **windowing**, which is a function applied to the edges of the image before the DFT to reduce sharp transitions between the edges of the image.

## The Fast Fourier Transform

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The Fast Fourier Transform is extensively used in image processing and computer vision. It is faster than a regular fourier transform. It is a type of Discrete Fourier Transform. The Wiener Filter, used for blurring, is defined in terms of the fourier transform.

### TODO INSERT Function

Instead of an integral, we now have a finite sum.

The FFT can be computed by dividing the original vector, calculating the FFT for each part recursively, and joining them together.

## 2D Fourier Transform

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It is essentially the wighted sums of sines and cosines in 2 dimensions.

## Centering Signals

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After doing a DFT, the resultant transformation might not have the things we are interested in at the centre. In that case, we can shift the Fourier Transform, resulting in a change of where the frequencies are to make it easier to understand.

Centering effectively swaps the 4 quadrants of the image diagonally.

## Displaying Transformations

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As elements are complex numbers, we can view the magnitudes directly. Displaying the magnitudes of Fourier transforms is called spectrum of the transform. Often, we have to apply a log transformation on the spectrum so its more processable.

## Image Filtering in the Frequency Domain

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- Image Filtering is performed using signal filters.
- Low pass filters have a smoothing/blurring effect.
  - Ideal, Butterworth, Gaussian.
- High pass filters have a sharpening effect.
  - Ideal, Butterworth, Gaussian.