# Chapter 7 : Numerical Integration

The general idea is to estimate $\int_a^b f(x)dx$

We know that the value of $\int_a^b f(x)dx$ is the area under the curve.

We also know that $\int_a^b f(x)dx = \sum_1^n f(C_k)\triangle x_k$ (Reemans Sum)

The idea is to replace Reemans Sum with other formulas called **Quadrature Formulas** to approximate $\int_a^b f(x)dx$, so

$$\int_a^b f(x)dx = Q[f] + E[f]$$

## Quadrature Formulas

### Closed Newton-Cotes Quadrature Formulas

1. Trapezoidal Rule --> $\int_a^b f(x)dx = \frac{h}{2}(f_0 + f_1) - \frac{h^3 f''(c)}{12}$, where $h$ is the distance between $x_0, x_1$
2. Simpsons Rule--> $\int_a^b f(x)dx = \int_{x_0}^{x_2} f(x)dx = \frac{h}{3}[f_0 + 4f_1 + f_2] - \frac{h^5 f^{(4)}(c)}{90}$, where $h = \frac{b-a}{2}$
3. Simpsons $\frac{3}{8}$ rule--> $\int_a^b f(x)dx = \int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}[f_0 + 3f_1 + 3f_2 + f_3] - \frac{3h^5 f^{(4)}(c)}{80}$, where $h = \frac{b-a}{3}$

## Degree Of Percision

Defn : given a $Q[f]$, the DOP of $Q[f]$ is the largest positive integer $n$ such that the formula is exact $\forall$ polynomials of degree $\leq n$, In other terms, the DOP of $Q[f]$ is $n$ if $E[1] = E[x] = E[x^2]... = E[x^n] = 0$, but $E[x^{n+1}] \neq 0$