Chapter 5 : Cubic Spline and Curve Fitting

Cubic Spline

Cubic Spline is one of the examples of what is called the piecewise polynomial interpolation.

Given $(x_0, y_0)......(x_n, y_n)$

We want to find a function $S(x)$

But what is the general form of $S(x)$?

well,

$S(x) = (S_0(x) =$نيدعبهلقنبيسفنلل ش$—). \;-$

We end up with $4n$ unknowns. For $S(x)$ to be a cublic spline, it needs to satisfy

1. $S(x)$ must pass through all of $(x_0, y_0)......(x_n, y_n)$, that is
   $S_0(x_0) = y_0, S_1(x_1) = y_1, ...S_n(x_n) = y_n$
2. $S(x)$ is connected at the conjunctions, that is
   $S_0(x_1) = S_1(x_1), S_1(x_2) = S_2(x_2)....S_{n-2}(x_{n-1}) = S_{n-1}(x_{n-1})$
3. $S(x)'$ is connected at the conjunctions, that is
   $S'_0(x_1) = S'_1(x_1), S'_1(x_2) = S'_2(x_2)....S'_{n-2}(x_{n-1}) = S'_{n-1}(x_{n-1})$
4. $S(x)''$ is connected at the conjunctions, that is
   $S''_0(x_1) = S''_1(x_1), S''_1(x_2) = S''_2(x_2)....S''_{n-2}(x_{n-1}) = S''_{n-1}(x_{n-1})$

   The sum of the conditions = $(n + 1) + 3(n - 1) = 4n - 2$

   So we are 2 assumtions short right now !

Now, According to the 2 missing conditions we add, there are 2 types of cubic splines.

1. Natural Cubic Spline

   a. $S''_0(x_0) = 0$

   b. $S''_{n-1}(x_n) = 0$

2. Clamped Cubic Spline

   a. $S'_0(x_0) = f'(x_0)$

   b. $S'_{n-1}(x_n) = f'(x_n)$

Curve Fitting

Say we have points $(x_1, y_1)...(x_n, y_n)$

we need to find a curve $f(x)$ that fits these points the most.

There are 3 types of error norms

1. Max Error = Let error $E = MAX_{1 \leq k \leq n} |e_k|$

2. Average Error = Let error $E = \frac{\sum_{k=1}^{n} |e_k|}{n}$
3. Root Mean Square error = Let error $E = \sqrt{\frac{\sum_{k=1}^{n} (e_k)^2}{n}}$

We will be using the third norm to find the $f(x)$ that fits the data the most. This $f(x)$ is called the **least square curve**

Lineraization

The general idea is to change the shape of the curve to linear. We do tis sby changing

$$y = f(x) \rightarrow \gamma = AX + B$$

we use the normal equations

$$A \sum X_k^2 + B \sum X_k = \sum X_k \gamma_k$$

and

$$A \sum X_k + nB = \sum \gamma_k$$