University of Stuttgart | ANALYTIC COMPUTING

Machine Learning (SS 23)

Assignment 3: Bayes

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This assignment sheet consists of 2 pages, plus the 2 tasks contained in the .ipynb notebook.

Submit your solution in ILIAS as a single PDF file.[1] Make sure to list full names of all participants, matriculation number, study program and B.Sc. or M.Sc on the first page. Optionally, you can *additionally* upload source files (e.g. PPTX files). If you have any questions, feel free to ask them in the exercise forum in ILIAS.

Submission is open until Monday, 15th May 2023, 12:00 noon.

[1] Your drawing software probably allows to export as PDF. An alternative option is to use a PDF printer. If you create multiple PDF files, use a merging tool (like pdfarranger) to combine the PDFs into a single file.

Maximum Likelihood vs. Bayes

An unfair coin is tossed seven times and the event (head or tail) is recorded at each iteration. The observed sequence of events is $\mathcal{D} = (x_1, x_2, \ldots, x_7) = (\text{ head, head, tail, tail, head, head, head })$. We assume that all tosses $x_1, x_2, \ldots$ have been generated independently following the Bernoulli probability distribution

$$P(x \mid \theta) = \begin{cases} \theta & \text{if} \quad x = \text{ head} \\ 1 - \theta & \text{if} \quad x = \text{ tail} \end{cases}$$

where $\theta \in [0, 1]$ is an unknown parameter.

1. State the likelihood function $P(\mathcal{D} \mid \theta)$, that depends on the parameter θ.
2. Compute the maximum likelihood solution $\hat{\theta}$, and evaluate for this parameter the probability that the next two tosses are "head", that is, evaluate $P\left(x_8 = \text{ head}, x_9 = \text{ head } \mid \hat{\theta}\right)$.
3. We now adopt a Bayesian view on this problem, where we assume a prior distribution for the parameter θ defined as:

   $$p(\theta) = \begin{cases} 1 & \text{if } 0 \leq \theta \leq 1 \\ 0 & \text{else} \end{cases}$$

   Compute the posterior distribution $p(\theta \mid \mathcal{D})$, and evaluate the probability that the next two tosses are head, that is,

   $$\int P\left(x_8 = \text{ head }, x_9 = \text{ head } \mid \theta\right) p(\theta \mid \mathcal{D}) d\theta$$