DMET 901: Computer Vision

Corner Detectors

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Corner Detectors

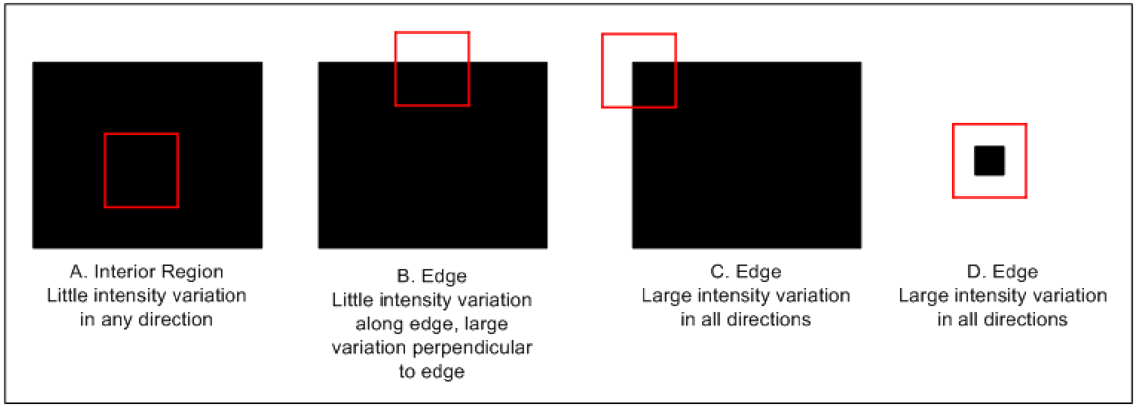
- What is a corner?
- It is the point where two lines meet forming an angular inclination.
  - Juction detection, JUDOCA.
- It is the area where any small shift, in perspective, in any direction causes a relatively large change in the scene.
  - Moravec Operator, Harris Corner Detector.
- In is the point with relatively small self esteem defined by similarity to neighborhoods.
  - SUSAN Corner Detector.

Moravec Operator

Moravec operator estimates the *cornerness* of a point by computing a measure of intensity variation in a given neighborhood w with a shift of u and v

$$V(x,y)_{w(u,v)} = \sum_{i=-1}^{1}\sum_{j=-1}^{1}\left(f(x+i,y+j) - f(x+u+i,y+v+j)\right)^2$$

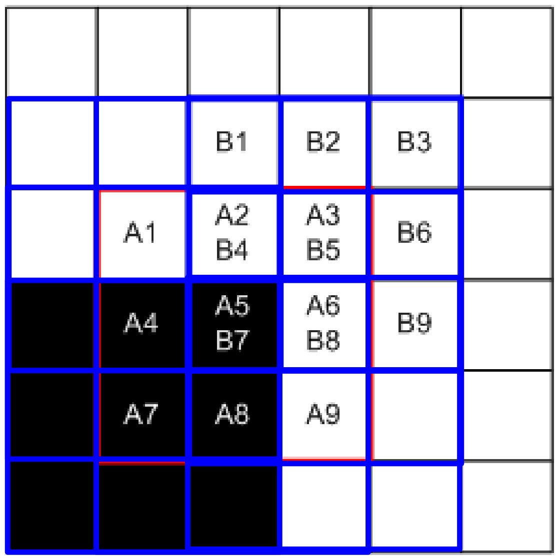
Why would Moravec operator work?



http://kiwi.cs.dal.ca/~dparks/CornerDetection/algorithms.htm

Moravec Operator

- It measures the cornerness by shifting a window around the considered pixel by 1 pixel in each of the 8 principal directions and calculating the corresponding V



$$V = \sum_{i=1}^{9} (A_i - B_i)^2 = 3 * 255^2$$

http://kiwi.cs.dal.ca/~dparks/CornerDetection/algorithms.htm

$$V = \begin{array}{cc} \begin{bmatrix} 3*255^2 \\ 2*255^2 \\ 3*255^2 \\ 2*255^2 \\ 5*255^2 \\ 2*255^2 \\ 3*255^2 \\ 2*255^2 \end{bmatrix} & \begin{array}{c} \textbf{Window } \boldsymbol{B} \textbf{ Centered at} \\ \text{A3} \\ \text{A2} \\ \text{A1} \\ \text{A4} \\ \text{A7} \\ \text{A8} \\ \text{A9} \\ \text{A6} \end{array} \end{array}$$

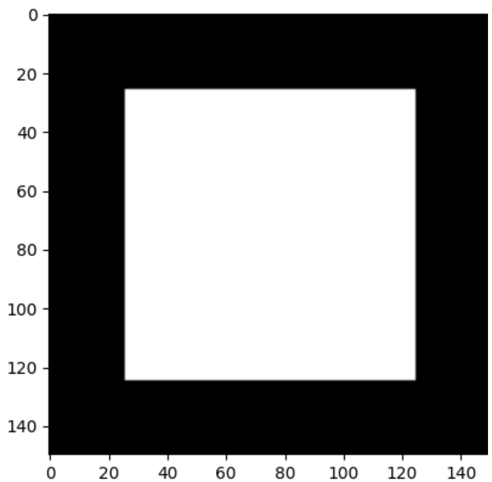
- The cornerness at a pixel (x, y)

$$C(x,y) = \min_{u,v} V(x,y)_{w(u,v)}$$

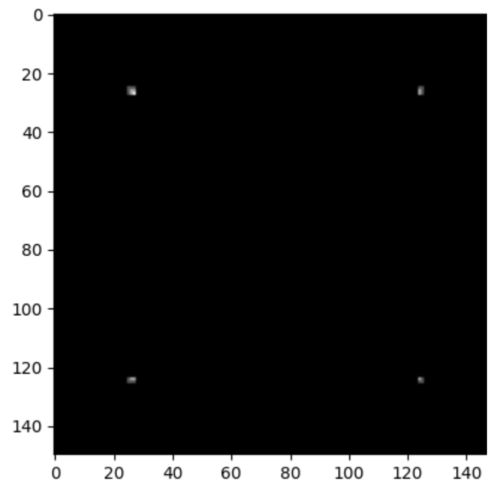
- Or Average?

Moravec Operator

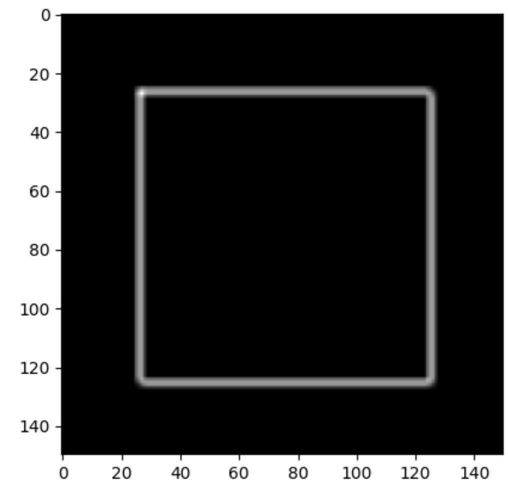
Input



Minimum



Average



Harris Corner Detector

- Improved upon Moravec operator

$$S_W(\Delta x, \Delta y) = \sum_{x_i \in W} \sum_{y_i \in W} \left(f(x_i, y_i) - f(x_i - \Delta x, y_i - \Delta y)\right)^2$$

- Similar to the goal of Moravec operator, we try to find the minimum value of S_W

- This could be found analytically if the shifted image patch is approximated by the first-order Taylor expansion

$$f(x_i - \Delta x, y_i - \Delta y) \approx f(x_i, y_i) + \left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y}\right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

- Substituting in the expression for S_W

$$S_W(\Delta x, \Delta y) = \sum_{x_i \in W} \sum_{y_i \in W} \left(f(x_i, y_i) - f(x_i, y_i) - \left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y}\right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}\right)^2$$

Harris Corner Detector

$$S_W(\Delta x, \Delta y) = \sum_{x_i \in W} \sum_{y_i \in W} \left(f(x_i, y_i) - f(x_i, y_i) - \left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y} \right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2$$

$$= \sum_{x_i \in W} \sum_{y_i \in W} \left(- \left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y} \right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2$$

$$= \sum_{x_i \in W} \sum_{y_i \in W} \left(\quad \left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y} \right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2$$

$$= \sum_{x_i \in W} \sum_{y_i \in W} [\Delta x, \Delta y] \left(\begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \right) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$= [\Delta x, \Delta y] \left(\sum_{x_i \in W} \sum_{y_i \in W} \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \right) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$= [\Delta x, \Delta y] \, A_W(x, y) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix},$$

Harris Corner Detector

- The goal now is to minimize

$$S_W(\Delta x, \Delta y) = [\Delta x, \Delta y]\, A_W(x, y) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

- This is equivalent to finding the eigenvector of A_w corresponding to the minimum eigenvalue
- What are the eigenvectors and eigenvalues?

  For any matrix L, the eigenvectors and eigenvalues are defined as

$$Lf = \lambda f \quad \rightarrow \quad f^T L f = \lambda$$

f is an eigenvector of L

λ is the eigenvalue corresponding to f

- For A_w, the eigenvector is $\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$ and the eigenvalue is the corresponding S_w

Harris Corner Detector

- Therefore, finding the eigenvalues of A_w would be sufficient to solve the minimization problem where

$$A(x,y)=\begin{bmatrix} \sum_{x_i\in W}\sum_{y_i\in W}\frac{\partial^2 f(x_i,y_i)}{\partial x^2} & \sum_{x_i\in W}\sum_{y_i\in W}\frac{\partial f(x_i,y_i)}{\partial x}\frac{\partial f(x_i,y_i)}{\partial y} \\ \sum_{x_i\in W}\sum_{y_i\in W}\frac{\partial f(x_i,y_i)}{\partial x}\frac{\partial f(x_i,y_i)}{\partial y} & \sum_{x_i\in W}\sum_{y_i\in W}\frac{\partial^2 f(x_i,y_i)}{\partial y^2} \end{bmatrix}$$

- Instead of computing the eigenvalues, Harris suggested using the following approximation

$$R(A)=\det(A)-\kappa\, trace^2(A)$$

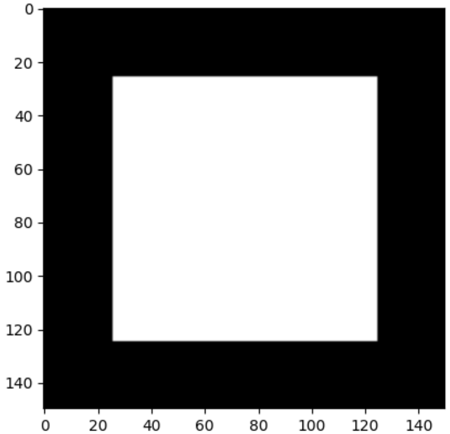
where det(A) is the determinant of the local structure matrix A

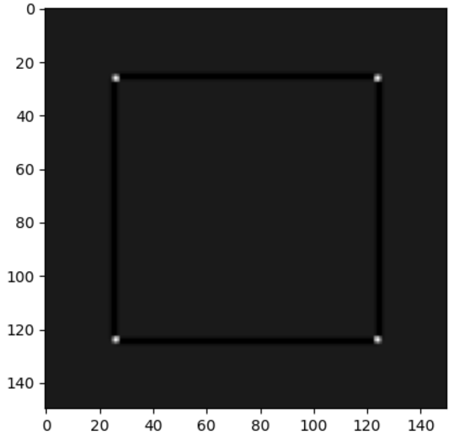
$trace\,(A)$ is the trace of matrix A (sum of elements on the diagonal)

κ is a tunable parameter

Harris Corner Detector

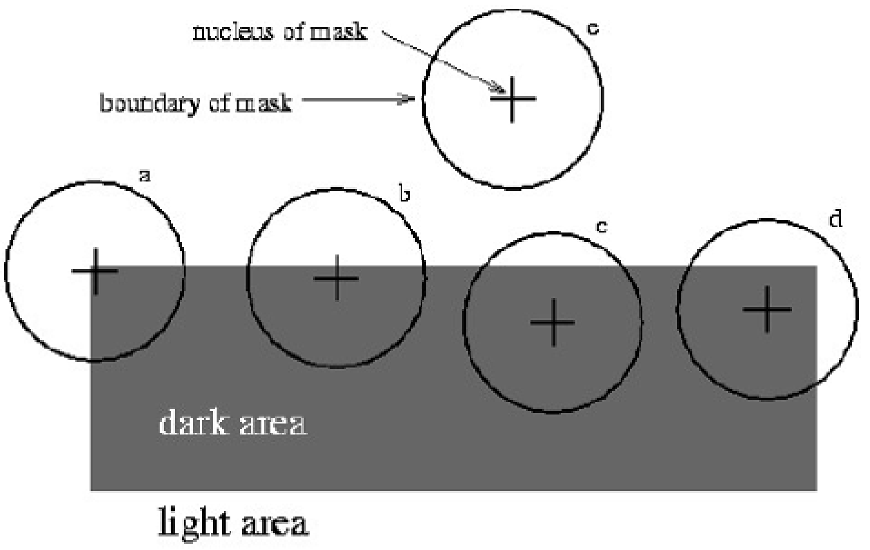
- Algorithm
  - Estimate intensity gradient in 2 perpendicular directions for each pixel
    $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$
  - For each pixel and a given neighborhood window
    - Calculate the local structure matrix A
    - Evaluate the response function $R(A)$





SUSAN Corner Detector

- **SUSAN** stands for **S**mallest **U**nivalue **S**egment **A**ssymilating **N**ucleus.
- A circular mask (having a center pixel which shall be known as the "nucleus") is shown at five image positions.
- If the brightness of each pixel within a mask is compared with the brightness of that mask's nucleus then an area of the mask can be defined which has the same (or similar) brightness as the nucleus.
- This area of the mask shall be known as the "USAN".
- Note that the USAN area is at a maximum when the nucleus lies in a flat homogeneous region. It falls to half of this maximum very near a straight edge, and falls even further when inside a corner.



SUSAN Corner Detector

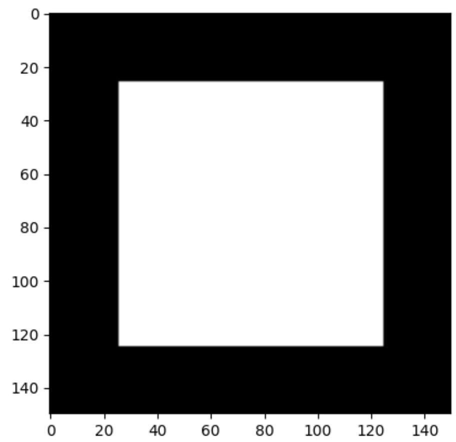
- Every pixel is compared to the nucleus using the comparison function:

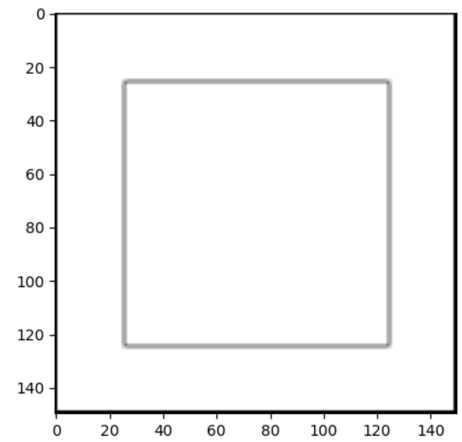
$$c(\overrightarrow{m}) = e^{-\left(\frac{I(\overrightarrow{m}) - I(\overrightarrow{m_0})}{t}\right)^6}$$

Where t is the brightness difference threshold.

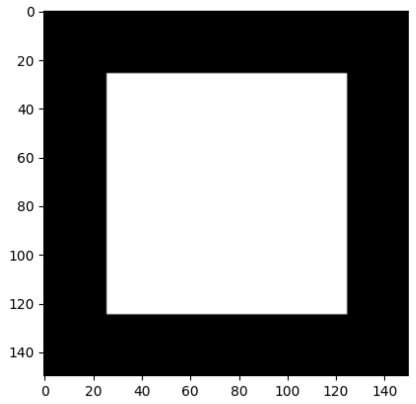
- For all $\overrightarrow{m}$ in the neighborhood of $\overrightarrow{m_0}$:

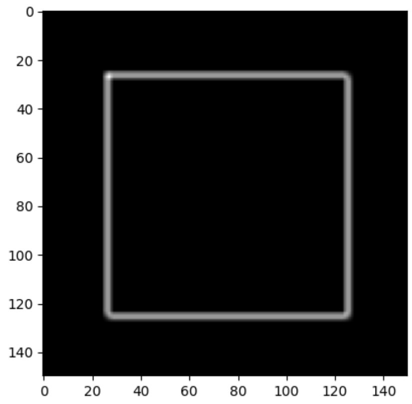
$$n(\overrightarrow{m_0}) = \sum_{\overrightarrow{m}} c(\overrightarrow{m})$$

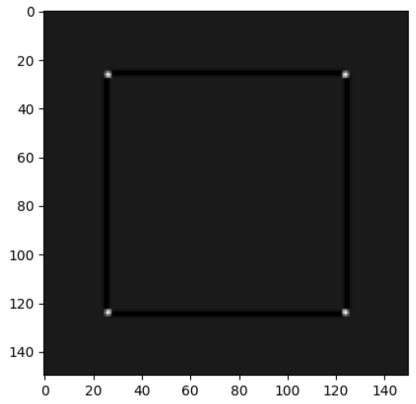


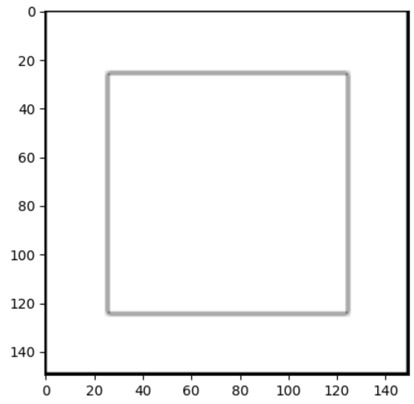


Corner Detectors Examples

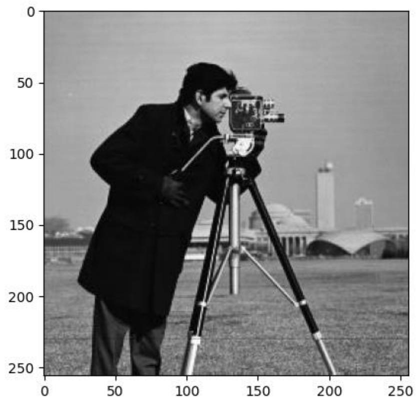


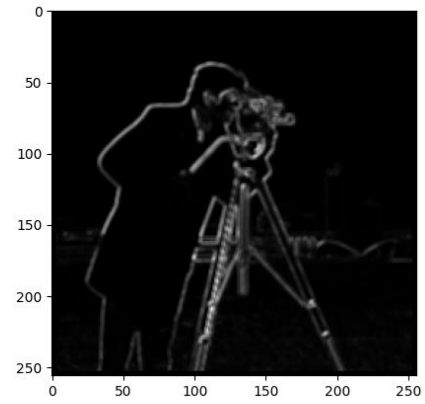


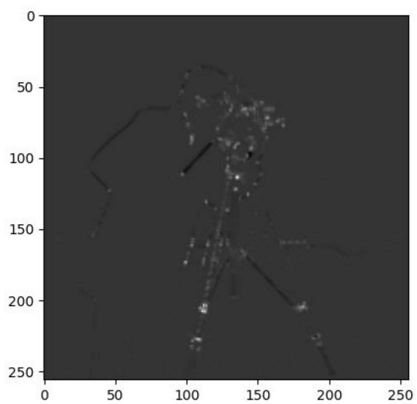


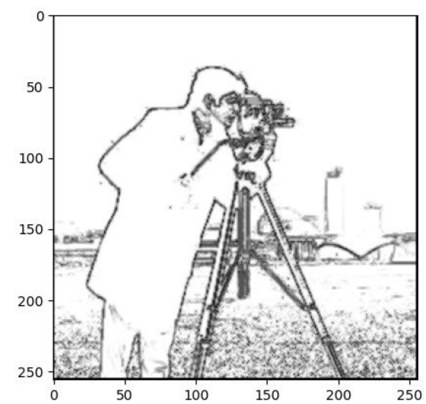


Corner Detectors Examples









Corner Detectors Examples