what is calculus?

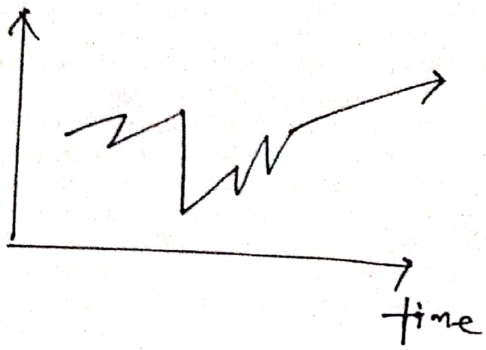
→ Calculus, a branch of mathmatics, developed by Newton and Leibniz, deals with the study of the rate of change

what is calculus used for? How to use calculus in real life.

calculus

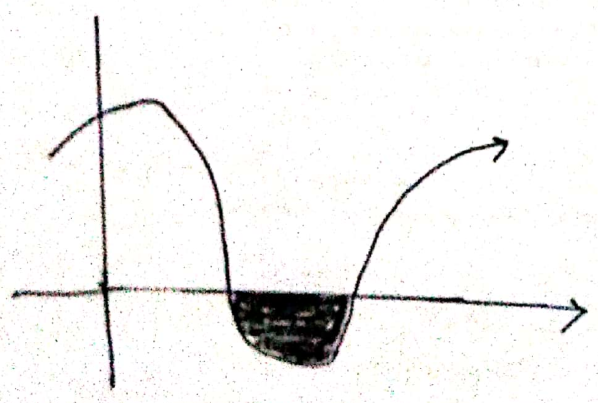
Integration + Differentiation

we learn about calculus in college and we know it includes integration and differentiation but what does it actually use for and how the language of calculus appears everywhere in modern science and Technology whether we are modeling the rise in the fall of the stock market or determining exactly when a space rocket will arrive into Earth's orbit it the language invented for the specific purpose of describing the dynamic nature of our universe to put it simply calculus the maths of motion and change. The

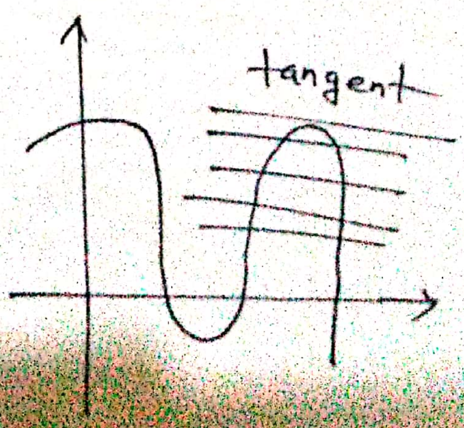


word calculus originates from the Latin word meaning pebble the Romans use pebbles to perform calculation on an abacus and the word became associated generally with computation just like the word calculator the beauty of calculus is not just in the maths alone it's in the way that calculus can form a connection or relationship and a language to describe the dynamic nature of our word there are unlimited uses and benefits of in any field calculus the language of motion and change and by using calculus we have the ability to find the effect of changing condition on a system.

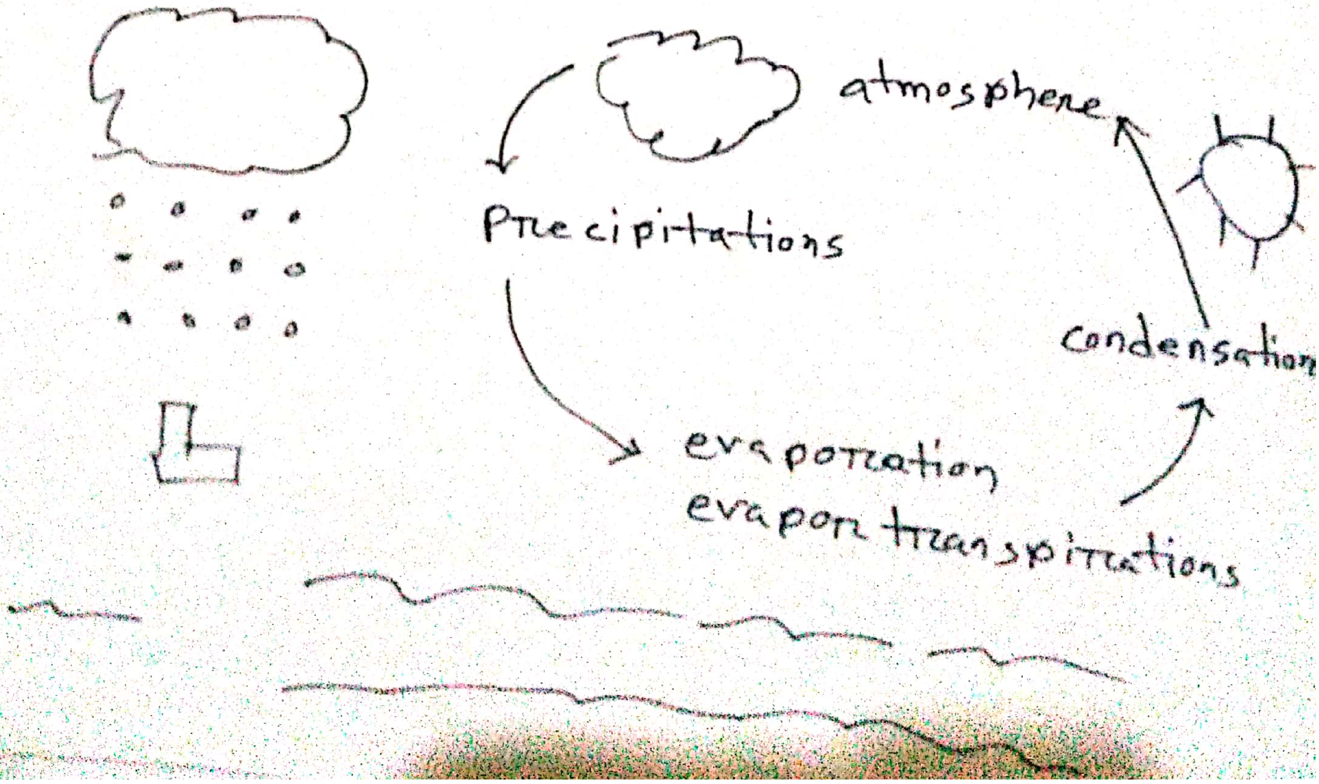
$$\int_a^b f(x)\,dx = F(b) - F(a)$$



$$\frac{d}{dx} f(x) = f'(x)$$



The weather for example in the atmosphere we have changing Temperature and changing pressure so by using differential equations meteorologists can indicate and predict weather for our benefit calculus holds incredible power over the physical words by modeling and controlling system it's the language of medical experts scientists, engineers, statisticians, physicists and economists.



weather Forecasting

I model the weather using calculus.

Atmospheric Dynamics

$\frac{d}{dx} f(x) = f'(x)$ $\int_a^b f(x)\,dx = f(b) - f(a)$ $C_v \frac{dT}{dt} + p \frac{d\alpha}{dt} = q + f$

if a quantity or a system is changing we can use a mathematical modeling of calculus to analyze a system find an optimal solution and predict the future motion, electricity, heat and light harmonics and acoustics astronomy radioactive decay reaction rates, both in death rates costs and revenue all those can be modeled beautifully using calculus.

calculus models:

Motion
electricity
heat
light
Harmonics
acoutics

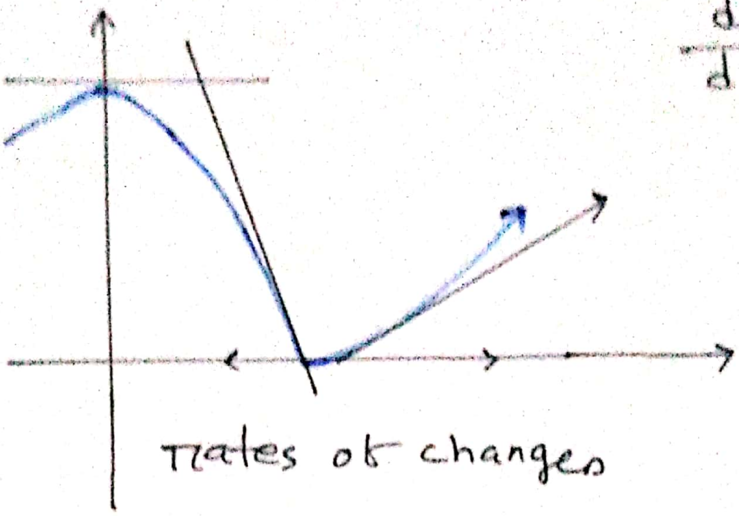
astronomy
radioactive decay
reaction rates
birth / death rates
costs
revenue.

The calculus we have two different branches the first branch is differential calculus and this involves the concept of the derivative of a function this branch of calculus studies the behavior and rate

calculus
↙ ↘
Differential   Integral

the derivative $f'(x)$ / g'

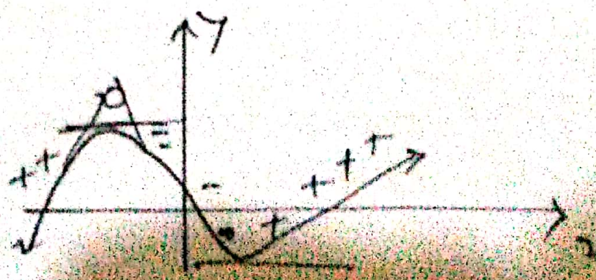
$$\frac{d}{dx} f(x) = f'(x)$$



rates of changes

at which a quantity like distance for example changes over when we use the process of differentiation we are essentially analyzing the changing rate of a quantity of a quantity and making predictions about it's behavior so.

Q what is happening to this quantity



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it's rate of change is positive 0, negative 0

$$\frac{d}{dx} 4x^3 - 2x + 7$$

$$= 12x^3 - 2$$

= rate of change

what happen when ~~x~~ $x = 2$?

$$\frac{d}{dx} = 4x^3 - 2x + 7$$

$$= 12x^2 - 2$$

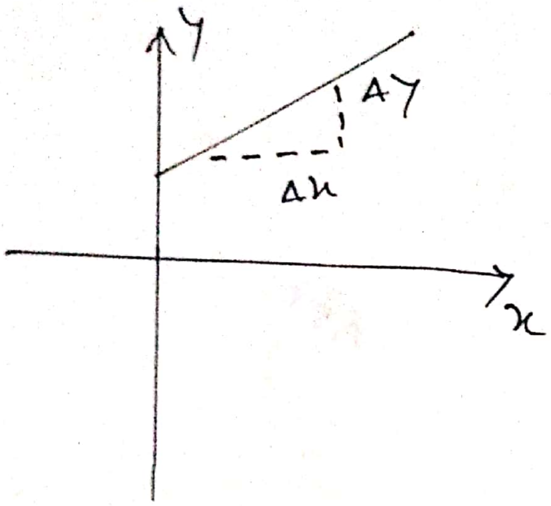
$$= 12(2)^2 - 2$$

$$= 46$$

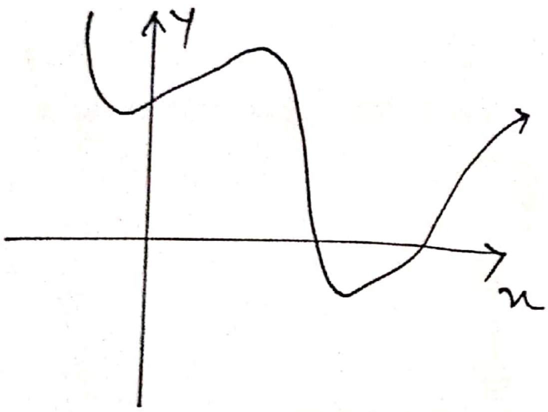
by finding the derivative we can find the exact instantaneous rate of change at any point we like. ~~it~~

If a function has a constant rate of change we get a straight line and it's easy enough to

Just find the rate of change using rise over run.



$$y' = \frac{rise}{run} = \frac{\Delta y}{\Delta x}$$



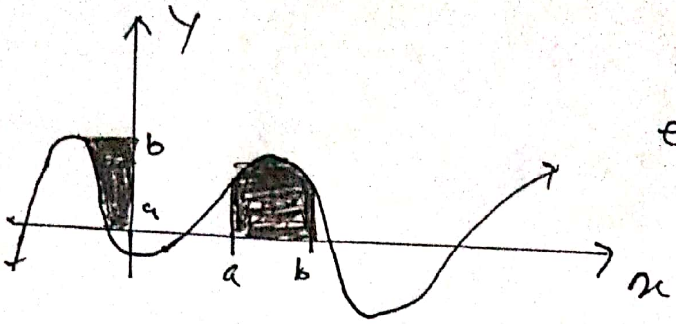
$$\frac{d}{dx} f(x) = f'(x)$$

sub $x =$ value into $f'(x)$ where you want to find the rate of change.

Integral calculus

Integral $\longleftrightarrow$ different

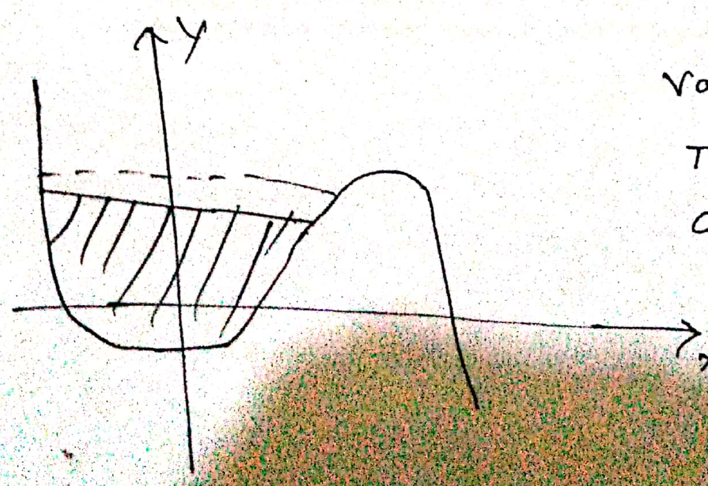
2D:



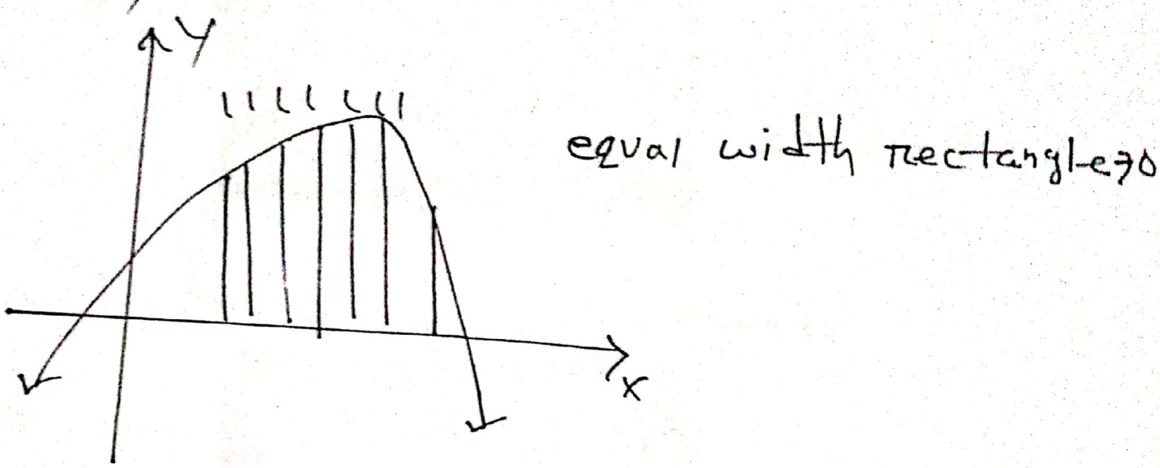
Area under curve enclosed by x or y axis.

integral is the reverse process of differentiation sometimes called anti differentiation with integration we can describle the area of a 2D region with a curved boundary or the volume of a 3D object with a curved boundary we

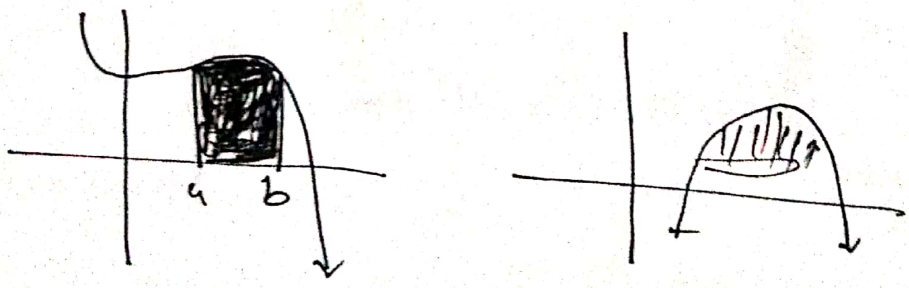
3D:



volume rotating a region of a curve around x or y axis integrate by breaking the region apart into thin unlimited vertical rectangles of equal width untill the width of the rectangle are virtually become 0 which is



This process = A limit



is called a limit this limiting process allows us to calculate areas and volumes with exact precision it we differentiate a function and then integrate it will always take us back to where we started both

$$\frac{d}{dx} 30x^3 - x^2$$

$$= 90x^2 - 2x$$ Differentiation

$$\int 90x^2 - 2x \, dx$$

$$= 30x^3 - x^2 + c$$ Integration

Fundamental theorem of calculus.

These branches differentiation and integration are connected together by something called the fundamental theorem of calculus.

This theorem created by Newton and Leibnitz states at differentiation and integration are inverse operations or opposite just like Yin Yin Yank black and white or matter and anti-matter

Take the square root for instance the opposite of taking the square root is squaring a number just like a differentiation is the opposite or inverse of integration ~~so~~

$\sqrt{x} \longleftrightarrow x^2$

opposites

Integration $\longleftrightarrow$ Differentiation

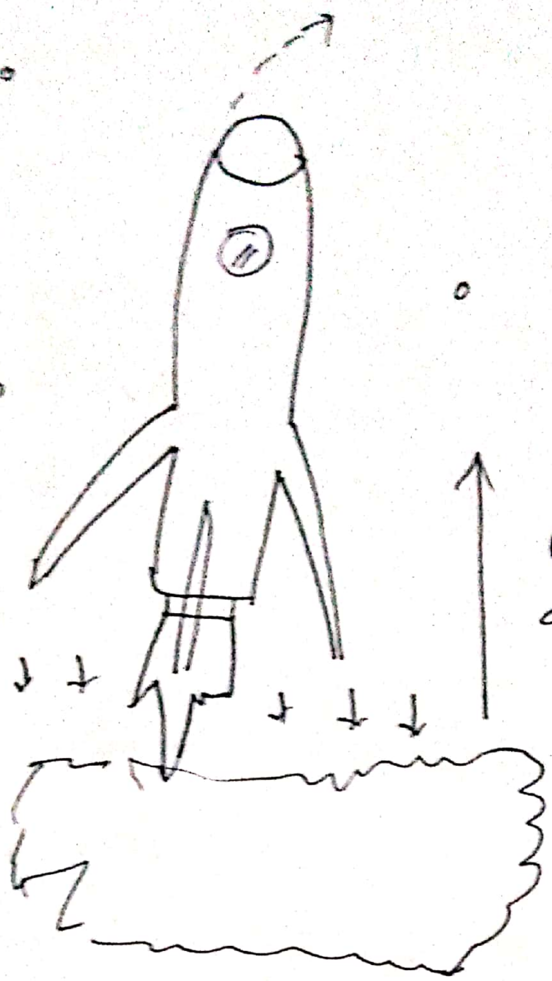
inverse operation
(opposites)

now that we know what calculus is would not be interesting to see how it can be used in aerospace to describle a rocket launch if ~~in~~ an object is in motion like a rocket we can use calculus to model it the trust of a rocket into space is based on the calculus of motion which physicists hen momentum is rocket physics we are applying Newton's second and third law to a rocket that has a variable mass how is the mass variable the Rocket mass is decreasing over two as the fuel propellant is being burned off as the rocket propellant ignites the rocket experience a very large acceleration so as the exhaust exits out the back of the rocket at a very high velocity this backwards acceleration exerts a push force on the

Thrust = Δp

(rate of change of momentum)

$P_1 = \frac{dmv}{dt}$



push force
acceleration

↓ high velocity

↓ blackwards
acceleration

Thrust : $p' = \frac{dmv}{dt}$

$F = ma$ (Newton's 2nd law)

F / thrust = 1st derivative of momentum

conservation of momentum

↑ REACTION
rocket acceleration

Fuel
↓ Action

Rocket propulsion
Newton's 3rd law

$FA = - FB$

increase in momentum

the Rocket in the opposite direction causing the Rocket to accelerate upwards the force acting on the rocket called the thrust is the rate of change of momentum which is the first derivative of momentum using calculus momentum or the amount of motion of the rocket p equal mass times velocity and so the rate of chang of momentum p- equal ~~(mv)~~

$p' = \frac{dmv}{dt}$ the Thrust of the rocket we can also write this as a physics equation

$F = ma$ (Newton's 2nd law)

and rewriting this from a calculus stan point F / thrust = 1st derivative of momentum

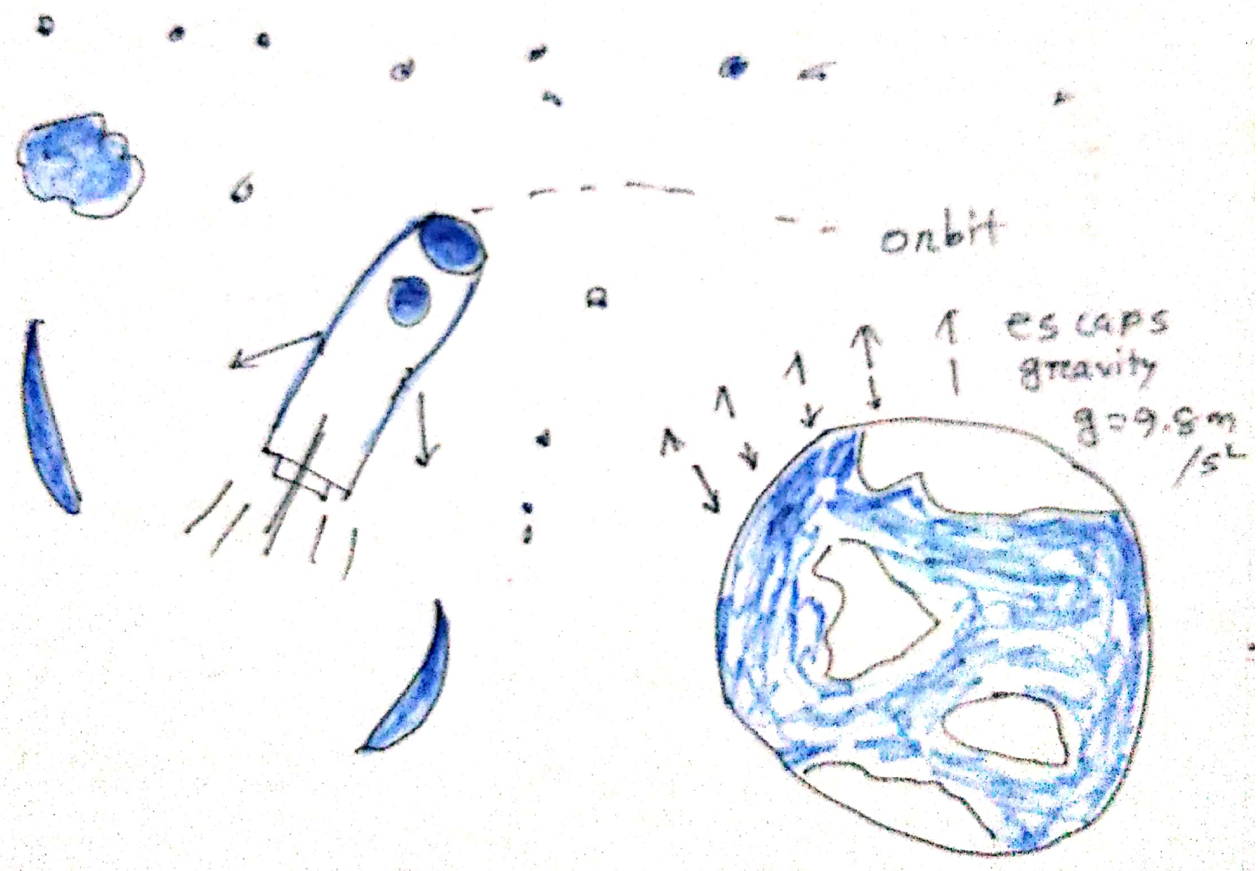
To put it simply the thrust of the rocket during a launch is the first derivative of momentum rocket

Rocket propulsion

Newton 3rd law conservation of momentum this dictates that if material is ejected back wards like the exhaust in a Rocket launch the forward momentum of the remaining rocket must increase because an isolated system can not change its net momentum in other words for every action there

there is an equal and opposite reaction newtons third law

After launch to achive the decired final orbital velocity around the earth or to escape from Earth's gravity the mass of the rocket



must be as small as possible and so the Rocket sheds mass by using different rocket stages separating it's parts such as the rocket boosters now that we have seen calculus applied in a physics word of aerospace let's see the benefits of calculus in the world of economics

microeconomics

A lot of people dream about running their own business would not be great if you could work out exactly how to maximize your business profits and help build a thriving company well calculus can be used to maximize profits and revenue for any Business in actual fact Calculus provides the language of microeconomics.

and the means by which economics can model and solve financial problem let's see how we can apply calculus to maximize your profits in your theoretical game business power

~~maximum~~

maximisation of Revenue

$R(x)$ = Revenue function

$R'(x)$ = marginal revenue

+ $\boxed{\text{game}}$ $R'(x) = \dfrac{\Delta R(x)}{\Delta x}$

$= \dfrac{\text{change in revenue}}{\text{change in number games}}$

How many units should we sell to maximise Revenue?

$\boxed{\text{batch of games}}$

Legend of Horus = $50

$$R'(x) = \frac{\Delta R(x)}{\Delta x} \rightarrow \frac{\text{change in Reve}}{\text{change in no. game}}$$

↑ $ Legend of Horus = $30

$$R'(x) = \frac{\Delta R(x)}{\Delta x} = \frac{50-30}{2-1} = 20$$

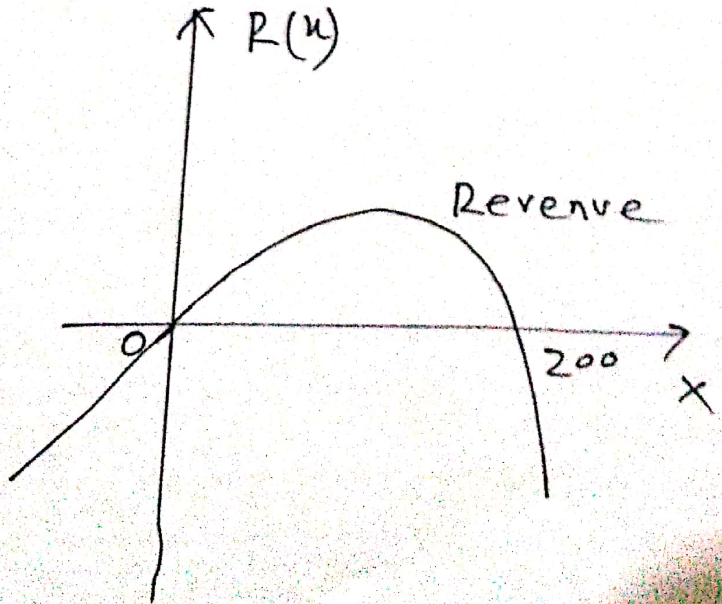
$$= \$20$$

The marginal revenue gained by producing the second video game is change in revenue so 50 - 30 = 20$ divided by the change in the quantity of video games 1 which equal 20$ but this is less than the price that you wanted to charge for an additional video game as you can see we have finally problem here and we need to model the revenue here using calculus to find the optimal maximize your revenue let's say we model the revenue for power power and produce the revenue function as our.

$$R(x) = 100x - \frac{1}{2}x^2 \rightarrow x\left(100 - \frac{1}{2}x\right)$$

$R(x)$ = Revenue of pow pow,

x = number of video game



last

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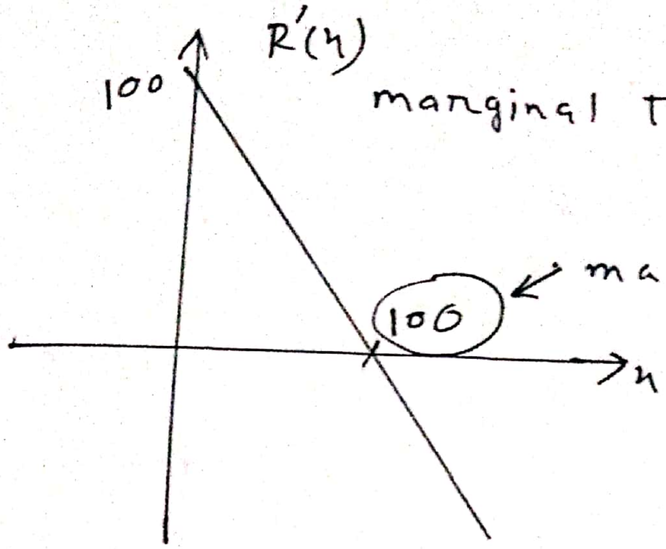
you can be sure that we can turn to calculus as a tool to model the problem and provided us with the answer

$R(x) = 100x - \frac{1}{2}x^2$ (Revenue)

$R'(x) = 100 - x$ (marginal Revenue)



marginal revenue fn

max revenue occur here $R'(x) = 0$ (first derivative)

$R'(x) = 100 - x$

let $R'(x) = 0$ to find max revenue

$100 - x = 0$

$\therefore x = 100$ (100 ~~game~~ video game)

$R(x) = 100x - \frac{1}{2}x^2$

$R(x) = 100(100) - \frac{1}{2}(100^2)$

$= \$\ 5000.$

maximum revenue occurs when we sell 100 video games.

easily maximize your business profit by using first derivative of revenue marginal revenue it's also known that a company products best results when production and sales continue on and to a marginal revenue equal marginal cost. now that we have seen the benefit of calculus, in aerospace and economics.

Calculus in Medicine

let's say that you are a doctor. Doctor and you would like to observe the progression of a China in one of your patients Tareq. Tareq has a small early onset tumor and you would like to see whether it's responding to a new innovative drug arrivederci tumor which has no-side-effects as a Doctor you would like to model the growth of John's Tumor using calculus to analyze the progression or reversion of his disease the function you have created to model the progression of growth of John's Tumor is an exponential function with respect to time v

$v(t) = v_0 e^{at}$

v = volume of the tumour

v_0 = initial volume of the tumour

a = constant

t = time

using first derivative

$$v'(t) = \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = SGR$$

$v'(t)$ = rate of change of tumour volume

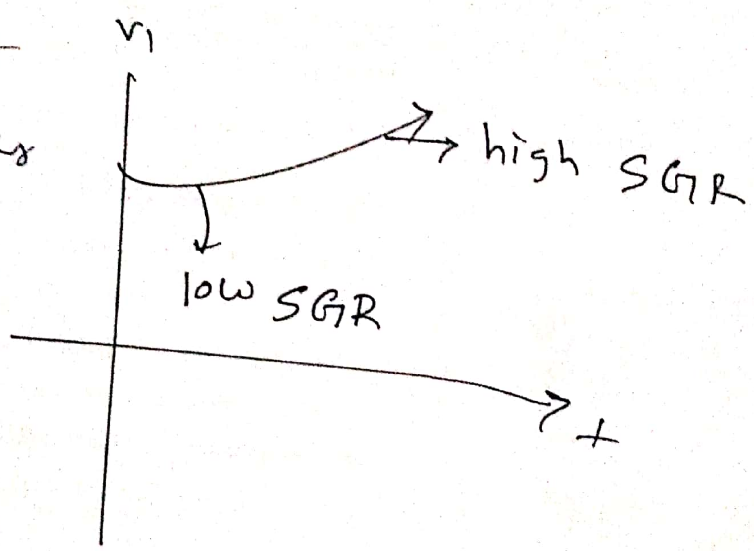
derivative v- gives important ~~in~~ about whether Tareq's Tumor is growing or shrinking and the rate at which it's doing so v-

V dash tells Dr. OR the relative change in China volume per unit of time.

Dr. QR differentiates the function and produces fee so specific growth rate equal V 0 time

$$V'(SGR) = V_0 \times a \times e^{at}$$
$$= V_0 a e^{at}$$

SGR tells me about the changes



to the power of 18 so V- if the tumor has a higher SGR or V- Doctor cure can interpret this as a rapidly growing tumor and then he can make decistion about the form of therapy or change in therapy to cure the tumor and get Tarez back to good health again if the SGR is low then dr. cure can assume that the new innovative drug arrivederci China is working the tumour is shrinking → ~~[illegible]~~ and discontinous Tarez on the current regime as we have seen the beauty and benefits of calculus. can be applied in any scenario of change or motion whether it be Aespace, economics, medice and more the benefit of calculus are endless and if we have any problem in any dynamic situation that involves change either on motion