Sampling and Reconstruction

Signals and Systems: Experiment 4

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Overview

1. Impulse-Train Sampling
2. Signal Reconstruction
3. Experiment Problem

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Impulse-Train Sampling

Consider a signal $x(t)$ and the periodic impulse train

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \tag{1}$$

Impulse-train sampling yields

$$x_p(t) = \sum_{k=-\infty}^{\infty} x(kT)\,\delta(t - kT) \tag{2}$$

Spectrum of the sampled signal is

$$X_p(jw) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(w - kw_s)) \tag{3}$$

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Signal Reconstruction

The sampled signal is

$$x_p(t) = \sum_{k=-\infty}^{\infty} x(kT)\,\delta(t - kT) \tag{4}$$

For band-limited interpolation, the interpolation function is given by

$$h(t) = \frac{w_c T}{\pi} Sa(w_c t) = \frac{w_c T}{\pi} \frac{\sin(w_c t)}{w_c t} \tag{5}$$

Band-limited interpolation produces

$$x_r(t) = \sum_{k=-\infty}^{\infty} x(kT) \left[\frac{w_c T}{\pi} \frac{\sin(w_c (t - kT))}{w_c (t - kT)} \right] \tag{6}$$

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Problem: Sampling and Reconstruction

Consider a signal $x(t)$ as

$$x(t) = e^{-0.5t}u(t) \tag{7}$$

and sample the signal with a sampling period of $T_s = 0.1$ s. For $t \in [0, 4]$ and $w \in [-100, 100]$:

(1) Plot $x(t)$ and $x(kT_s)$;
(2) Plot $|X(jw)|$;
(3) Plot $|X_p(jw)|$;
(4) Plot $x_r(t)$;
(5) Plot the interpolation error $|x(t) - x_r(t)|$.

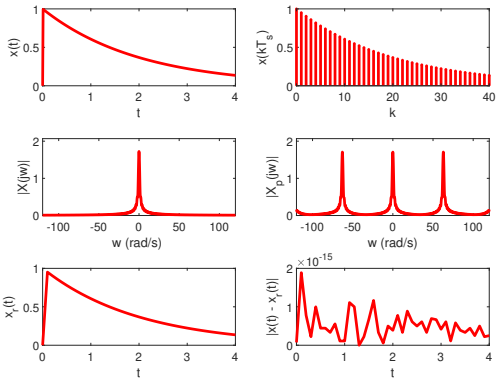

Figure 3.1: Illustration of sampling and reconstruction.

Thank You!