Fundamentals of Information Theory

Introduction to Information Theory

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Outline

- What is information?
- How was information theory born?
  - Technical background
- What are the contributions of information theory?
  - Theory vs. Engineering
- Who is Claude Shannon?
  - Life of a Master of Theory

01

What is information?

学习目标

1. 认识信息的复杂性
2. 说出信息的3个层面

Information Age——What is information?

微助教讨论

What is information?

吴冠中，山水画，桂林印象

- **Eliminate Redundancy?**

What is information?

- **Extract essential characteristics?**

What is information?

- **Symbolic presentation?**

What is information?

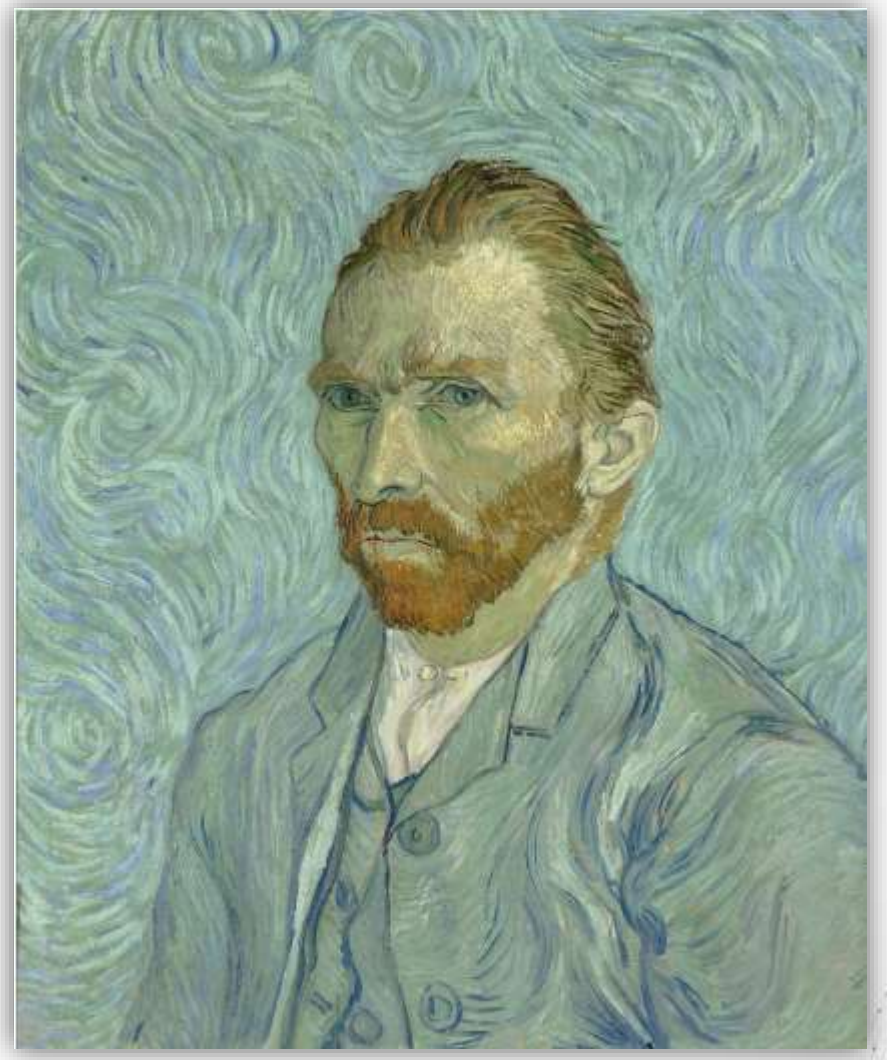
飞流直下三千尺，
疑是银河落九天。

李白《望庐山瀑布》

枯藤老树昏鸦，
小桥流水人家。

马致远《天净沙·秋思》

What is information?



- **Inspiration?**

What is information?

- Emotional?
- Subjective?
- Collateral information?

What do you see in the drawing below?

"There are a thousand Hamlets in a thousand people's eyes"

对信息的感性认识

第二次听天气预报说明天是下雨，得到多少信息？

A. 没有得到信息　　B. 还是得到了信息

- 感觉上信息量应为零，因为已经知道这个消息了，是确定性的事件。
- 但又似乎不完全是，因为又得到了一次确认，对真实性更有把握了。

对信息的感性认识

一位渔民和一个工人同时收听天气预报说明天是下雨，谁得到了更有价值的信息？价值多出多少？

A. 渔民多　　B. 工人多　　C. 一样多

- 感觉上是渔民，天气情况对他更重要。
- 价值多出多少无法回答。

对信息的感性认识

新闻里说中国男乒乓球队战胜了巴西男乒乓球队，和中国男足战胜了巴西男足，分别获得多少信息？

A. 前者多　　B. 后者多　　C. 一样多

- 感觉上似乎是后者信息量大些，因为这种几率非常小，人们感觉很惊讶。
- 不过对于不懂体育的人，似乎又没多大区别。
- 说不太清分别得到多少信息。

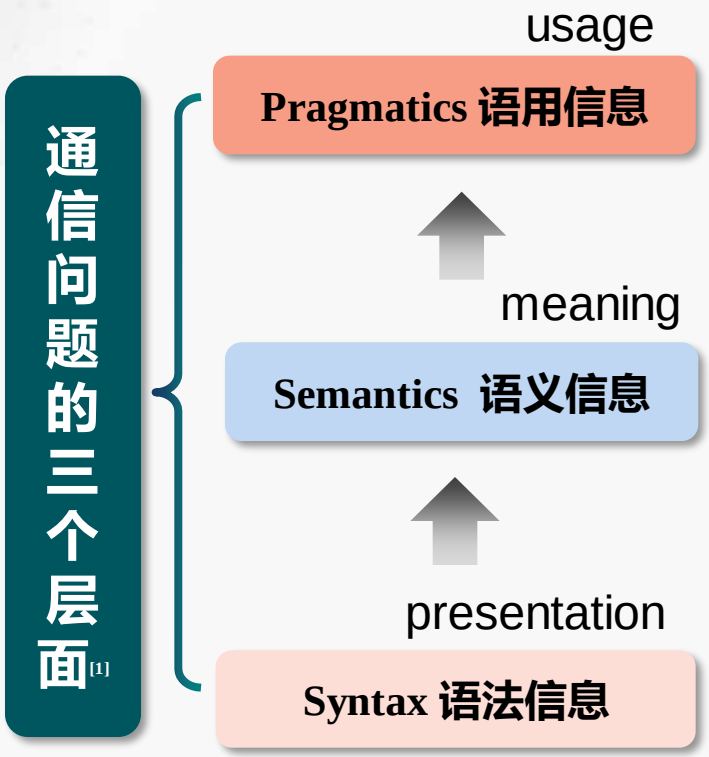
Information seems very complicated...

- 信息既有其**客观性**，又有其**主观性**。

- **客观性**：所有人所听到的消息都是一样的。
  - 天气预报说今天会下雨。
  - 电视报道说中国足球队战胜了巴西足球队。

- **主观性**：与接收人的兴趣、职业等主观因素有关。
  - 兴趣：听到体育新闻的人是否爱好体育的例子。
  - 职业：渔民和工人听海浪预报的例子。

Information seems very complicated...



- Meaning within a certain context
- Highly subjective, closely dependent on background & context

- Semantics are hard to represent, extract (sense) and compute (inference)
- Can be affected by language, background, personalities, etc.

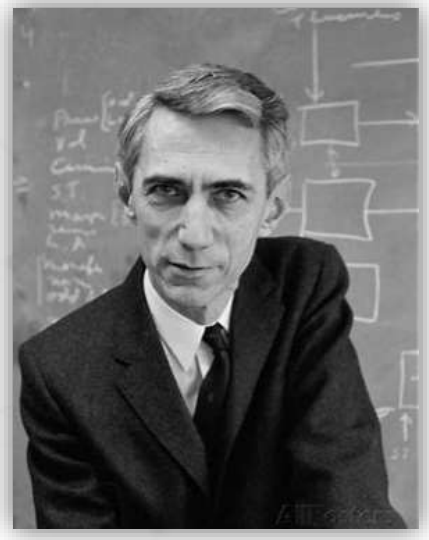
- Easy to represent
- Objective

- **No universal definition**
- Lack of complete, clear, universally recognized concept of information
- **Difficult to measure** how much information

[1] Weaver W. Recent contributions to the mathematical theory of communication[J]. ETC: a review of general semantics, 1953: 261-281.

How to define information?

How to measure information?



Claude E. Shannon, “A Mathematical Theory of Communications,” Bell System Technical Journal, July & October 1948.

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- **How was information theory born?**
  - **Technical background**
- What are the contributions of information theory?
  - Theory vs. Engineering
- Who is Claude Shannon?
  - Life of a Master of Theory

02

Technical Background

学习目标

1. 概述1948之前通信技术发展背景
2. 说出≥3个关于通信当时尚未解答的问题

information theory

1948

Long before Shannon's information theory

古人都是怎么通信的?　　兵者，国之大事，死生之地，存亡之道，不可不察也。

烽火通信系统

- 优势：**传输速度快**（汉代烽火速度为一昼夜580公里、明代烽炮速度一昼夜可达3500余公里）
- 劣势：**信息量**极为**有限**、传输**成本极高**，仅能作为**警报**使用 → **紧急下发公文怎么办?**

Long before Shannon's information theory

古人都是怎么进行**大信息量**的通信的?

一骑红尘妃子笑，无人知是荔枝来。

驿传通信

- 优势：**传输速度较快、**可用于**传递通用的、一般性的信息**
- 劣势：**成本极高**、受地形影响

Long before Shannon's information theory

古代的**普通人**该如何通信呢?

烽火连三月，**家书抵万金**。

一怀愁绪，几年离索，
山盟虽在，**锦书难托**。

Long before Shannon's information theory

古人都是怎么通信的?

“儿已平安到达，勿念。”

飞鸽传书

- 优势：**传输速度快、成本较低、民用通信**
- 劣势：**单向**通信、**可靠性低**

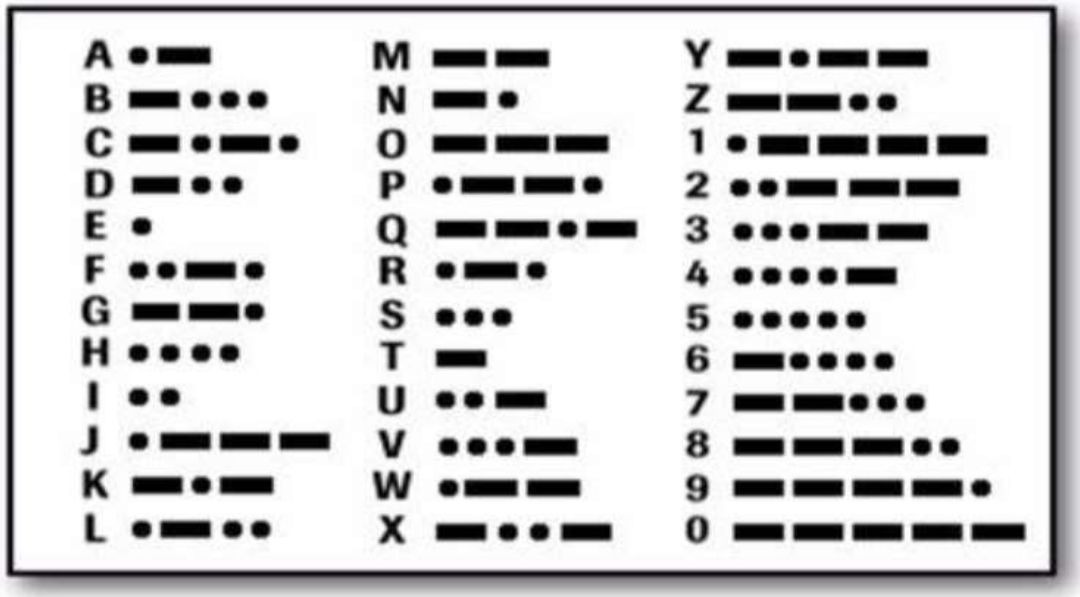
Key Requirements of Communications Systems

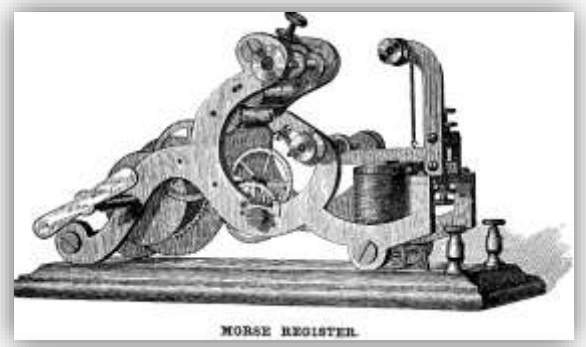
Speed

Efficiency

Reliability

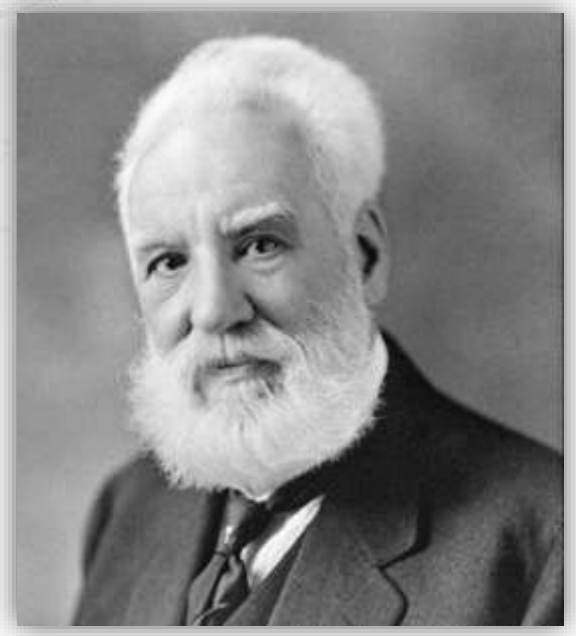
Development of Modern Communication Technologies



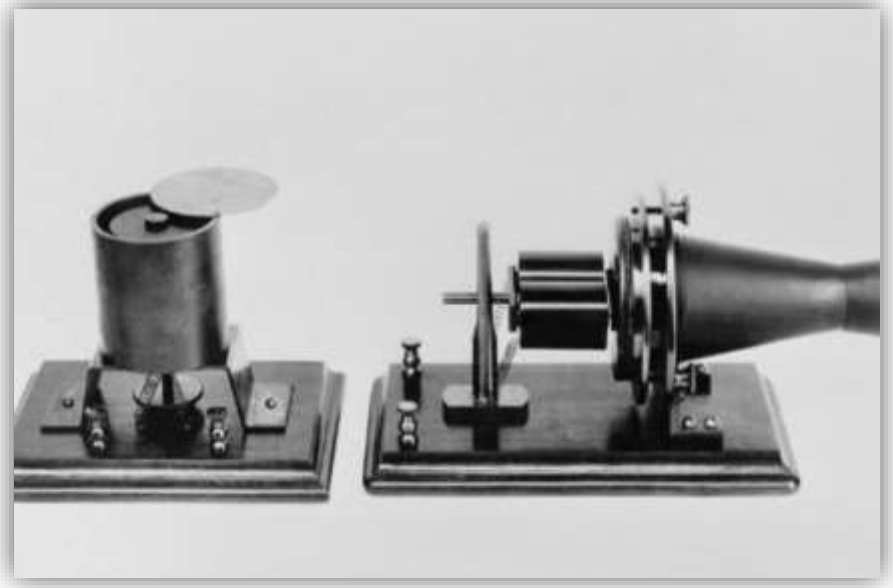


- 1838, Morse, Telegraph & Morse codes
- 1839, Telegraph Operator in UK
- 1844, Telegraph Operator in US

Development of Modern Communication Technologies



A. G. Bell



贝尔电话构思模型

最早期打电话情形

- 1875, Alekander Graham Bell, Telephone

美国波士顿法院路109号铜牌上面镌刻有:“1875年6月2日 电话诞生在此“

Development of Modern Communication Technologies

1892年，纽约至芝加哥的电话线路开通，贝尔在开通现场演示

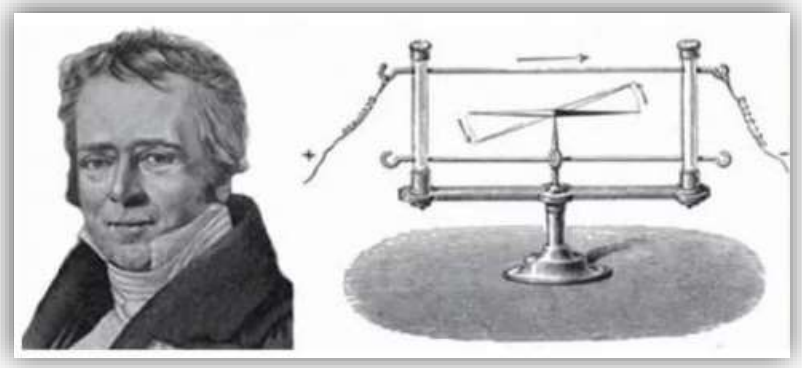
当时的时髦职业——接线员

人工电话接线

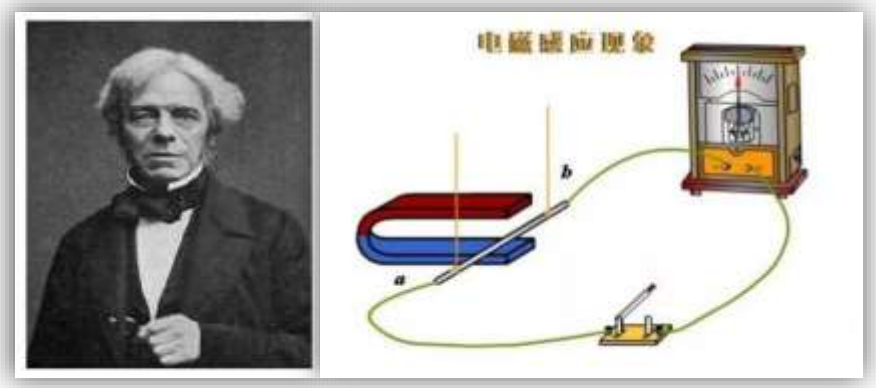
1889年，纽约曼哈顿街上的电话线

- Switching/Multiplexing

Wired to Wireless: The Beginning

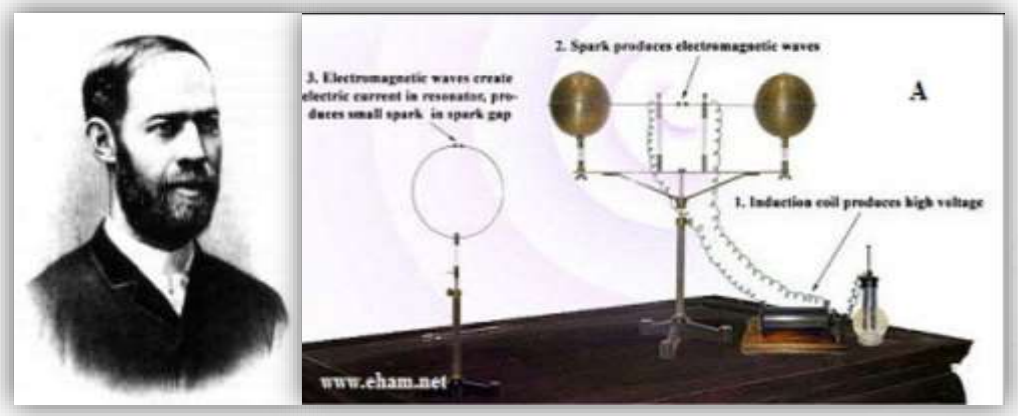


1820年7月，奥斯特发布一个细小的发现：在电流线周围，小磁针发生了环形偏。**电可以产生磁。**



1831年，法拉第通过电磁感应实验，发现**了磁可以产生电**

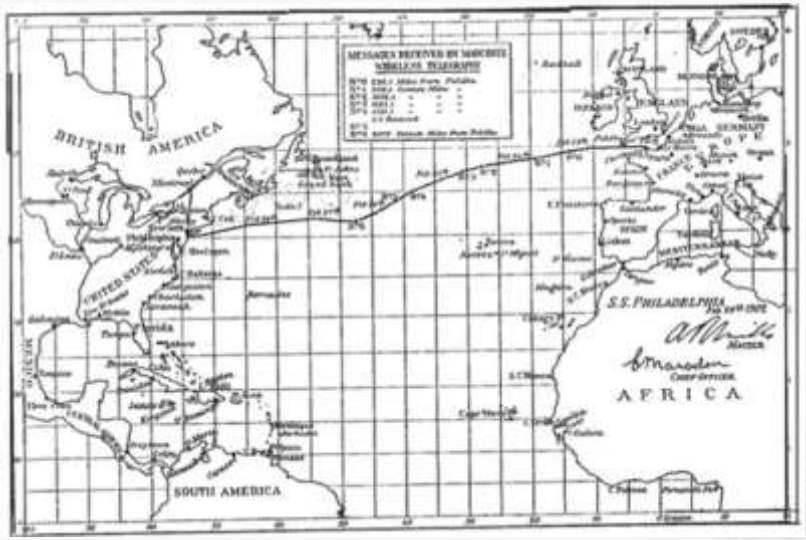
1861年，麦克斯韦建立了**麦克斯韦方程**，并预言了电磁波的存在，建立电磁理论



1888年，赫兹通过实验**证明了电磁波的存在**

Wired to Wireless: The Development

- Guglielmo Marconi 1874.4.25—1937.7.20
- 对物理、电学有浓厚兴趣，热爱做实验
- 1895年，在自家庄园成功进行了约3公里的无线电信号传输
- 1899年，无线电信号首次穿越英吉利海峡
- 1900年，马可尼为其"调谐式无线电报"取得了著名的第7777号专利
- 1901年，无线电报首次穿越大西洋，3000公里
- 1909年，马可尼获得诺贝尔物理学奖
- 1912年，泰坦尼克号事件中，无线电再次发挥了作用，最终使得711人获救。



史上第一次跨大西洋电报通讯图解，出自马可尼的诺贝尔奖讲座

Technical preparation before Shannon's information theory

- Telegraph(Morse, 1830's);
- Telephone(Bell, 1876);
- Wireless Telegraph(Marconi, 1887);
- AM Radio(early 1900's);
- SSB modulation(Carson, 1922);
- Television(1925-1927);
- Telex(1931);
- FM Radio(Armstrong, 1936);
- Pulse modulation(PCM) (Reeves, 1937-1939);
- Vocoder(Dudley, 1939);
- Spread Spectrum(1940's), ….

All the technologies are developed based on specific **engineering** requirement.

The **theoretical** basis remains **largely unknown**!

Implications and Unsolved Questions

- Telegraph(Morse, 1830's);
- Telephone(Bell, 1876);
- Wireless 1887);
- AM Rad
- SSB mo 2);
- Televisic
- Telex(19
- FM Rad
- Pulse m ves, 1937-1939);
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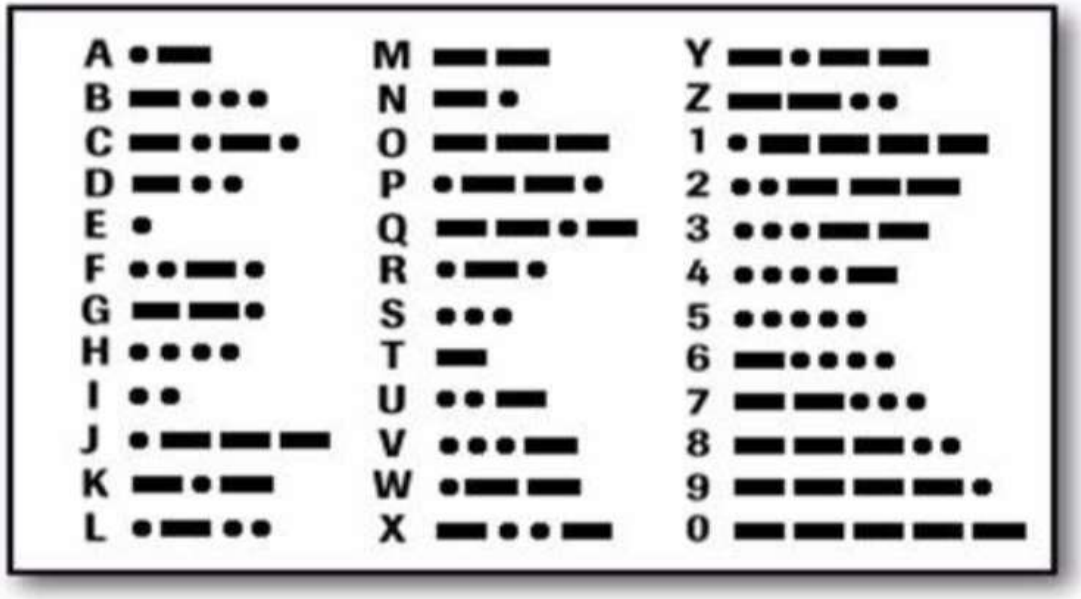
Source coding scheme



higher frequency, shorter code

Is Morse code optimal?

What is the most efficient code?

Implications and Unsolved Questions

- Telegraph(Morse, 1830's);
- Te
- W 887);
- AM
- SS);
- Te
- Te
- FM
- Pulse modulation(PCM) (Reeves, 1937-1939);
- Vocoder(Dudley, 1939);
- Spread Spectrum(1940's), ….

Noise is inevitable!

Channel is unreliable!!

All the information can be damaged!!!

- High transmission rate, low reliability
- High reliability, low transmission rate

Can we overcome noise?

Can we achieve reliable communication?

Optimal Tradeoff between reliability and efficiency?

Implications and Unsolved Questions

- Telegraph(Morse, 1830's);
- Telep
- Wirel 1887);
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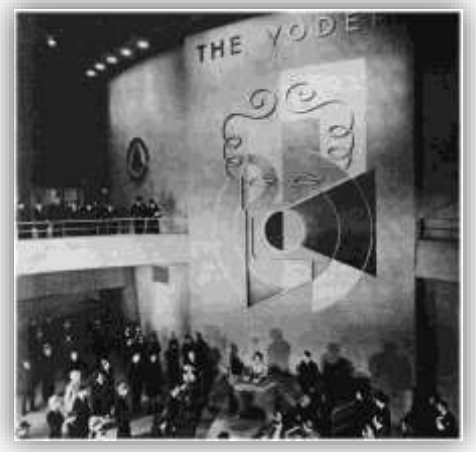
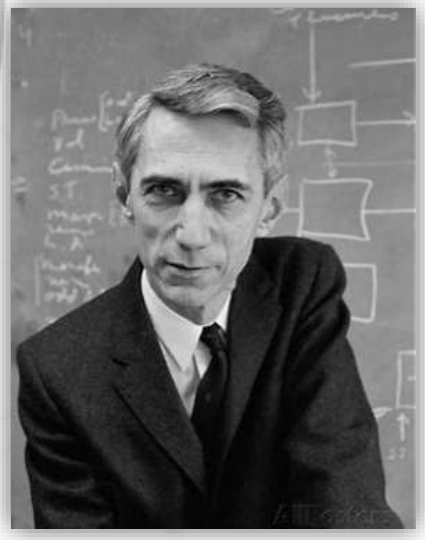


We can sacrifice the fidelity of voice signals to obtain a lower compression rate..

Optimal tradeoff between fidelity and rate?

What are the contributions of information theory?



Claude E. Shannon, “A Mathematical Theory of Communications,” Bell System Technical Journal, July & October 1948.

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- What is information?
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  - Technical background
- **What are the contributions of information theory?**
  - **Theory vs. Engineering**
- Who is Claude Shannon?
  - Life of a Master of Theory

03

Key Contributions

学习目标

1. 说出≥3个信息论的主要贡献
2. 画出香农提出的通信系统模型
3. 说出香农信息的定义及及其优缺点
4. 概述理论与应用的相互作用
5. 说出≥4个信息论所应用的领域

Fundamental Contributions of Information Theory

- **Before** 1948, what do we know about communication systems?
  - A technique, vague/experimental understanding
  - No unified definition on information
  - No theoretical framework
  - Intuition: Faster you transmit, More errors you have.

- **After** 1948, what do we know about communication systems?
  1. A simple yet unified model of communication systems
  2. Quantitative measurement of information
  3. Fundamental Limits of communications
     - data compression limit
     - information transmission limit

Shannon's Communication System Model

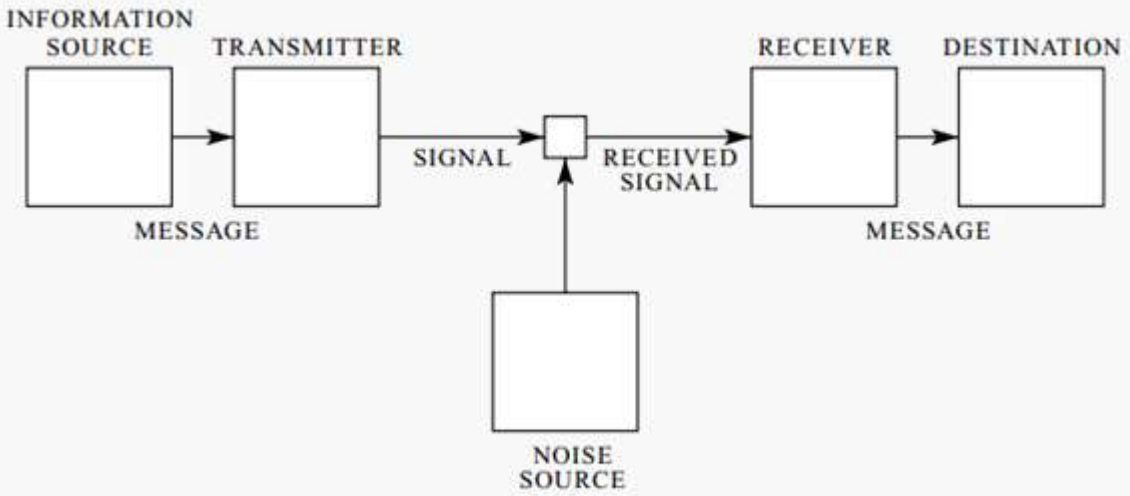


Fig. 1—Schematic diagram of a general communication system.

- A **unified** abstraction model for all kinds of communications
- **Decompose** complicated systems into individual segments
  - Information source
  - Channel
- Use **mathematical** tools to study each segment

Revisiting: Information seems very complicated…

- 信息既有其**客观性**，又有其**主观性**。
  - **客观性**：所有人所听到的消息都是一样的。
  - **主观性**：与接收人的兴趣、职业等主观因素有关。

- Characteristics of information
  - Syntax (语法): presentation
  - Semantics (语义): meaning
  - Pragmatics (语用): utility

- **No universal definition**
- **Difficult to measure** how much information

Shannon's Bold Assumption on Information

"Frequently the messages have meaning; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem."

- 通信的任务只是在接收端把发送端发出的消息从形式上复制出来。
- 从工程实现的角度，通信系统并不需要对消息的语义作处理和判断。

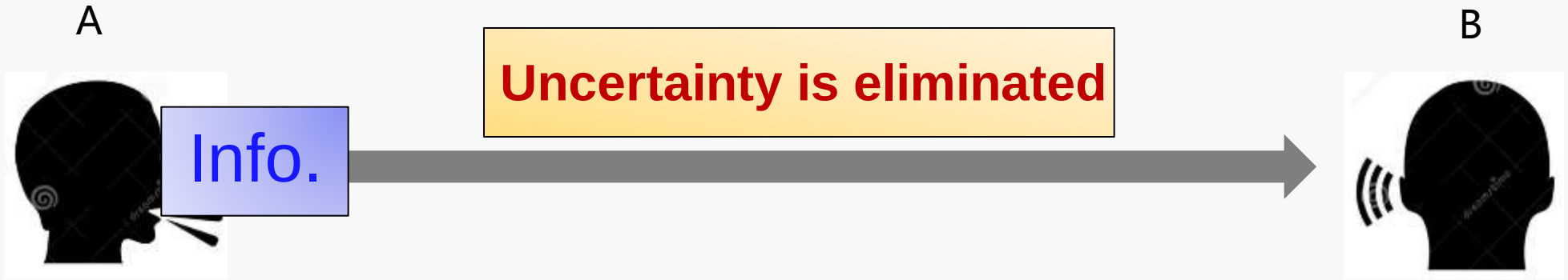
- ✓ **只考虑统计特性**，便于数学分析
- ✓ 使得对**信息的量化成为可能**
- ✓ 是信息论中**最重要的假设**之一

- ❖ 忽略了人的主观情感
- ❖ 只是现实世界信息的一种简化
- ❖ 使用场景受限，很多场景不满足需要

C. E. Shannon, "A mathematical theory of communication," Bell System Technical Journal, vol. 27, no. 4, pp. 623–656, Oct. 1948.

Shannon's Perspective on Information

"The significant aspect is that the actual message is one selected from a set of possible messages."



- 例：A每天向B发送武汉的明日天气。
  - 通信前，问B：武汉明天会天晴吗？B答：不知道。
  - A告诉B：武汉明日晴。
  - 通信后，问B：武汉明天会天晴吗？B答：是的。

Shannon Information：The uncertainty eliminated after communication.

根据香农信息的定义，重新回答之前的问题

第二次听天气预报说明天是下雨，得到多少信息？

A. 没有得到信息　　B. 还是得到了信息

信息量 = 听到消息之前的不确定度 – 听到消息之后的不确定度

0　　0

根据香农信息的定义，重新回答之前的问题

一位渔民和一个工人同时收听天气预报说明天是下雨，谁得到了更有价值的信息？价值多出多少？

A. 渔民多　　B. 工人多　　一样多

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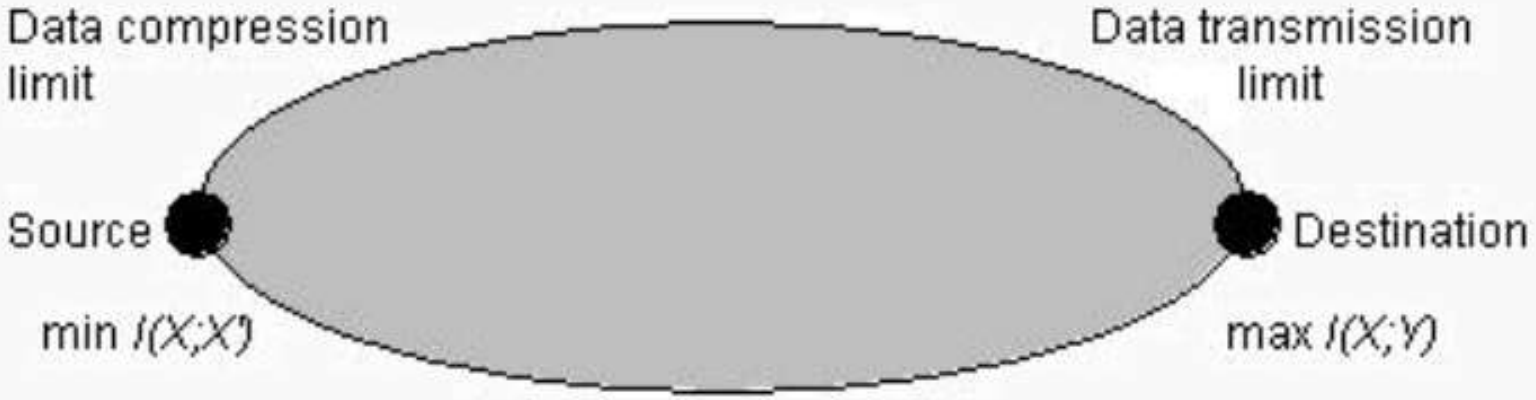
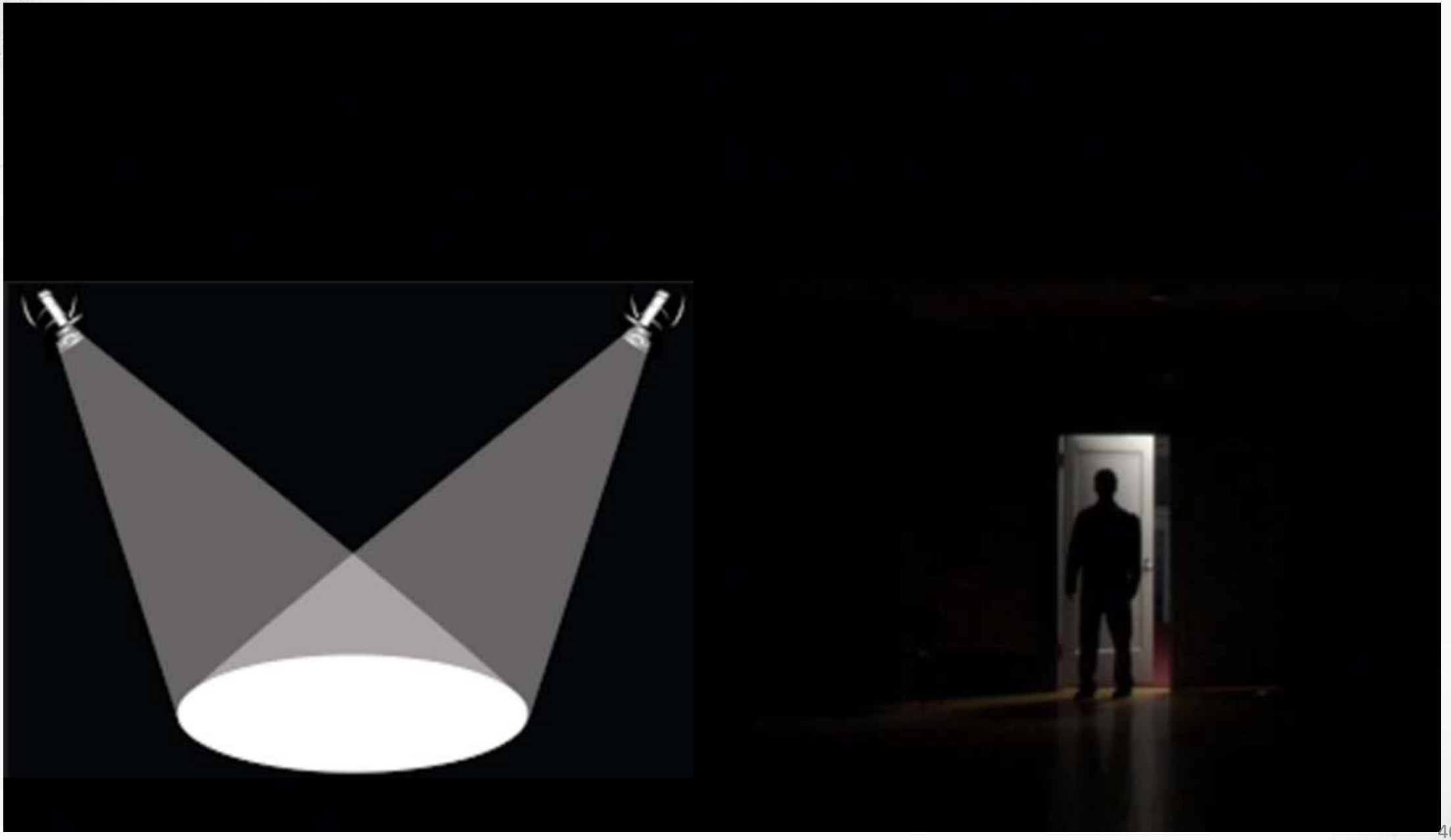
     - data compression limit
     - information transmission limit

Limits of Communications

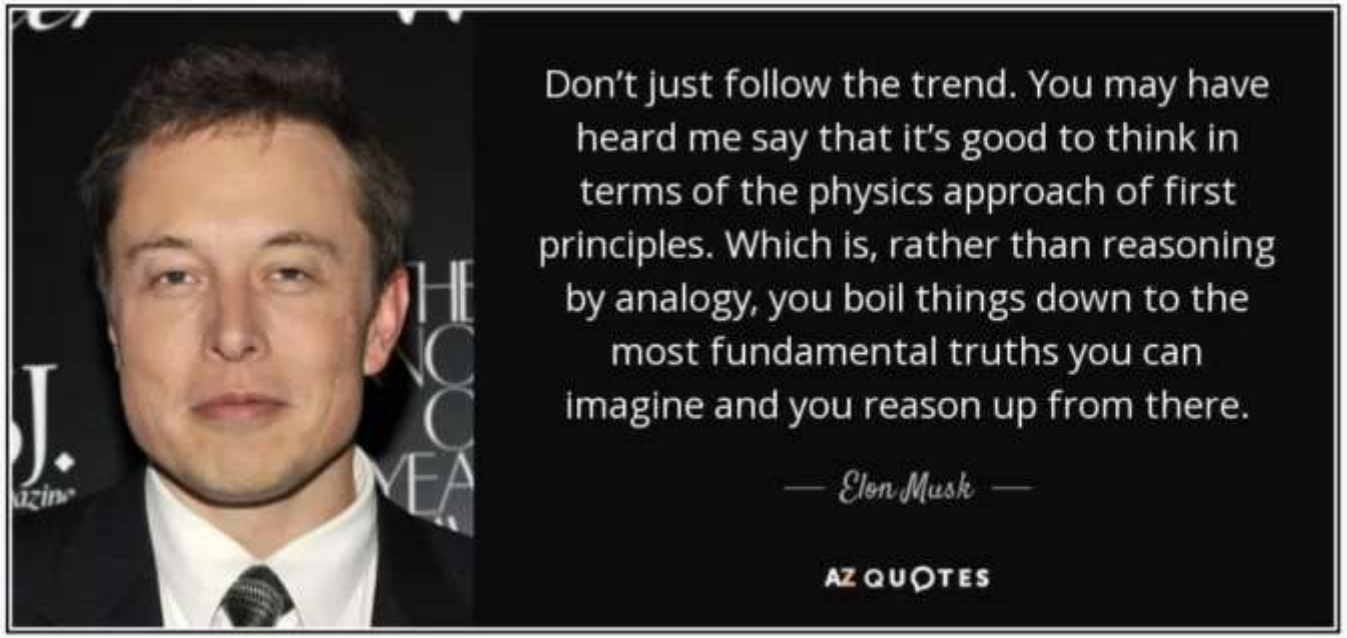


- Discover **mathematical** laws in communicating or manipulating information
- **Three coding theorems** invented by Shannon
  - Source coding theorem: How well can we compress the source?
  - Channel coding theorem: What is the maximum information rate?
  - Rate distortion theorem: What is the optimal tradeoff between rate and distortion?



Insights: First Principles Reasoning 第一性原理

“不要随波逐流。你可能听过我说，**用第一性原理的物理方法来思考是件好事。**也就是说，不是通过类比推理，而是将事情**归结为你能想象的最基本的事实**，然后从那里开始推理。”



“**在每个系统探索中都存在第一性原理。**第一性原理是基本的命题和假设，不能被省略和删除，也不能被违反。”——亚里士多德

化繁为简

探寻本质

寻找极限

What is Mathematical Modeling?

- “Mathematical modeling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application.”

-- Arnold Neumaier

应用问题　　数学建模　　理论指导

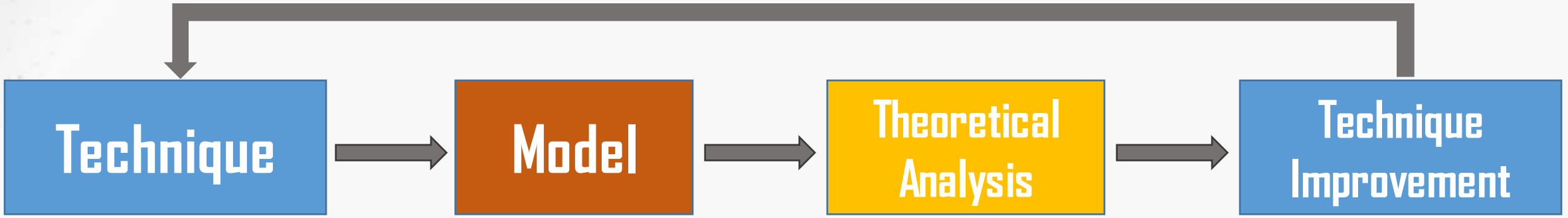
How to do Mathematical Modeling?

- Ask the **right** question
  - Fundamental

- Make **appropriate** assumptions
  - Simplify the model
  - **Capture the essence**
  - Every model has its limitations and boundaries.

- **Properly** formulate the problem
  - Always start from some **simple (naive) examples**

Insights: Theory vs. Engineering



- Theory
  - Mathematical models to analyze the system
  - Theoretical performance limits
  - Insights to practical system design
    - Directions to improve the performance

- Engineering
  - System Design
    - How to approach the theoretical limits in practical systems?
  - System Evaluation
    - Compare the practical system to the reference system

Applications in Communications

- Major application area: **coding**
- Three coding theorems invented by Shannon
  - Source coding theorem: How well can we compress the source?
  - Channel coding theorem: What is the maximum information rate?
  - Rate distortion theorem: What is the optimal tradeoff between rate and distortion?
- Practical methods have been inspired and invented after Shannon:
  - Source coding: Huffman codes, Lempel-Ziv (gzip)…
  - Channel coding: Hamming, Reed-Solomon, Viterbi, Turbo, Polar…
  - Rate-distortion coding: vocoders, minidiscs, MP3, JPEG, MPEG....

Hamming code (1954)

R. Hamming

BCH code (1954)

R. C. Bose

Convolutional code (1955)

P. Elias

Viterbi code (1967) 2/3G

A. Viterbi

LDPC code (1967)

R. G. Gallager

Turbo code (1993) 3/4G

C. Berrou

A. Glavieux

Polar code (2007) 5G

E. Arikan

Insights: Theory and Applications

任正非2020.5.21长篇访谈：5G标准源自土耳其科学家的一篇数学论文

“大家今天讲5G标准对人类社会有多么厉害，怎么会想到，5G标准是源于十多年前土耳其Arikan教授的一篇数学论文?

Arikan教授发表这篇论文两个月后，被我们发现了，我们就开始以这个论文为中心研究各种专利，一步步研究解体，共投入了数千人。

十年时间，我们就把土耳其教授数学论文变成技术和标准。我们的5G基本专利数量占世界27%左右，排第一位。”

Insights: Theory and Applications

中国要踏踏实实在数学、物理、化学、神经学、脑科学……各方面努力去改变，我们可能在这个世界上能站起来

“任正非如是说：以中国为中心建立理论基地要突破美国的重围，眼前这个方式比较难，因为中国在基础理论上不够，这些年好一些了。我曾在全国科学大会上讲了数学的重要性，听说现在数学毕业生比较好分配了。我们有几个人愿意读数学的?

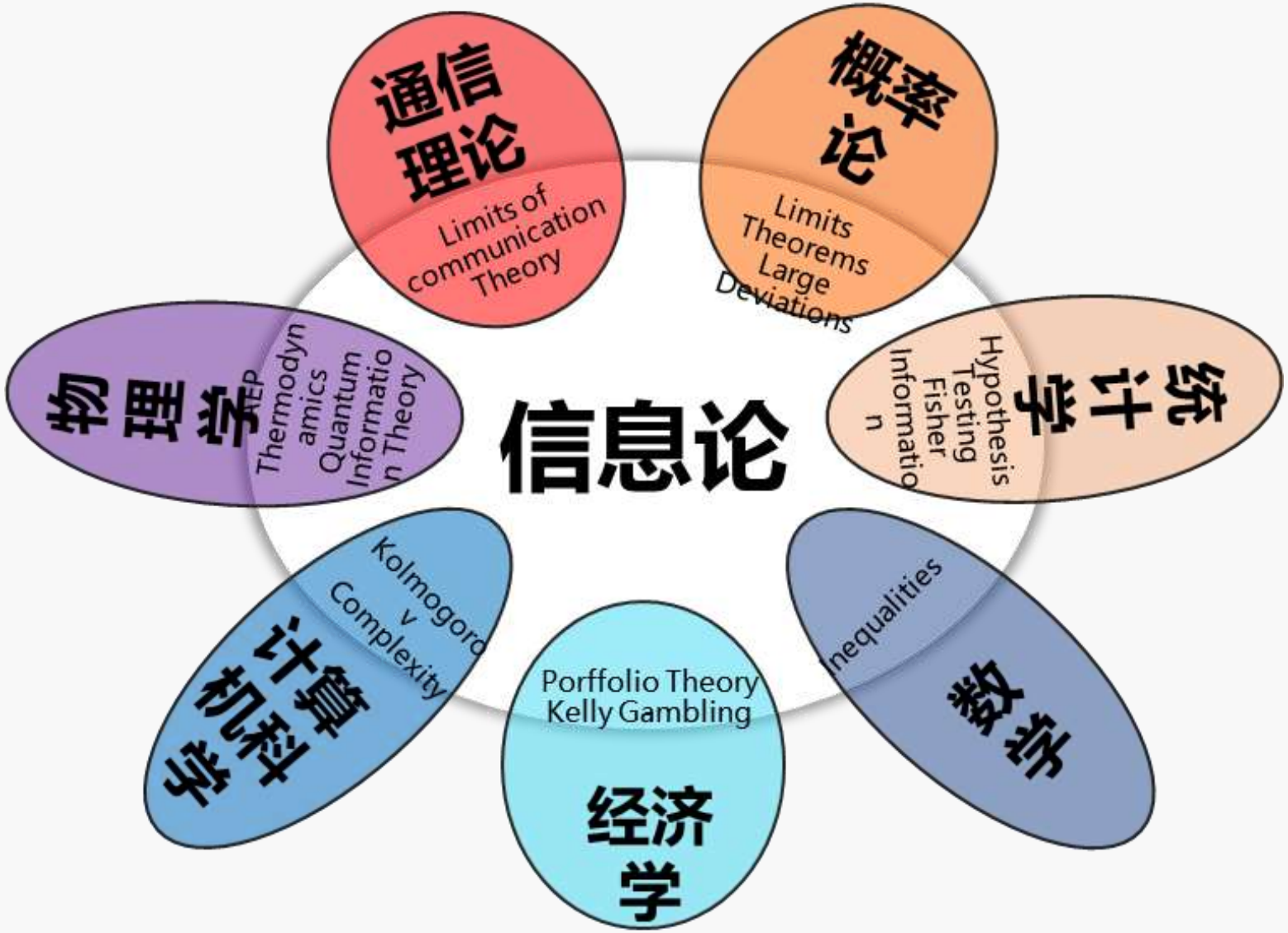
我不是学数学的，我曾经说，我退休以后想找一个好大学，学数学。校长问我，学数学干什么？我说，想研究热力学第二定律。他问，研究用来做什么？我说，想研究宇宙起源。他说，我很欢迎你！但是我到现在还不能退休，还去不了。我们那时是工科学生，学的是高等数学，最浅的数学。”

“一切新产品和新工艺都不是突如其来、自我发育和自我生长起来的。它们皆源自新的科学原理和科学概念。新科学原理和科学概念则必须来自最纯粹科学领域持续不懈的艰难探索。如果一个国家最基础的前沿科学知识依赖他人，其产业进步必然异常缓慢，其产业和世界贸易竞争力必然极其孱弱。”

Impact after Shannon's information theory

- The Internet (Ethernet, DSL, WiFi, telephone line modems, etc.)
- The Web (Google, EBay, Amazon, MySpace, Facebook, YouTube, etc.)
- Cellular Communications (GSM, IS-95, cdma2000, WCDMA, 3GPP, etc.)
- The Personal Computer (Mac, Windows, etc.)
- Bluetooth
- VHS, CD, DVD, BlueRay
- Digital TV & radio (DirectTV, XM Radio, Sirius, Digital CATV, HDTV, TiVo)
- JPEG, MPEG, mp3
- GPS
- DSP chips; Flash Memory
- MIMO, MUD
- TCM, Turbo codes, Raptor codes, Fountain codes

Impact after Shannon's information theory



Information Theory in 6G: Semantic Communications

- 《语义通信及语义认知网络架构研究》白皮书
- 2021年以来，IMT-2030 6G推进组在已发布的系列白皮书中均将语义通信和语义认知网络定义为6G的潜在技术和架构之一。
- IMT-2030《6G智能内生网络架构》白皮书中(“智能内生的关键技术”章节)和《6G网络架构愿景与关键技术展望》白皮书中，明确提出“语义通信：语义驱动、万物智联”。
- 6G网络将不再仅聚焦信号的传输和复现，而将具备语义认知、识别、分析、理解和推理能力。语义认知与通信将成为6G内生智能的基石，推动通信网络架构从传统的数据驱动向语义驱动的范式转变。

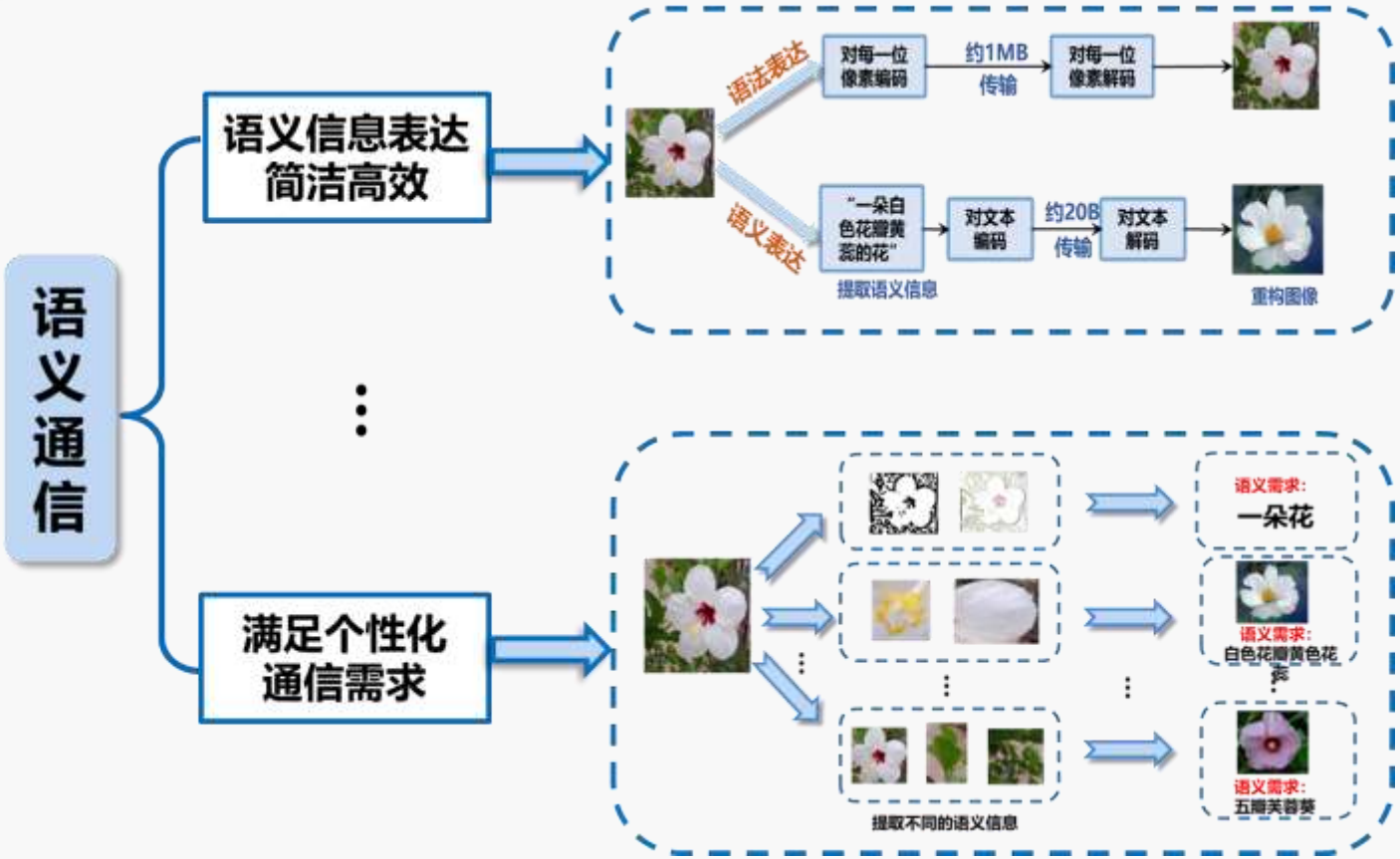
5G愿景：万物互联[1]

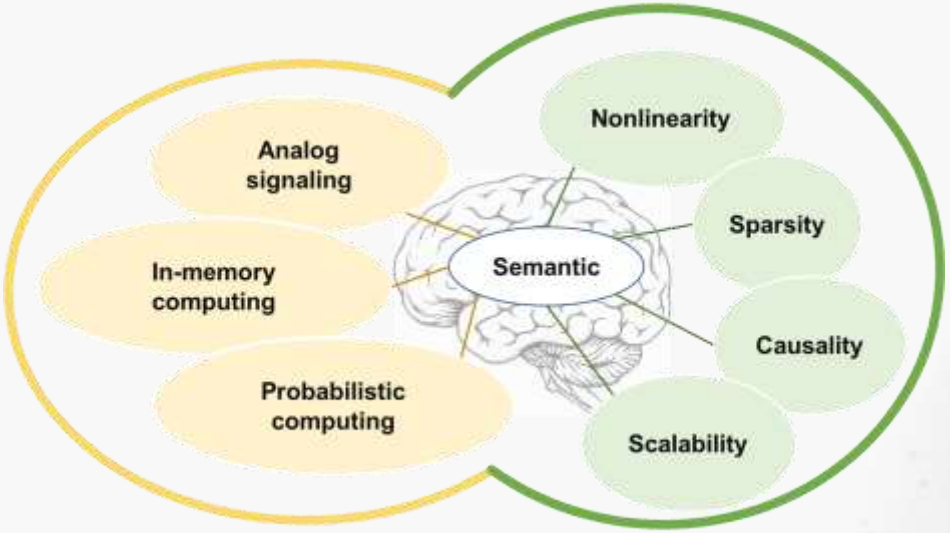
6G愿景：万物智联[2]

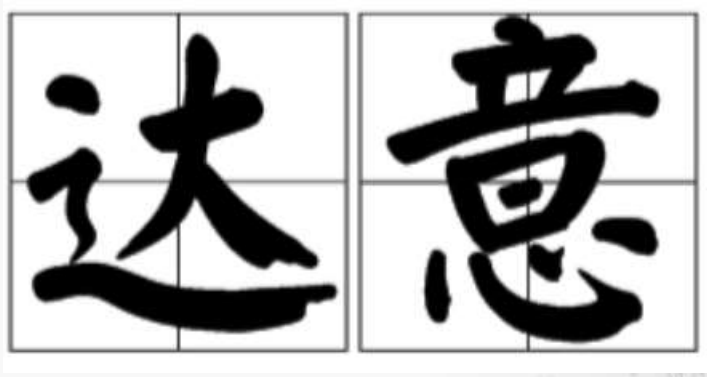
Information Theory in 6G: Semantic Communications

Information Theory in 6G: Semantic Communications

语义通信主要解决“传输的符号如何精准传达含义”的问题，通过提取语义信息，将大大语义通信效率提升，同时，语义通信考虑信息含义及用户对信息的需求，支撑个性化服务需求。







Course Project #2: Advanced Topic 高级选题

- **游戏规则**
- 总分：5分
- 主题：选取一篇语义信息论领域近年来的高水平论文，进行深入阅读与调研
- 成果：一份调研报告&综述PPT
  - 报告：中/英文、pdf
  - PPT：英文，模板见微助教
  - Deadline: TBD
- 每个人选择不同的论文，严禁抄袭，违者0分。

Summary

- What is information?

1. 认识信息的复杂性
2. 说出信息的3个层面

- How was information theory born?
  - Technical background

1. 概述1948之前通信技术发展背景
2. 说出≥3个关于通信当时尚未解答的问题

- What are the contributions of information theory?
  - Theory vs. Engineering

1. 说出≥3个信息论的主要贡献
2. 画出香农提出的通信系统模型
3. 说出香农信息的定义及及其优缺点
4. 概述理论与应用的相互作用
5. 说出≥4个信息论所应用的领域

- **Who is Claude Shannon?**
  - **Life of a Master of Theory**

→ **Homework 1**

Thank you!

My Homepage

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多选题：关于香农信息，以下说法正确的是?

A. 不考虑收信者主观感受的不同 

B. 只考虑信息的统计特性，便于数学分析 

C. 不符合人们对信息的直观理解，没有什么用  符合，对信息论很重要

D. 是对现实世界信息的一种抽象和简化 

E. 具有通用性，在所有场景下都能使用  具有局限性，很多场景下不满足要求

F. 信息量多少与消息的不确定度直接相关