## Information Theory Exam 2024

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## 1 True/False Problems

**Tips:** Explain your conclusion.

- 1. g(X) is a function of the random variable X,  $H(g(X)) \geq H(X)$ .
- 2.  $I(X;Y|Z) \ge I(X;Y)$ .
- 3. Entropy is non-negative.
- 4. A prefix code with code lengths  $\{1, 3, 3, 3, 4, 5\}$  can be achieved.
- 5. Given the Markov transition matrix, is its maximum entropy rate log<sub>2</sub> 3?

$$\begin{pmatrix} 1-p & p/2 & p/2 \\ p/2 & 1-p & p/2 \\ p/2 & p/2 & 1-p \end{pmatrix}$$

## 2 Short Problems

- 1. State Shannon's first theorem and explain its meaning.
- 2. A car's license plate ends with a digit from 0 to 9. The probabilities of the last digit being 0, 1, and 2 are  $\frac{1}{20}$ ,  $\frac{1}{8}$ , and  $\frac{1}{8}$  respectively. The probabilities of the other digits are equal. Suppose you ask the car owner if the last digit of their license plate is a specific number, and they will answer "yes" or "no". On average, how many questions do you need to ask to determine the last digit of the license plate?
- 3. Write down the upper bound of the optimal expected code length, and explain how to decrease the expected code length.
  - 4. Write down the form of the rate-distortion function.
  - 5. Compare channel capacity and rate-distortion function.
- 6. State why the maximum transmission rate is equal to the channel capacity (Hint: Use AEP).

## 3 Long Problems

1. X and Y are random variables. Let  $Z = X \oplus Y$ , where Z donate Modulo 2 additions. The joint distribution of X and Y is as follows:

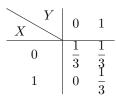


Table 1: Joint Probability Distribution

- (a) Find H(X), H(Y), and H(Z).
- (b) Find H(X|Z) and H(Z|X).
- (c) Find H(X,Y,Z) and H(X,Y), and explain your results.
- 2. A source emits symbols with the probability distribution  $\{0.45, 0.25, 0.12, 0.05, 0.05, 0.04, 0.03, 0.01\}$ .
- (a) Derive the binary Shannon code for this distribution and calculate the coding efficiency.
- (b) Derive the binary Huffman code for this distribution, draw the Huffman tree, and calculate the coding efficiency.
- (c) Derive the ternary Huffman code for this distribution and calculate the coding efficiency.
  - 3. Prove the following:
  - (a)  $L(C,X) \geq H_D(X)$ , and explain when we can get equality.
  - (b)  $H(X) \ge \log |\mathcal{X}|$ , and explain when we can get equality.
  - 4. Write down Shannon's formula and explain the meaning of each symbol:
- (a) What can you observe from this formula? Please state at least three points.
- (b) For an Additive White Gaussian Noise (AWGN) channel with a channel capacity C=150 kbps, bandwidth B=30 kHz, and signal power S=0.5 W, if the bandwidth approaches infinity and you want to maintain the same transmission rate, what will be the required signal power?
- 5. A military unit uses a flock of pigeons for communication. Each pigeon can carry 8 bits of information, and one pigeon is released every 5 minutes. Each pigeon takes 3 minutes to reach its destination.
  - (a) Calculate the channel capacity: how many bits per hour?
- (b) If a proportion  $\alpha$  of the pigeons are shot down by the enemy, and the information receiver can know which pigeons are shot down, calculate the channel capacity.
- (c) If the enemy becomes more clever and releases pigeons carrying random information when they shoot down the original pigeons, calculate the channel capacity. (Construct your model and explain it).