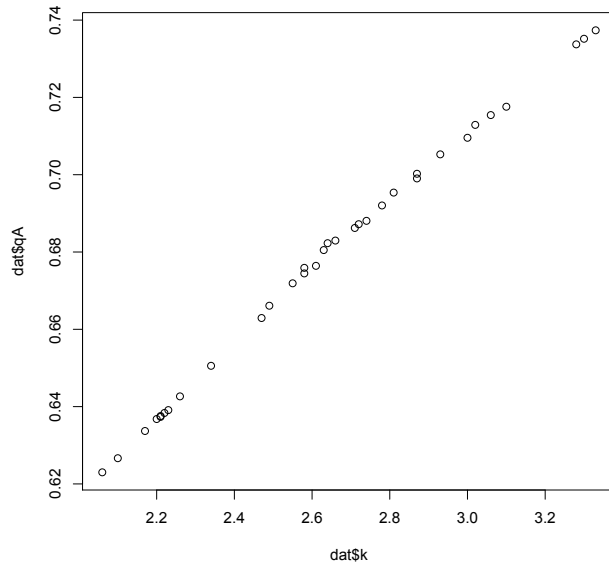


Solow-Swan Model Replication

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By using the data provided in Chart 4 from the Solow report, which excludes the 7 war years(1943-1949) he mentioned, we can plot the data in a graph where k is the explanatory variable and qA as the responsive variable. The result is as the



same as the Solow report showed. The detailed regression analysis is in the appendices.

By using the data provided in the Table 2 in the Solow Report that includes the war years (1943-1949), We can use both OLS and 20% trimmed least squares to re-estimate equation from (4a) to (4e). The results

are shown in the following two tables.

OLS			
	α	β	r
4a	0.44511698	0.08962653	0.9283
4b	0.452369	0.238146	0.9435
4c	0.919672	0.618630	0.9492
4d	-0.72274	0.35160	0.9407
4e	-0.03219	0.91496	0.9496

20% trimmed LS			
	α	β	r
4a	0.4382	0.0909	0.9962
4b	0.4476	0.2397	0.9991
4c	0.916813	0.618699	0.9929
4d	-0.730007	0.354042	0.9992
4e	-0.03646	0.91527	0.9961

*When running the 20% trimmed least squares in R, we use $\alpha=80\%$ (trimmed upper 10% and lower 10%) in the "ltsReg" function. The details are in the appendices.

We choose the 3 best out the 5 cases for the highest r value for both OLS and 20% trimmed LS. They are 4e, 4c, 4b for OLS and 4d, 4b, 4a for 20% trimmed LS. When R does the 20% trimmed least square, it does not tell me which observations it drops. However, based on the value from the tables above, we can observe that the result from 20% least square is almost as identical as the report from Solow. The result from OLS table is similar but has some variance. We can draw a conclusion that the 20% trimmed least square method drop the 7 war years (1934-1949) data from the OLS method since the war years data are dropped in the Solow report.

The summary report in R contains the standard errors for the regression coefficients. The standard errors for the regression coefficients reported from the regression analysis for 4a using OLS method is 0.01058 (α) and 0.00399 (β) and for 4a using 20% trimmed least square method is 0.0026409 (α) 0.0009917 (β).

Bibliography

- Robert M. Solow. “Technical Change and the Aggregate Production Function” *The review of Economics and Statistics*, Vol. 39, No. 3.(Aug., 1957), pp. 312-320;
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- Daron, Acemoglu. “14.452 Economic Growth: Lectures 2 and 3: The Solow Growth Model.”(November 1 and 3, 2011);
economics.mit.edu/files/7181
- Stanisic, Dragana. “Lecture 6. Explaining Economic Growth.” Lecture(Spring 2014); home.cerge-ei.cz/dragana/L6.pdf
- Ross, Ihaka., and Robert Gentleman. R. Computer Statistical Software. Version 3.2.3

Appendices (R runs)

```
> qA =
c(0.623,0.626653103,0.633692458,0.637341153,0.639097744,0.636788049,
0.642651297,0.650557621,0.637558685,0.638353765,0.662921348,0.674462
114,0.671902269,0.666103128,0.676421405,0.688065844,0.695374801,0.70
0241741,0.705263158,0.712887439,0.715427658,0.735171261,0.737357259,
0.733722871,0.71758877,0.709553159,0.699036323,0.687193842,0.6862190
81,0.692041522,0.682959049,0.680503145,0.675903614,0.682282282)
> k =
c(2.06,2.1,2.17,2.21,2.23,2.2,2.26,2.34,2.21,2.22,2.47,2.58,2.55,2.4
9,2.61,2.74,2.81,2.87,2.93,3.02,3.06,3.3,3.33,3.28,3.1,3,2.87,2.72,2
.71,2.78,2.66,2.63,2.58,2.64)
> dat = data.frame(qA,k)
> attach(dat)
The following objects are masked _by_ .GlobalEnv:
```

k, qA

The following objects are masked from dat (pos = 3):

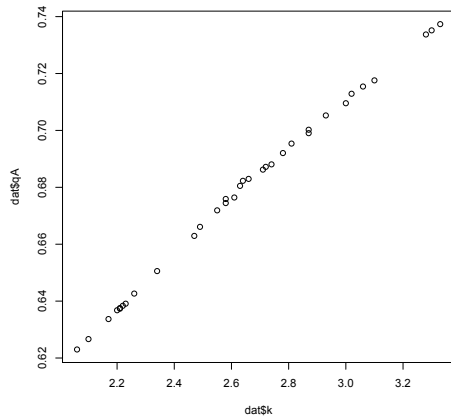
k, qA

```
> dat
      qA      k
1 0.6230000 2.06
2 0.6266531 2.10
3 0.6336925 2.17
4 0.6373412 2.21
5 0.6390977 2.23
6 0.6367880 2.20
7 0.6426513 2.26
8 0.6505576 2.34
9 0.6375587 2.21
10 0.6383538 2.22
11 0.6629213 2.47
12 0.6744621 2.58
13 0.6719023 2.55
14 0.6661031 2.49
15 0.6764214 2.61
16 0.6880658 2.74
17 0.6953748 2.81
18 0.7002417 2.87
19 0.7052632 2.93
20 0.7128874 3.02
21 0.7154277 3.06
22 0.7351713 3.30
23 0.7373573 3.33
24 0.7337229 3.28
25 0.7175888 3.10
26 0.7095532 3.00
27 0.6990363 2.87
28 0.6871938 2.72
29 0.6862191 2.71
30 0.6920415 2.78
31 0.6829590 2.66
```

```

32 0.6805031 2.63
33 0.6759036 2.58
34 0.6822823 2.64
> plot(dat$k, dat$qA)

```



```

> cov(dat)
           qA          k
qA 0.001081356 0.01185089
k   0.011850887 0.13037193
> cor(dat)
           qA          k
qA 1.0000000 0.998101
k   0.998101 1.000000
> results = lm(k~qA)
> results

```

```

Call:
lm(formula = k ~ qA)

```

```

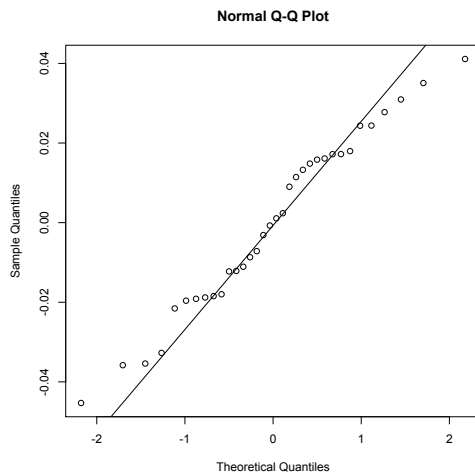
Coefficients:
(Intercept)          qA
    -4.792         10.959

```

```

> qqnorm(results$res)
> qqline(results$res)

```



```
> summary(results)
```

```
Call:
```

```
lm(formula = k ~ qA)
```

```
Residuals:
```

	Min	1Q	Median	3Q	Max
	-0.045308	-0.018338	0.000181	0.016931	0.041110

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-4.79202	0.08117	-59.04	<2e-16 ***
qA	10.95928	0.11956	91.66	<2e-16 ***

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.02259 on 32 degrees of freedom
```

```
Multiple R-squared:  0.9962, Adjusted R-squared:  0.9961
```

```
F-statistic: 8402 on 1 and 32 DF, p-value: < 2.2e-16
```

```
>
```

```
> qA =
```

```
c(0.623,0.626653103,0.633692458,0.637341153,0.639097744,0.636788049,
0.642651297,0.650557621,0.637558685,0.638353765,0.662921348,0.674462
114,0.671902269,0.666103128,0.676421405,0.688065844,0.695374801,0.70
0241741,0.705263158,0.712887439,0.715427658,0.735171261,0.737357259,
0.733722871,0.71758877,0.709553159,0.699036323,0.687193842,0.6862190
81,0.692041522,0.682959049,0.680503145,0.675903614,0.682282282,0.697
399527,0.69812362,0.700540541,0.686828717,0.686601495,0.691001698,0.
704809287)
```

```
> k =
```

```
c(2.06,2.1,2.17,2.21,2.23,2.2,2.26,2.34,2.21,2.22,2.47,2.58,2.55,2.4
9,2.61,2.74,2.81,2.87,2.93,3.02,3.06,3.3,3.33,3.28,3.1,3,2.87,2.72,2
.71,2.78,2.66,2.63,2.58,2.64,2.62,2.63,2.66,2.5,2.5,2.55,2.7)
```

```
> fit = lm(qA~k)
```

```
> summary(fit)
```

```
Call:
```

```
lm(formula = qA ~ k)
```

```
Residuals:
```

	Min	1Q	Median	3Q	Max
	-0.0067476	-0.0053704	-0.0026208	-0.0005645	0.0177007

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.44512	0.01058	42.08	<2e-16 ***
k	0.08963	0.00399	22.46	<2e-16 ***

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.008322 on 39 degrees of freedom
```

```
Multiple R-squared:  0.9283, Adjusted R-squared:  0.9264
```

F-statistic: 504.6 on 1 and 39 DF, p-value: < 2.2e-16

```
> fit=lm(qA~log(k))
> summary(fit)
```

```
Call:
lm(formula = qA ~ log(k))
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-0.004783 -0.003875 -0.003207 -0.001525  0.016249
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.452369   0.009028   50.11  <2e-16 ***
log(k)       0.238146   0.009328   25.53  <2e-16 ***
---

```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 0.007383 on 39 degrees of freedom
Multiple R-squared: 0.9435, Adjusted R-squared: 0.9421
F-statistic: 651.8 on 1 and 39 DF, p-value: < 2.2e-16

```
> summary(ltsReg(x=(-1/k), y=qA, intercept = TRUE, alpha =1))
```

```
Call:
ltsReg.default(x = (-1/k), y = qA, intercept = TRUE, alpha = 1)
```

```
Residuals (from reweighted LS):
    Min       1Q   Median       3Q      Max
-0.006293 -0.004743 -0.003060  0.002657  0.014608
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
Intercept  0.919672   0.008914  103.17  <2e-16 ***
(-1/k)     0.618630   0.022928   26.98  <2e-16 ***
---

```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 0.007006 on 39 degrees of freedom
Multiple R-Squared: 0.9492, Adjusted R-squared: 0.9479
F-statistic: 728 on 1 and 39 DF, p-value: < 2.2e-16

```
> fit=lm(log(qA)~log(k))
> summary(fit)
```

```
Call:
lm(formula = log(qA) ~ log(k))
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-0.006937 -0.005841 -0.004525 -0.002555  0.024907
```


Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-0.72274	0.01368	-52.83	<2e-16 ***
log(k)	0.35160	0.01414	24.87	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01119 on 39 degrees of freedom

Multiple R-squared: 0.9407, Adjusted R-squared: 0.9392

F-statistic: 618.6 on 1 and 39 DF, p-value: < 2.2e-16

```
> ltsReg(x=(-1/k), y=log(qA), intercept = TRUE, alpha =0.8)
```

Call:

```
ltsReg.default(x = (-1/k), y = log(qA), intercept = TRUE, alpha = 1)
```

Coefficients:

	(-1/k)
Intercept	-0.03219
	0.91496

Scale estimate 0.01018

```
> summary(ltsReg(x=(-1/k), y=log(qA), intercept = TRUE, alpha =1))
```

Call:

```
ltsReg.default(x = (-1/k), y = log(qA), intercept = TRUE, alpha = 1)
```

Residuals (from reweighted LS):

Min	1Q	Median	3Q	Max
-0.008477	-0.006564	-0.004525	0.001520	0.022507

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
Intercept	-0.03219	0.01312	-2.454	0.0187 ***
(-1/k)	0.91496	0.03374	27.118	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01031 on 39 degrees of freedom

Multiple R-Squared: 0.9496, Adjusted R-squared: 0.9483

F-statistic: 735.4 on 1 and 39 DF, p-value: < 2.2e-16

```
> ltsReg(k, qA, intercept = TRUE, alpha = 0.8)
```

Call:

```
ltsReg.default(x = k, y = qA, intercept = TRUE, alpha = 0.8)
```

Coefficients:

	k
Intercept	0.4382
	0.0909

Scale estimate 0.003001

```
> summary(ltsReg(k, qA, intercept = TRUE, alpha = 0.8))
```

```

Call:
ltsReg.default(x = k, y = qA, intercept = TRUE, alpha = 0.8)

Residuals (from reweighted LS):
      Min       1Q       Median       3Q      Max
-0.003512 -0.001502  0.000000  0.001187  0.004135

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
Intercept  0.4381702   0.0026409   165.92  <2e-16 ***
k          0.0909006   0.0009917    91.66  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.002057 on 32 degrees of freedom
Multiple R-Squared: 0.9962, Adjusted R-squared: 0.9961
F-statistic: 8402 on 1 and 32 DF, p-value: < 2.2e-16

> ltsReg(x=log(k), y=qA, intercept = TRUE, alpha =0.8)

Call:
ltsReg.default(x = log(k), y = qA, intercept = TRUE, alpha = 0.8)

Coefficients:
Intercept      log(k)
    0.4476      0.2397

Scale estimate 0.00145

> summary(ltsReg(x=log(k), y=qA, intercept = TRUE, alpha =0.8))

Call:
ltsReg.default(x = log(k), y = qA, intercept = TRUE, alpha = 0.8)

Residuals (from reweighted LS):
      Min       1Q       Median       3Q      Max
-1.453e-03 -4.241e-04 -4.934e-05  3.227e-04  2.136e-03

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
Intercept  0.447626    0.001225   365.5  <2e-16 ***
log(k)     0.239708    0.001262   190.0  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0009937 on 32 degrees of freedom
Multiple R-Squared: 0.9991, Adjusted R-squared: 0.9991
F-statistic: 3.61e+04 on 1 and 32 DF, p-value: < 2.2e-16

> ltsReg(x=(-1/k), y=qA, intercept = TRUE, alpha =0.8)

Call:

```

```
ltsReg.default(x = (-1/k), y = qA, intercept = TRUE, alpha = 0.8)
```

Coefficients:

```
Intercept      (-1/k)
      0.9168      0.6187
```

Scale estimate 0.004114

```
> summary(ltsReg(x=(-1/k), y=qA, intercept = TRUE, alpha =0.8))
```

Call:

```
ltsReg.default(x = (-1/k), y = qA, intercept = TRUE, alpha = 0.8)
```

Residuals (from reweighted LS):

```
      Min      1Q      Median      3Q      Max
-0.0034059 -0.0018538 -0.0001748  0.0004827  0.0065266
```

Coefficients:

```
      Estimate Std. Error t value Pr(>|t|)
Intercept  0.916813    0.003610   253.97  <2e-16 ***
(-1/k)     0.618699    0.009271    66.74  <2e-16 ***
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 0.00282 on 32 degrees of freedom

Multiple R-Squared: 0.9929, Adjusted R-squared: 0.9926

F-statistic: 4454 on 1 and 32 DF, p-value: < 2.2e-16

```
> ltsReg(x=log(k), y=qA, intercept = TRUE, alpha =0.8)
```

Call:

```
ltsReg.default(x = log(k), y = qA, intercept = TRUE, alpha = 0.8)
```

Coefficients:

```
Intercept      log(k)
      0.4476      0.2397
```

Scale estimate 0.00145

```
> summary(ltsReg(x=log(k), y=qA, intercept = TRUE, alpha =0.8))
```

Call:

```
ltsReg.default(x = log(k), y = qA, intercept = TRUE, alpha = 0.8)
```

Residuals (from reweighted LS):

```
      Min      1Q      Median      3Q      Max
-1.453e-03 -4.241e-04 -4.934e-05  3.227e-04  2.136e-03
```

Coefficients:

```
      Estimate Std. Error t value Pr(>|t|)
Intercept  0.447626    0.001225   365.5  <2e-16 ***
log(k)     0.239708    0.001262   190.0  <2e-16 ***
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0009937 on 32 degrees of freedom
Multiple R-Squared: 0.9991, Adjusted R-squared: 0.9991
F-statistic: 3.61e+04 on 1 and 32 DF, p-value: < 2.2e-16

```
> ltsReg(x=(-1/k), y=log(qA), intercept = TRUE, alpha =0.8)
```

Call:
ltsReg.default(x = (-1/k), y = log(qA), intercept = TRUE, alpha = 0.8)

Coefficients:
Intercept (-1/k)
-0.03646 0.91527

Scale estimate 0.004479

```
> summary(ltsReg(x=(-1/k), y=log(qA), intercept = TRUE, alpha =0.8))
```

Call:
ltsReg.default(x = (-1/k), y = log(qA), intercept = TRUE, alpha = 0.8)

Residuals (from reweighted LS):

Min	1Q	Median	3Q	Max
-0.0040792	-0.0021775	-0.0001453	0.0005056	0.0075624

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
Intercept	-0.03646	0.00393	-9.278	1.37e-10 ***
(-1/k)	0.91527	0.01009	90.683	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.003071 on 32 degrees of freedom
Multiple R-Squared: 0.9961, Adjusted R-squared: 0.996
F-statistic: 8223 on 1 and 32 DF, p-value: < 2.2e-16

```
>
```