DEPARTMENT OF
ECONOMETRICS
AND BUSINESS
STATISTICS

CHUN MINSOO

MONASH University

MONASH BUSINESS SCHOOL

Multiple Linear Regression analysis

Part 2

ETW2001 Foundations of Data Analysis and Modeling

Outline

- ❑ **Regression – Part 2**
  - ❑ Residual analysis
  - ❑ Functional forms
  - ❑ Hypothesis testing
  - ❑ Prediction

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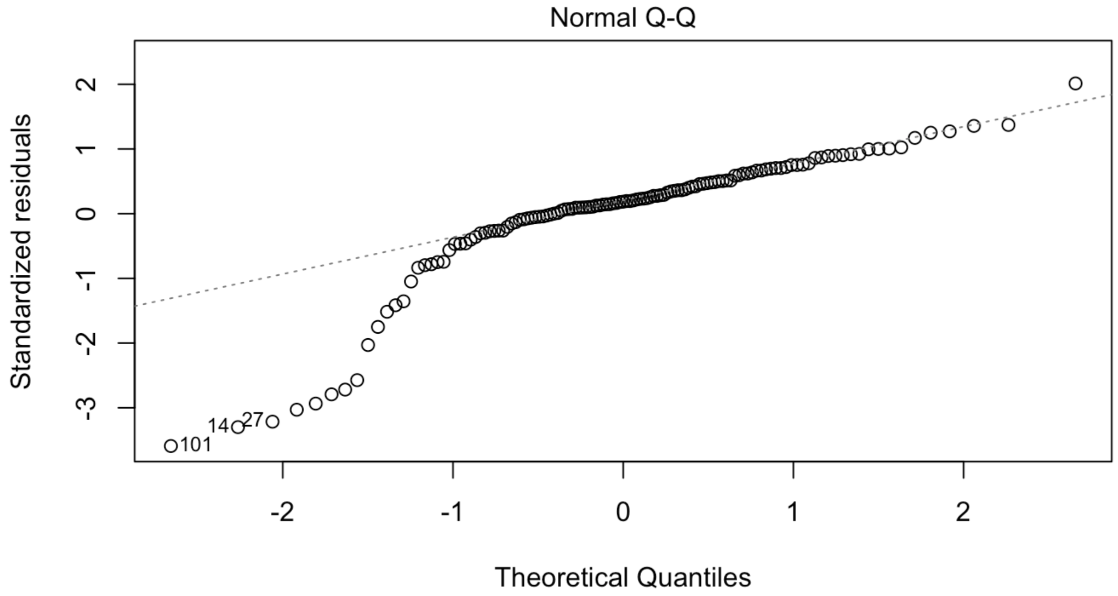
Residual diagnostics

- So far, we focused on interpretation and hypothesis testing of estimated coefficient.
- For rigorous modelling, you need to ensure the ‘robustness’ of the model.
- One way to improve the robustness is to check multicollinearity. (from previous week)
- Another diagnostic check is to examine patterns of residuals.
- Residual, $\hat{e}\ or\ \hat{u} =$ y (actual observation) - $\hat{y}$ (predicted value)
- There is a difference between error term and residual term.
    - Error term: all the excluded variables from your population model.
    - Residual term: discrepancy between actual and predicted value.
- You can visualize residuals to detect how it behaves
    1. Q-Q plot
    2. Residual vs fitted values plot
    3. Residual histogram

Residual diagnostics

Visualization of residuals

1. Q-Q plot



- Q-Q plot visualizes the normality of the variable.
- In this case, it examines the normality of the residuals.
- The residual follows a normality along the diagonal line.

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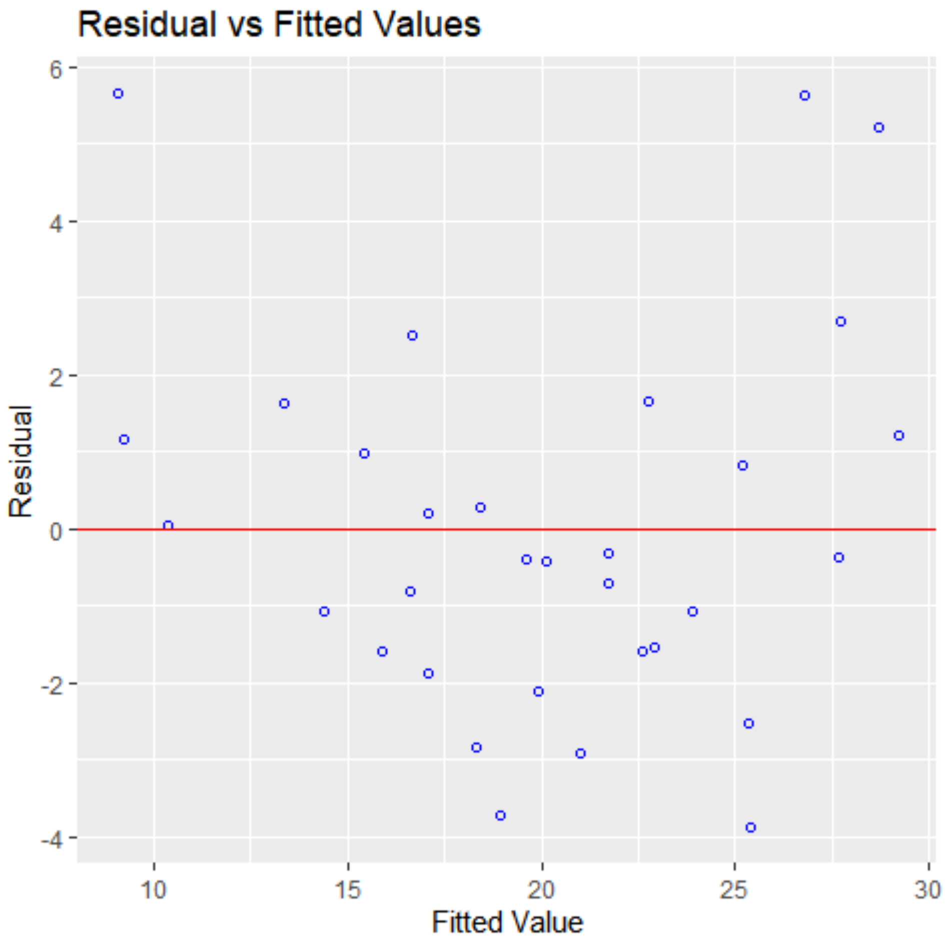
Residual diagnostics

Visualization of residuals
2. Residual vs fitted values plot



- x-axis: fitted value, y-axis: residuals.
- The residual 'behaves' well if the residuals are randomly scattered around the horizontal line, at 0.
- Any observed pattern (linear or non-linear) might indicate significant variable is omitted.
- The observed pattern in the residual plot influences the standard error of the estimated coefficients. Hence, it will affect the result of hypothesis testing.

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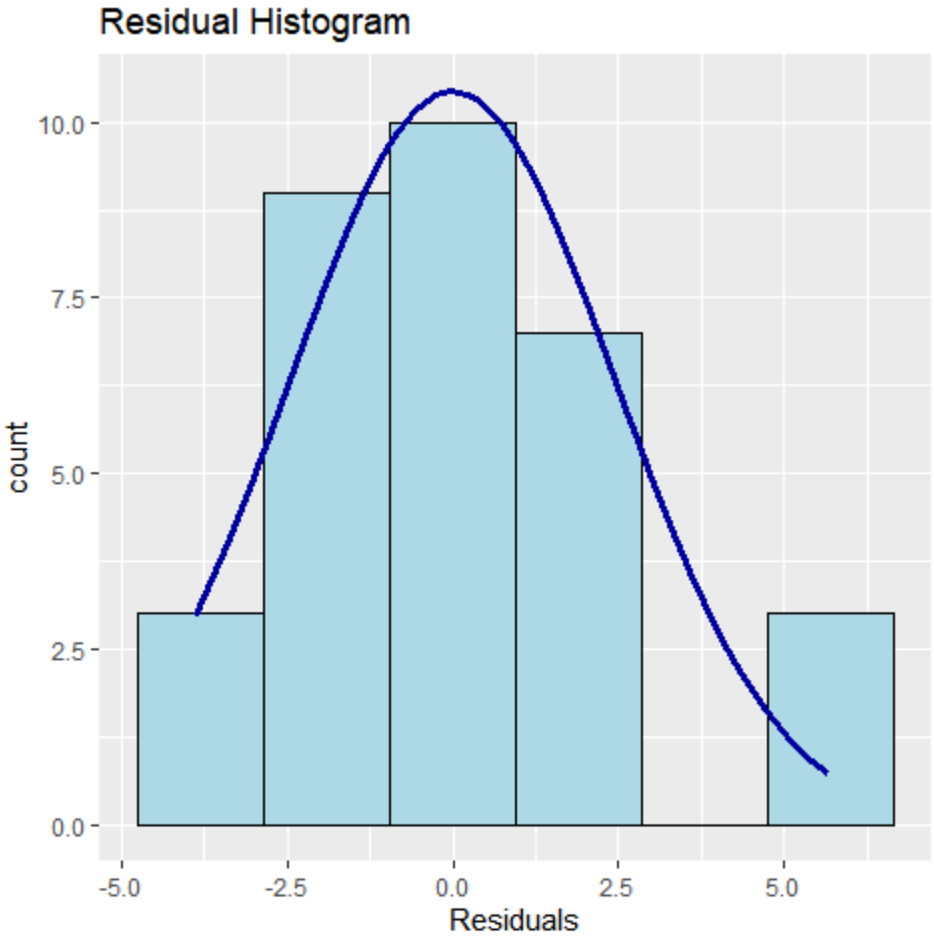
Residual diagnostics

Visualization of residuals

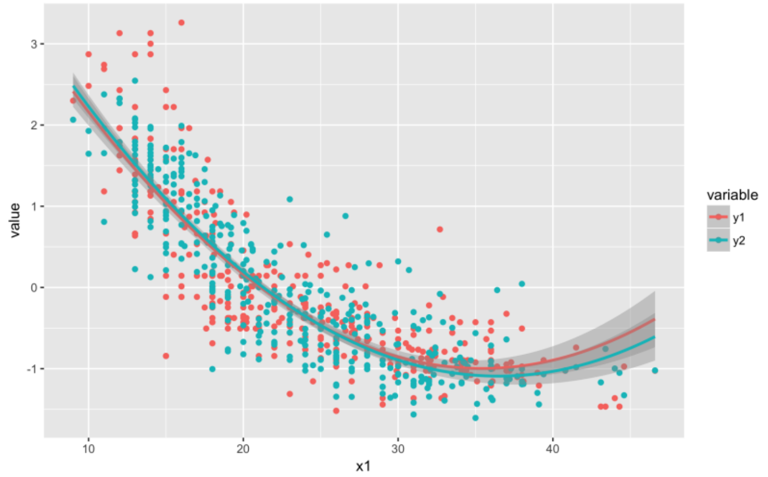
3. Residual histogram



- This is another visualization to observe normality of residuals.
- If the residuals are not distributed around 0, it also indicates omitted variable bias.
- We will not cover formal statistical test for the diagnostic analysis in this unit. ETW2510 will teach you further techniques 😀 (it is called heteroskedasticity).

Functional forms – quadratic term

- So far, we include variables as it is from the dataset.
- You can modify variables mathematically to improve the predictability.
- For instance:



- The scatterplot indicates quadratic relationship between independent and dependent variables.
- A typical regression model may not provide accurate prediction.

- It affects both estimation and interpretation.
- Note that our usual interpretation takes increases in 1 unit of x, y is estimated to change by its corresponding $\hat{\beta}$.
- However, the interpretation is no longer the same as the variable x does not take a linear form.

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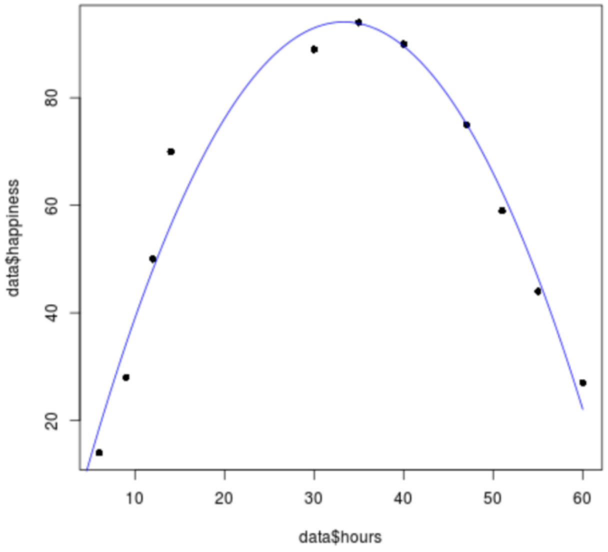
Functional forms – quadratic term

- Quadratic term: applying x^2 as independent variable.
- Here, the level of satisfaction increases until around 30 hours, reaching the maximum.
- If we use No. of hours to predict the happiness using linear function, the effect of time on happiness may not be significant – the best fit would be horizontally constant.
- Hence, the appropriate regression model would be:

$$happiness_{\mathrm{i}} = \beta_0 + \beta_1 hours_i + \beta_2 hours_i^2 + u_i$$



- Here, we can do a bit of maths to compute the turning point:

$$\frac{\delta happiness}{\delta hours} = \hat{\beta}_1 + 2 \cdot \hat{\beta}_2 \cdot hours_i = 0$$

- The pattern increases, then decreases. Here, we can estimate the direction of the estimated coefficient.

$$\hat{\beta}_1 = +ve, and\ \hat{\beta}_2 = -ve$$

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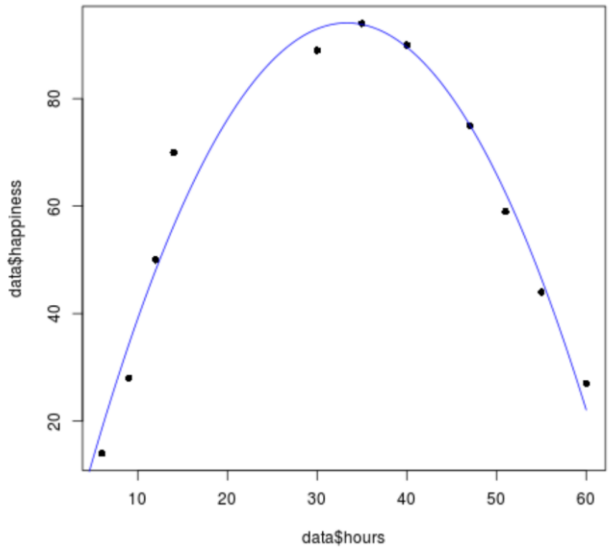
Functional forms – quadratic term

- Continuing from the previous example, the usual interpretation of constant increase does not work.

$$happiness_{\mathrm{i}} = \beta_0 + \beta_1 hours_i + \beta_2 hours_i^2 + u_i$$



- The estimated coefficient $\hat{\beta}_1$ would be relatively larger than the estimated coefficient $\hat{\beta}_2$.
- As the value of x (hours) increases in the beginning, the effect of estimated coefficient $\hat{\beta}_1$ dominates as the effect of $\hat{\beta}_2$ is relatively small ($\hat{\beta}_1 hours_i > \hat{\beta}_2 hours_i^2$).
- However, as hours increases further, the effect of $\hat{\beta}_2$ increases at a faster rate due to the quadratic term, diminishing the effect of hours on happiness. ($\hat{\beta}_1 hours_i < \hat{\beta}_2 hours_i^2$)
- The incremental effect of each hour on happiness is not constant, thus, the usual interpretation does not apply.
- Usually, we focus on the pattern, and when the effect reach out either maximum or minimum.

Functional forms – quadratic term

- Let's look at one example,

$$\underset{}{\widehat{wage}_{\mathrm{i}}} = \underset{(0.19)}{3.73} + \underset{(0.066)}{0.298} exper_i - \underset{(0.019)}{0.0061} exper_i^2$$

- The aggregate effect of experience cannot be interpreted directly.

The turning point:

$$0.298 - 2 * 0.0061 * exper = 0$$

$$exper = \frac{-0.298}{2 * -0.0061}$$

$$= 24.426$$

This indicates that the effect of experience decreases average income after 24 years of experience.

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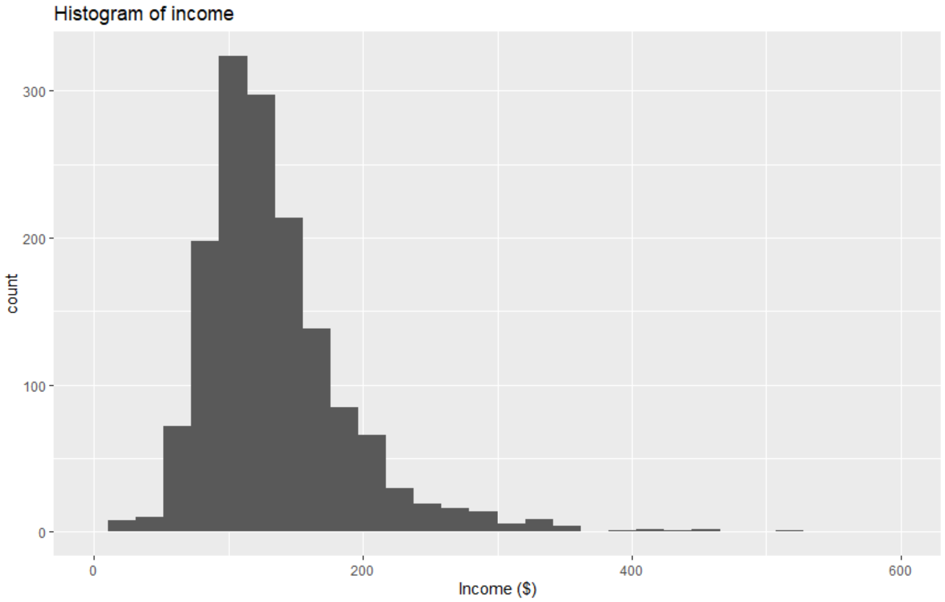
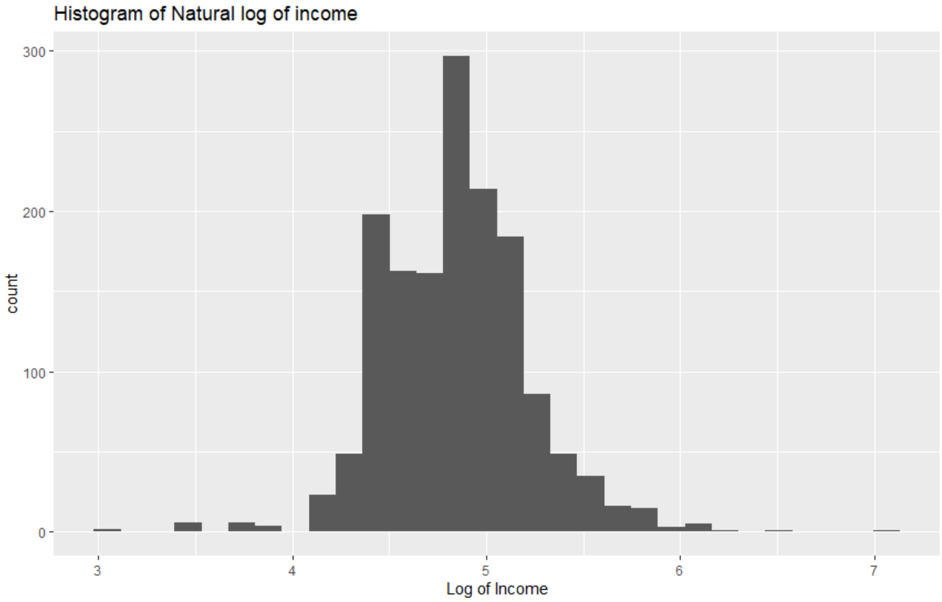
Functional forms – natural log

- Natural logarithm: applying natural $\log_e x$, $(\ln x)$.

- In statistics/econometrics, log(x) usually refers to natural logarithm.

- It is quite common to apply log function to the variables for few reasons:
    1. When the scale of variable is too large.
    2. When the distribution of the variable is not normal.
    3. When the relationship appears to be non-linear.
    4. If you wish to interpret the variables in terms of %.





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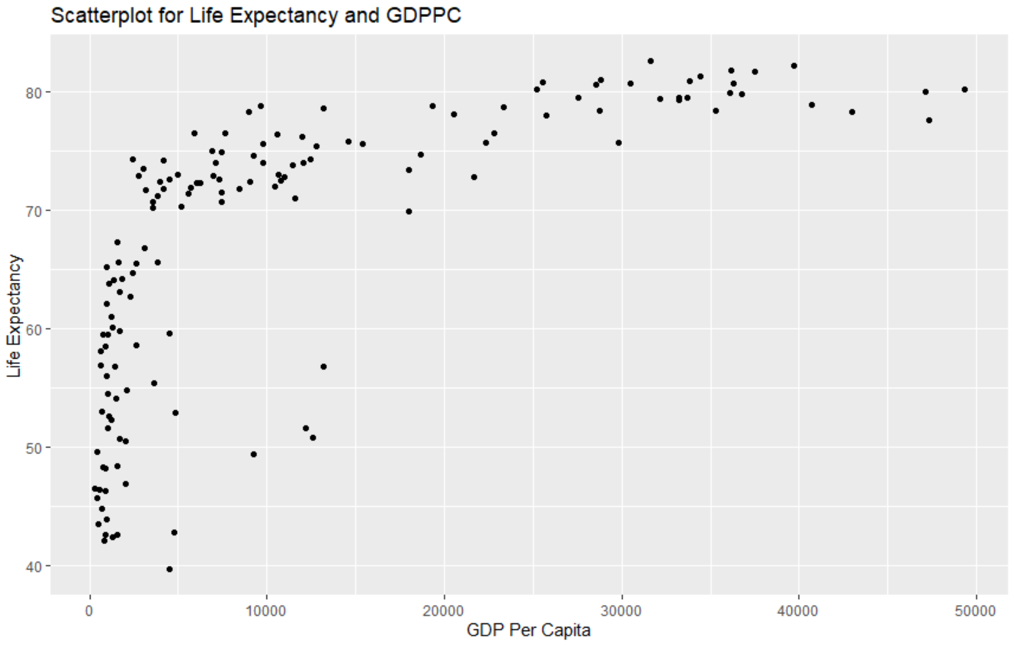
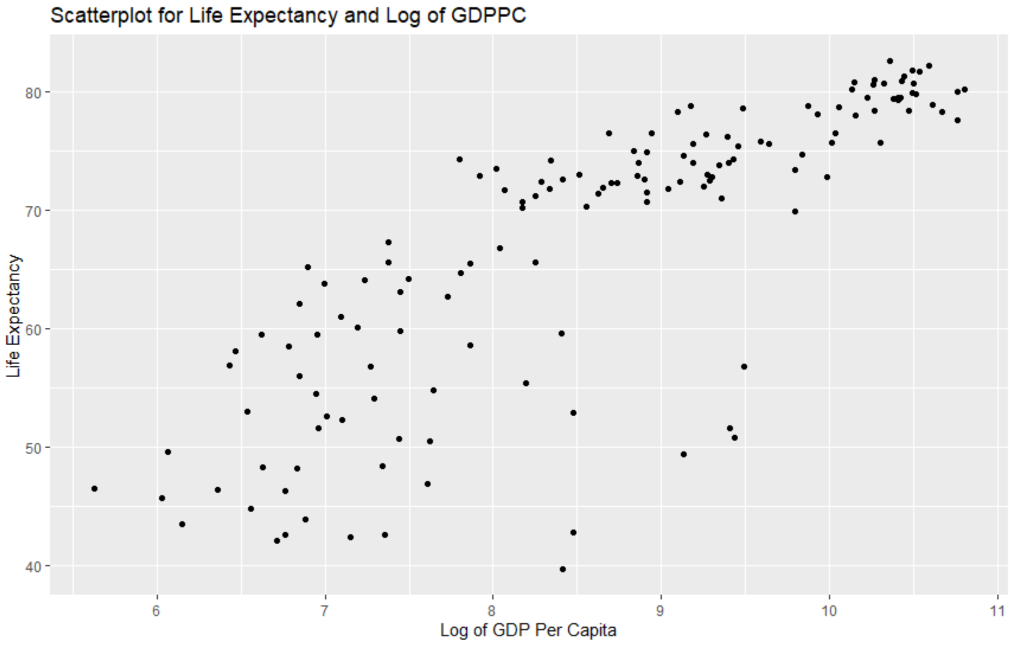
Functional forms – natural log

- Diagrams below show the relationship between GDP per capita and life expectancy.
- One used the data as it is (left) and the other applied natural log on GDP per capita (right).
- For the goodness fit of the model, which one would be better?





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Functional forms – natural log

- Applying natural log alters how we interpret the estimated coefficient.
- The following table shows how we interpret.

Model	Estimated Equation	Interpretation
Level – Level	$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$	One unit change in X leads to $\hat{\beta}$ unit change in Y
Log – Level	$Log(\hat{Y}) = \hat{\beta}_0 + \hat{\beta}_1 X$	One unit change in X leads to $100 * \hat{\beta}$ percent change in Y
Level – Log	$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 Log(X)$	One percent change in X leads to $\hat{\beta}/100$ unit change in Y
Log – Log	$Log(\hat{Y}) = \hat{\beta}_0 + \hat{\beta}_1 Log(X)$	One percent change in X leads to $\hat{\beta}$ percent change in Y

Functional forms – natural log

- Let’s look at one example.

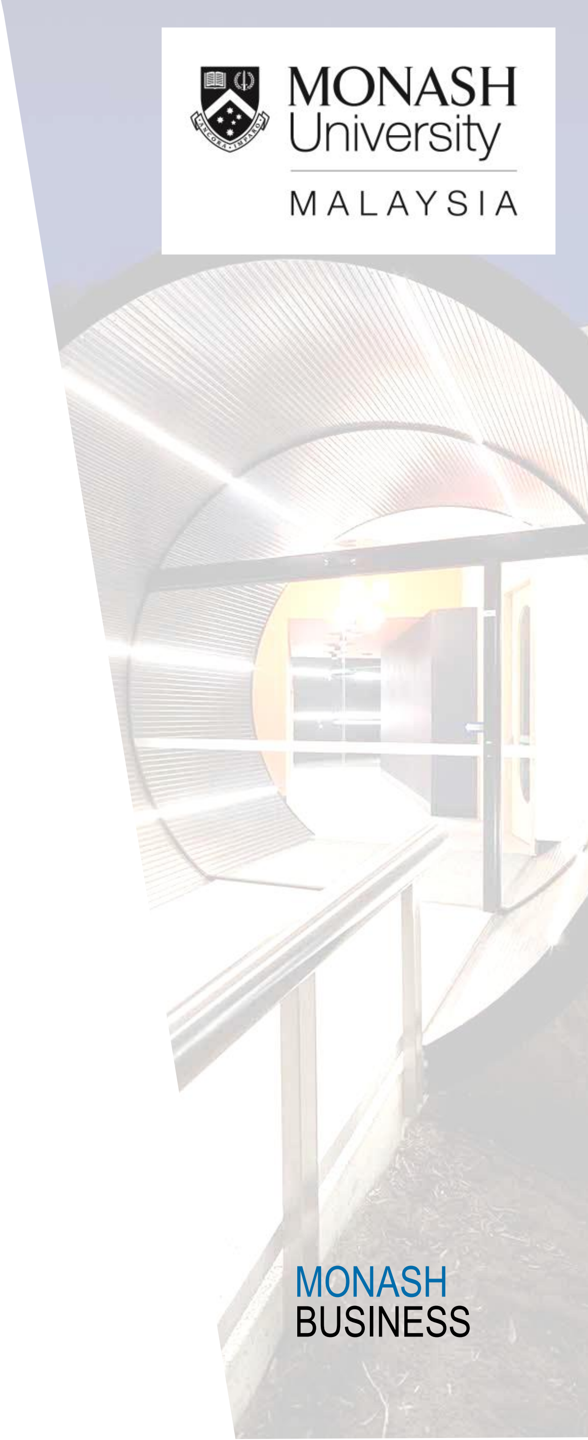
- $\widehat{Log(price)} = 9.23 - 0.718\log(nox) + 0.306rooms$

  $\qquad\qquad(0.19)\quad(0.066)\qquad\qquad(0.019)$

  - price = housing price
  - nox = amount of nitrogen oxide (pollution)
  - Rooms: number of rooms

- When the nitrogen oxide level increases by 1%, the housing price is expected to decrease by 0.72%, on average while holding the number of rooms constant.

- When the number of rooms increases by 1 unit, the housing price is expected to increase by 30.6% (100*0.306), on average while holding the nitrogen oxide level constant.

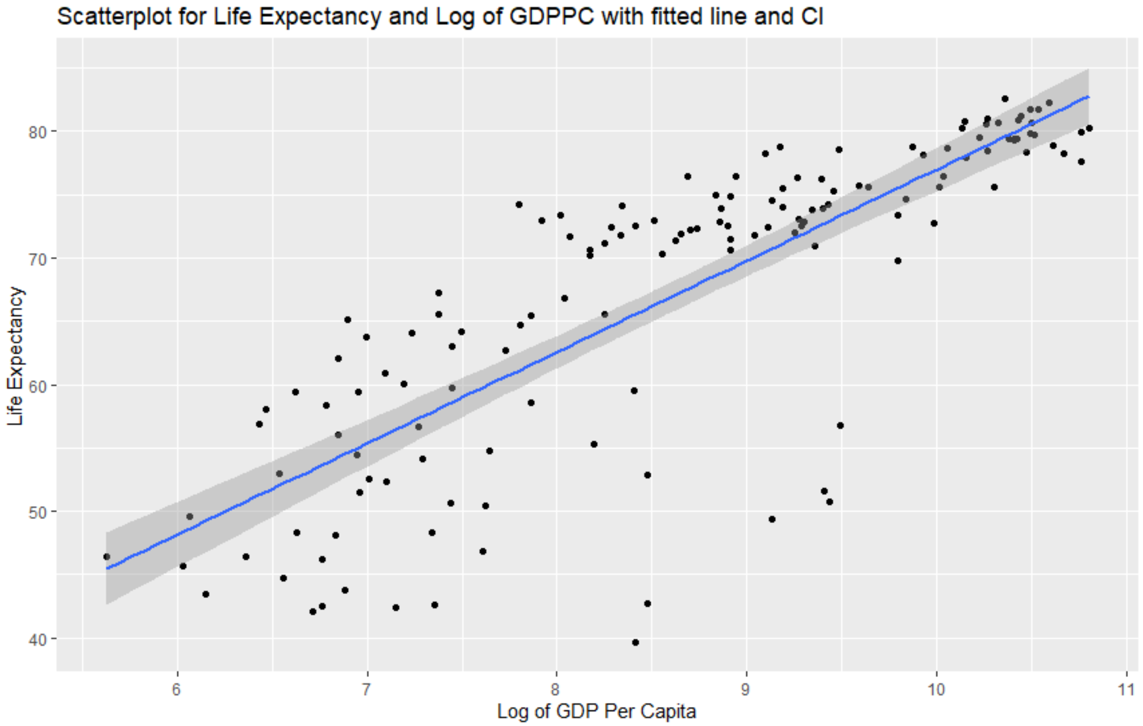


Prediction

- Once your model seems robust, making predictions is fairly simple.

- Using your estimated model, just substitute the desired values into independent variables, and the $\hat{y}$ is your predicted value for dependent variable.

- In a simple linear regression, you can visualize the prediction line with its confidence interval.



- From the scatterplot in slide 12, the fitted line is added showing the changes in life expectancy for changes in log of GDP per capita.

- The grey area shows the 95% confidence interval of the prediction.

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Prediction

- It would be difficult to visualize predicted value for multiple linear regression.

- There are two ways to predict for the estimated equation below:

$$\widehat{LifeExp}_i = 4.589 + 7.217\log(gdpPercap_i) + 0.000000005449pop_i$$

- Let's predict for a country that GDP per capita is \$21,583 and population is 13 mil.

1. You can substitute the values into the estimated model manually.

$$4.589 + 7.217\log(21583) + 0.000000005449 * 13000000$$
$$= 76.68$$

2. Alternatively, you can create a data frame of the values using tibble()

```
predict_data <- tibble(gdpPercap=21583, pop=13000000)
predict(lm2, predict_data)
```

Using predict(regression model, values for indep. Variable) to automatically substitute the values for you.

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