The F distribution

The **F-statistic** is a random variable that has an F distribution.

Here are the steps required to compute an **F-statistic**:

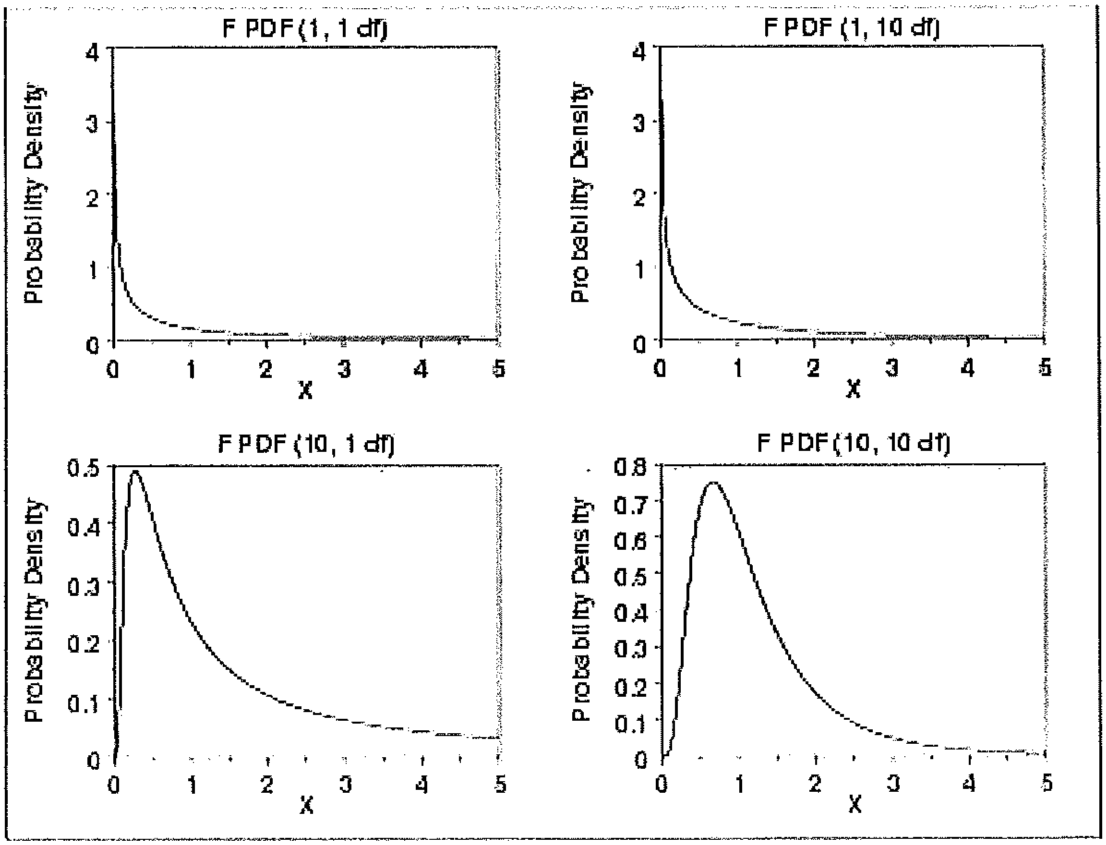
- Select a random sample of size n_1 from a normal population, having a standard deviation equal to σ_1.
- Select an independent random sample of size n_2 from a normal population, having a standard deviation equal to σ_2.
- The F-statistic is the ratio of s_1^2/σ_1^2 and s_2^2/σ_2^2. Thus

$$F = \frac{s_1^2 / \sigma_1^2}{s_2^2 / \sigma_2^2}$$

The F Distribution

- The distribution of all possible values of the F-statistic is called an **F distribution**, with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degrees of freedom.
- The curve of the F distribution depends on the degrees of freedom, v_1 and v_2. When describing an F distribution, the number of degrees of freedom associated with the standard deviation in the numerator of the F-statistic is always stated first. Thus, F(5, 9) would refer to an F distribution with $v_1 = 5$ and $v_2 = 9$ degrees of freedom; whereas F(9, 5) would refer to an F distribution with $v_1 = 9$ and $v_2 = 5$ degrees of freedom. Note that the curve represented by F(5, 9) would differ from the curve represented by F(9, 5).

Typical F curves



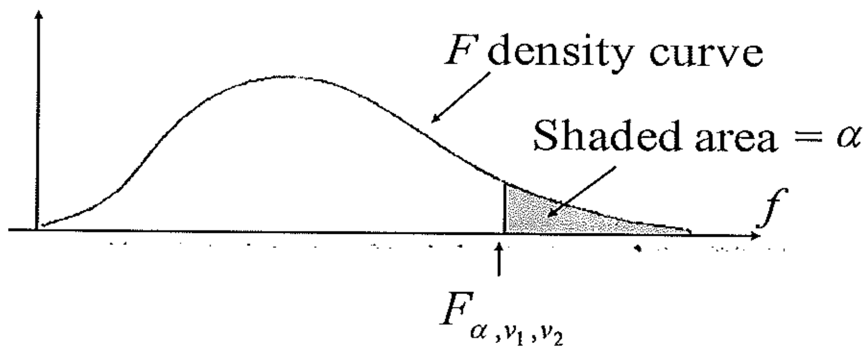
Note that:

The F-values are all non-negative

- The distribution is non-symmetric
- The mean is approximately 1
- There are two independent degrees of freedom, one for the numerator, and one for the denominator.
- There are many different F distributions, one for each pair of degrees of freedom.

The probability that 2 independent random samples produce an F-value greater than some specified value equals the area under the curve to the right of this value. We use F-tables to read these values.

The F Distribution Density Curve Property



The F-Test

The F-test is designed to test if two population variances are equal.

We test H_o: $\sigma_1^2 = \sigma_1^2$ against H_a: $\sigma_1^2 \neq \sigma_1^2$

All hypothesis testing is done under the assumption the null hypothesis is true

If the null hypothesis is true, then the F-test statistic given above can be simplified (dramatically) to:

$$\boxed{F = \frac{s_1^2}{s_2^2}}$$

This ratio of sample variances will therefore be the test statistic used.

There are several different F-tables. Each one has a different level of significance. So, find the correct level of significance first, and then look up the numerator degrees of freedom and the denominator degrees of freedom to find the critical value above which you will reject the null hypothesis.

You will notice that all the tables only give levels of significance for right tail tests. Because the F distribution is not symmetric, and there are no negative values, you may not simply take the opposite of the right critical value to find the left critical value. The way to find a left critical value is to reverse the degrees of freedom, look up the right critical value, and then take the reciprocal of this value. For example, the critical value with 0.05 on the left with 12 numerator and 15 denominator degrees of freedom is found of taking the reciprocal of the critical value with 0.05 on the right with 15 numerator and 12 denominator degrees of freedom.

Avoiding Left Critical Values

Left critical values are often avoided altogether. You can force the F-test into a right tail test by placing the sample with the large variance in the numerator and the smaller variance in the denominator. It does not matter which sample has the larger sample size, only which sample has the larger variance.

The numerator degrees of freedom will be the degrees of freedom for whichever sample has the larger variance (since it is in the numerator) and the denominator degrees of freedom will be the degrees of freedom for whichever sample has the smaller variance (since it is in the denominator).

If a two-tail test is being conducted, you still have to divide alpha by 2, but you only look up and compare the right critical value.

F-Test for Equal Population Variances

One-Tailed Test

$$H_o : \sigma_1^2 = \sigma_2^2$$

$$H_a : \sigma_1^2 < \sigma_2^2$$

(or >)

Test Statistic:

$$F = \frac{s_2^2}{s_1^2}$$

OR

$$F = \frac{s_1^2}{s_2^2} \text{ when}$$

$$H_a : \sigma_1^2 > \sigma_2^2$$

Rejection Region

$$F > F_\alpha$$

Two-Tailed Test

$$H_o : \sigma_1^2 = \sigma_2^2$$

$$H_a : \sigma_1^2 \neq \sigma_2^2$$

$$F = \frac{Larger\ sample\ variance}{Smaller\ sample\ Variance}$$

$$= \frac{s_1^2}{s_2^2} \text{ when } s_1^2 > s_2^2$$

OR

$$\frac{s_2^2}{s_1^2} \text{ when } s_2^2 > s_1^2$$

$$F > F_{\frac{\alpha}{2}}$$

Where F_α and $F_{\frac{\alpha}{2}}$ are based on ν_1 - numerator degrees of freedom and

ν_2 = denominator degrees of freedom; ν_1 and ν_2 are the degrees of freedom for the numerator and denominator sample variances respectively.

Assumptions

1) **Both sampled populations are normally distributed**
2) **The samples are random and independent**

APPLICATIONS

16. The average price of a new automobile in 1991 was \$16,700, an increase of 4.3% over the 1990 average price of \$16,012 (*U.S. News & World Report,* September 9, 1991). Assume that samples of 121 new 1991 automobiles showed a sample standard deviation in price of \$4200 and 121 new 1990 automobiles showed a sample standard deviation in price of \$3850. Using a .05 level of significance, can it be concluded that the variance in prices of new automobiles also increased in 1991?

17. Most individuals are aware of the fact that the average annual repair cost for an automobile depends on the age of the automobile. For example, the average annual repair cost (*Consumer Reports 1992 Buyers Guide*) for automobiles 4 years old (\$400) is almost twice as large as the average annual repair cost for automobiles 2 years old (\$220). A researcher is interested in studying the variance of the annual repair costs to see if the variance in the repair costs also increases with the age of the automobile. A sample of 25 automobiles 4 years old showed a sample standard deviation for annual repair costs of \$170 while a sample of 25 automobiles 2 years old showed a sample standard deviation for annual repair costs of \$100.

a. State the null and alternative hypotheses for the research hypothesis that the variance in annual repair costs is larger for the older automobiles.

b. Using a .01 level of significance, what is your conclusion? Discuss the reasonableness of your findings.

18. The Educational Testing Service has conducted studies designed to identify differences between the scores of males and females on the Scholastic Aptitude Test (*Journal of Educational Measurement,* Spring 1987). For a sample of females, the standard deviation of test scores was 83 on the verbal portion of the SAT. For a sample of males, the standard deviation was 78 on the same test. Assume that standard deviations were based on random samples of 121 females and 121 males. Do the data indicate there are differences between the variances of females and males on the verbal portion of the SAT? Use $\alpha = .05$.

19. Independent random samples of parts manufactured by two suppliers provided the following results.

Supplier	Sample Size	Sample Variance of Part Sizes
Durham Electric	41	$s_1^2 = 3.8$
Raleigh Electronics	31	$s_2^2 = 2.0$

The firm making the supplier-selection decision is prepared to use the Durham supplier unless the test results show that the Raleigh supplier provides a significantly lower variance in part sizes. Use $\alpha = .05$, and conduct the statistical test that will help the firm select a supplier. Which supplier do you recommend?

20. The following sample data have been collected from two independent random samples.

Population	Sample Size	Sample Mean	Sample Variance
A	$n_A = 25$	$\bar{x}_A = 40$	$s_A^2 = 5$
B	$n_B = 21$	$\bar{x}_B = 50$	$s_B^2 = 11$

Test for the equality of the variances of population A and population B. Use $\alpha = 10$. What is your conclusion?

21. Two secretaries are each given eight typing assignments of equal difficulty. The sample standard deviations of the completion times were 3.8 minutes and 5.2 minutes, respectively. Do the data suggest that there is a difference in the variability of completion times for the two secretaries? Test the hypothesis at a .10 level of significance.

22. A research hypothesis is that the variance of stopping distances of automobiles on wet pavement is substantially greater than the variance of stopping distances of automobiles on dry pavement. In the research study, 16 automobiles traveling at the same speed are tested with respect to stopping distances on wet pavement and then tested with respect to stopping distances on dry pavement. On wet pavement, the standard deviation of stopping distances was 32 feet. On dry pavement, the standard deviation was 16 feet.
    a. At a .05 level of significance, do the sample data justify the conclusion that the variance in stopping distances on wet pavement is greater than the variance in stopping distances on dry pavement?
    b. What are the implications of your statistical conclusions in terms of driving safety recommendations?