IIUM cat-us-trophy

Contents

.vimrc

```
set ai ts=4 sw=4 st=4 noet nu nohls
syntax enable
filetype plugin indent on
map <F6> :w<CR>:!g++ % -g && (ulimit -c unlimited; ./a.out < ~/input.txt) <CR>
map <F5> <F6>
colo pablo
map <F12> :!gdb ./a.out -c core <CR>
```

template.cpp

```
#include<cstdio>
#include<sstream>
#include<cstdlib>
#include<cctype>
#include<cmath>
#include<algorithm>
#include<set>
#include<queue>
#include<stack>
#include<list>
#include<iostream>
#include<string>
#include<vector>
#include<cstring>
#include<map>
#include<cassert>
#include<climits>
using namespace std;

#define REP(i,n) for(int i=0; i<(n); i++)
#define FOR(i,a,b) for(int i=(a); i<=(b); i++)
#define FORD(i,a,b) for(int i=(a); i>=(b); i--)
#define FORIT(i, m) for (__typeof((m).begin()) i=(m).begin(); i!=(m).end(); ++i)
#define SET(t,v) memset((t), (v), sizeof(t))
#define ALL(x) x.begin(), x.end()
#define UNIQUE(c) (c).resize( unique( ALL(c) ) - (c).begin() )

#define sz(v) int(v.size())
#define pb push_back
#define VI vector<int>
#define VS vector<string>
```

```
typedef long long LL;
typedef long double LD;
typedef pair<int,int> pii;

#define D(x) if(1) cout << __LINE__ <<" "<< #x " = " << (x) << endl;
#define D2(x,y) if(1) cout << __LINE__ <<" "<< #x " = " << (x) \
     <<", " << #y " = " << (y) << endl;
```

Combinatorics

Mathematical Sums

$\sum_{k=0}^{n} k = n(n+1)/2$ $\quad$ $\sum_{k=a}^{b} k = (a+b)(b-a+1)/2$

$\sum_{k=0}^{n} k^2 = n(n+1)(2n+1)/6$ $\quad$ $\sum_{k=0}^{n} k^3 = n^2(n+1)^2/4$

$\sum_{k=0}^{n} k^4 = (6n^5+15n^4+10n^3-n)/30$ $\quad$ $\sum_{k=0}^{n} k^5 = (2n^6+6n^5+5n^4-n^2)/12$

$\sum_{k=0}^{n} x^k = (x^{n+1}-1)/(x-1)$ $\quad$ $\sum_{k=0}^{n} kx^k = (x-(n+1)x^{n+1}+nx^{n+2})/(x-1)^2$

Binomial coefficients

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	1												
1	1	1											
2	1	2	1										
3	1	3	3	1									
4	1	4	6	4	1								
5	1	5	10	10	5	1							
6	1	6	15	20	15	6	1						
7	1	7	21	35	35	21	7	1					
8	1	8	28	56	70	56	28	8	1				
9	1	9	36	84	126	126	84	36	9	1			
10	1	10	45	120	210	252	210	120	45	10	1		
11	1	11	55	165	330	462	462	330	165	55	11	1	
12	1	12	66	220	495	792	924	792	495	220	66	12	1
	0	1	2	3	4	5	6	7	8	9	10	11	12

$\binom{n}{k} = \frac{n!}{(n-k)!k!}$

$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$

$\binom{n}{k} = \frac{n}{n-k}\binom{n-1}{k}$

$\binom{n}{k} = \frac{n-k+1}{k}\binom{n}{k-1}$

$\binom{n+1}{k} = \frac{n+1}{n-k+1}\binom{n}{k}$

$\binom{n}{k+1} = \frac{n-k}{k+1}\binom{n}{k}$

$\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$

$\sum_{k=1}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$

$\binom{m+n}{r} = \sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k}$

$\binom{n}{k} = \prod_{i=1}^{k}\frac{n-k+i}{i}$

Catalan numbers $C_n = \frac{1}{n+1}\binom{2n}{n}$. $\quad C_0 = 1$, $C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$. $\quad C_{n+1} = C_n\frac{4n+2}{n+2}$.
$C_0, C_1, \ldots = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, \ldots$
C_n is the number of: properly nested sequences of n pairs of parentheses; rooted ordered binary trees with $n+1$ leaves; triangulations of a convex $(n+2)$-gon.

Derangements . Number of permutations of $n = 0, 1, 2, \ldots$ elements without fixed points is $1, 0, 1, 2, 9, 44, 265, 1854, 14833, \ldots$ Recurrence: $D_n = (n-1)(D_{n-1} + D_{n-2}) = nD_{n-1} + (-1)^n$. Corollary: number of permutations with exactly k fixed points is $\binom{n}{k}D_{n-k}$.

Stirling numbers of 1^{st} kind . $s_{n,k}$ is $(-1)^{n-k}$ times the number of permutations of n elements with exactly k permutation cycles. $\left[{n \atop k}\right] = |s_{n,k}| = |s_{n-1,k-1}| + (n-1)|s_{n-1,k}|$ $\quad s(0,0) = 1$ and $s(n,0) = s(0,n) = 0$.

Stirling numbers of 2^{nd} kind . $S_{n,k}$ is the number of ways to partition a set of n elements into exactly k non-empty subsets. $\left\{{n \atop k}\right\} = S_{n,k} = S_{n-1,k-1} + kS_{n-1,k}$. $S_{n,1} = S_{n,n} = 1$.

Bell numbers . B_n is the number of partitions of n elements. $B_0, \ldots = 1, 1, 2, 5, 15, 52, 203, 877, \ldots$
$B_{n+1} = \sum_{k=0}^{n}\binom{n}{k}B_k = \sum_{k=1}^{n+1} S_{n,k}$. Bell triangle: $B_r = a_{r,1} = a_{r-1,r-1}$, $a_{r,c} = a_{r-1,c-1} + a_{r,c-1}$.

Eulerian numbers . $E(n,k) = \left\langle{n \atop k}\right\rangle$ is the number of permutations with exactly k descents $(i : \pi_i < \pi_{i+1})$ / ascents $(\pi_i > \pi_{i+1})$ / excedances $(\pi_i > i)$ / $k+1$ weak excedances $(\pi_i \geq i)$.
Formula: $E(n,m) = (m+1)E(n-1,m) + (n-m)E(n-1,m-1)$. $E(n,0) = E(n,n-1) = 1$.
$E(n,m) = \sum_{k=0}^{m}(-1)^k\binom{n+1}{k}(m+1-k)^n$.

Double factorial . Permutations of the multiset $\{1,1,2,3,\ldots n,n\}$ such that for each k, all the numbers between two occurrences of k in the permutation are greater than k. $(2n-1)!! = \prod_{k=1}^{n}(2k-1)$.

Eulerian numbers of 2^{nd} kind . Related to Double factorial, number of all such permutations that have exactly m ascents. $\left\langle\!\left\langle {n \atop m} \right\rangle\!\right\rangle = (2n-m-1)\left\langle\!\left\langle {n-1 \atop m-1} \right\rangle\!\right\rangle + (m+1)\left\langle\!\left\langle {n-1 \atop m} \right\rangle\!\right\rangle$. $\left\langle\!\left\langle {n \atop 0} \right\rangle\!\right\rangle = 1$

Multinomial theorem . $(a_1+\cdots+a_k)^n = \sum \binom{n}{n_1,\ldots,n_k} a_1^{n_1}\ldots a_k^{n_k}$, where $n_i \geq 0$ and $\sum n_i = n$. $\binom{n}{n_1,\ldots,n_k} = M(n_1,\ldots,n_k) = \frac{n!}{n_1!\ldots n_k!}$. $M(a,\ldots,b,c,\ldots) = M(a+\cdots+b,c,\ldots)M(a,\ldots,b)$

RMQ DP

```
int make_dp(int n) { // N log N
    REP(i,n) H[i][0]=i;
    for(int l=0,k; (k=1<<l) < n; l++) for(int i=0;i+k<n;i++)
        H[i][l+1] = A[H[i][l]] > A[H[i+k][l]] ? H[i+k][l] : H[i][l];
} // query log N almost O(1)
int query_dp(int a, int b) {
    for(int l=0;;l++)if(a+(1<<l+1) > b) {
        int o2 = H[b-(1<<l)+1][l];
        return A[H[a][l]]<A[o2] ? H[a][l]:o2;
}   }
```

Suffix arrays

```
const int N = 100 * 1000 + 10;
char str[N]; bool bh[N], b2h[N];
int rank[N], pos[N], cnt[N], next[N], lcp[N];
bool smaller(int a, int b) { return str[a]<str[b];}
void suffix_array(int n) {
    REP(i,n)pos[i]=i, b2h[i]=false;
    sort(pos,pos+n,smaller);
    REP(i,n) bh[i]=!i||str[pos[i]] != str[pos[i-1]];
    for(int h=1;h<n;h*=2) {
        int buckets=0;
        for(int i=0,j; i<n; i=j) {
            j=i+1;
            while(j<n && !bh[j])j++;
            next[i]=j;
            buckets++;
        }
        if(buckets==n)break;
        for(int i=0;i<n;i=next[i]) {
            cnt[i] = 0;
            FOR(j, i, next[i]-1) rank[pos[j]]=i;
        }
        cnt[rank[n-h]]++;
        b2h[rank[n-h]]=true;
        for(int i=0;i<n;i=next[i]) {
            FOR(j, i, next[i]-1) {
                int s = pos[j]-h;
                if(s>=0){
                    rank[s] = rank[s] + cnt[rank[s]]++;
                    b2h[rank[s]]=true;
            }   }
            FOR(j, i, next[i]-1) {
                int s = pos[j]-h;
```

```
                if(s>=0 && b2h[rank[s]])
                    for(int k=rank[s]+1;!bh[k] && b2h[k]; k++) b2h[k]=false;
        }   }
        REP(i,n) pos[rank[i]]=i, bh[i]|=b2h[i];
}   }
void get_lcp(int n) {
    lcp[0]=0;
    int h=0;
    REP(i,n) if(rank[i]) {
        int j=pos[rank[i]-1];
        while(i+h<n && j+h<n && str[i+h] == str[j+h]) h++;
        lcp[rank[i]]=h;
        if(h)h--;
}   }
```

Graph algorithms (LCA, SCC)

Tarjan's offline LCA

```
function TarjanOLCA(u)
    MakeSet(u); u.ancestor := u;
    for each v in u.children do
        TarjanOLCA(v); Union(u,v); Find(u).ancestor := u;
    u.colour := black;
    for each v such that {u,v} in P and v.color==black do
        print "LCA", u, v, Find(v).ancestor
```

Tarjan's Strong Connected Components

```
procedure tarjan(v)
  index = count; v.lowlink = count++; S.push(v);color[v] = 1;
  for all (v, v2) in E do
     if (!color[v2])
        tarjan(v2); v.lowlink = min(v.lowlink, v2.lowlink);
     else if (color[v2]==1)
        v.lowlink = min(v.lowlink, v2.lowlink);
  if (v.lowlink == index)
    do { v2 = S.top(); S.pop(); print v2; color[v2]=2; } while (v2 != v);
for all v in V do if(!color[v]) tarjan(v);
```