Cours 8 : algorithmes de tris

Jean-Stéphane Varré

Université Lille 1

jean-stephane.varre@lifl.fr

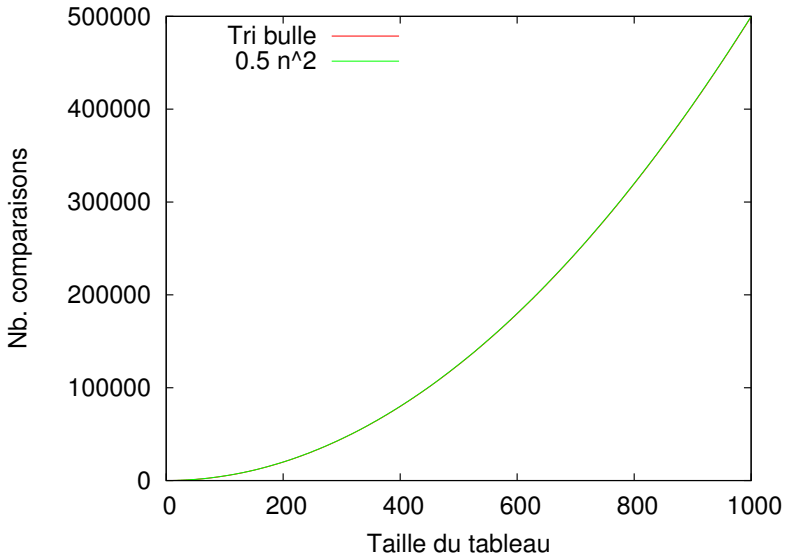
Au menu

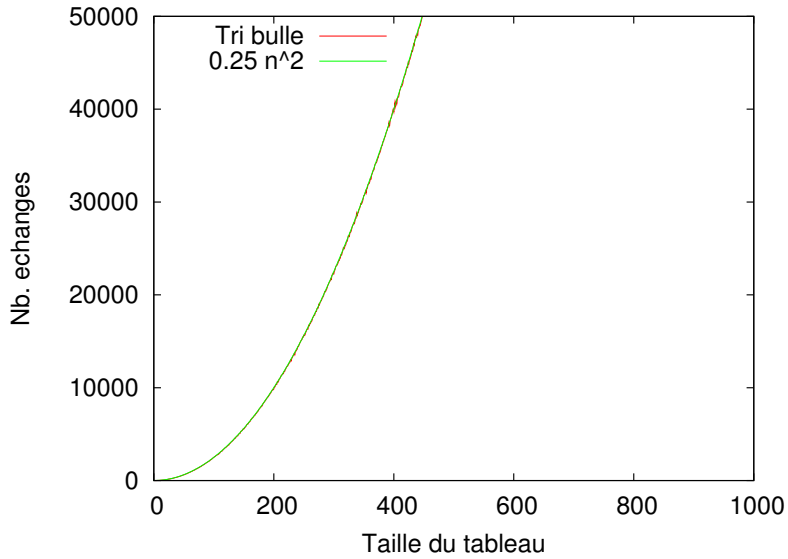
- Tri bulle
- Tri insertion
- Tri tas (HeapSort)
- Tri fusion (MergeSort)
- Tri rapide (QuickSort)
- Quelle est la complexité du meilleur algorithme de tri ?
- Trier en $\mathcal{O}(n)$, c'est possible !?

Implémentation

```
procedure triBulle (var t : TABLEAU; var nbcmp : CARDINAL; var nbech : CARDINAL);
var
   i, j : CARDINAL;
begin
   for i := high(t) downto low(t) + 1 do begin
      for j := low(t) to i-1 do begin
         inc(nbcmp);
         if t[j] > t[j+1] then begin
            inc(nbech);
            echanger(t,j,j+1);
         end {if};
      end {for};
   end {for};
end {triBulle};
```

Temps de calcul - tri bulle

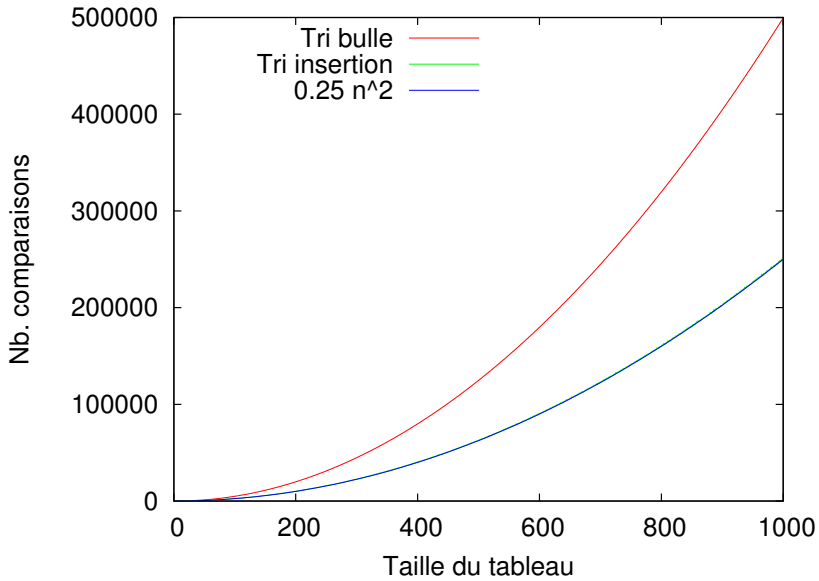


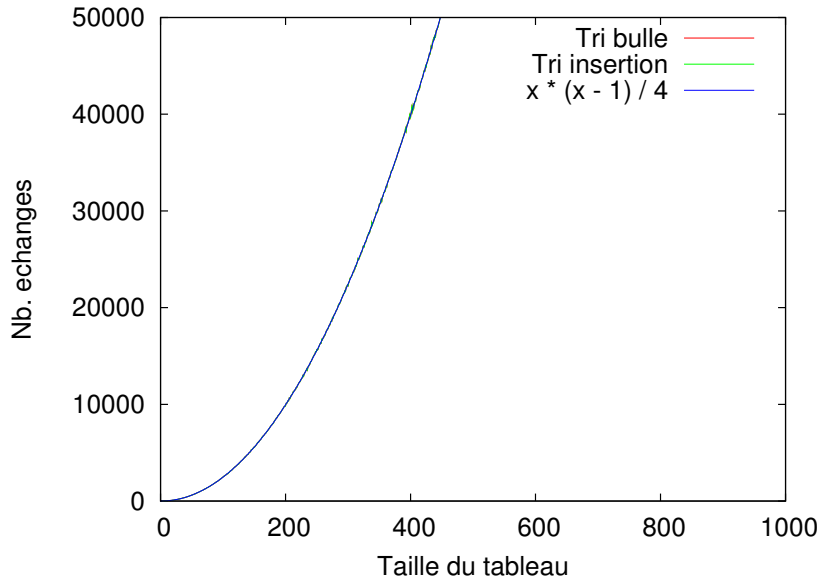


Implémentation

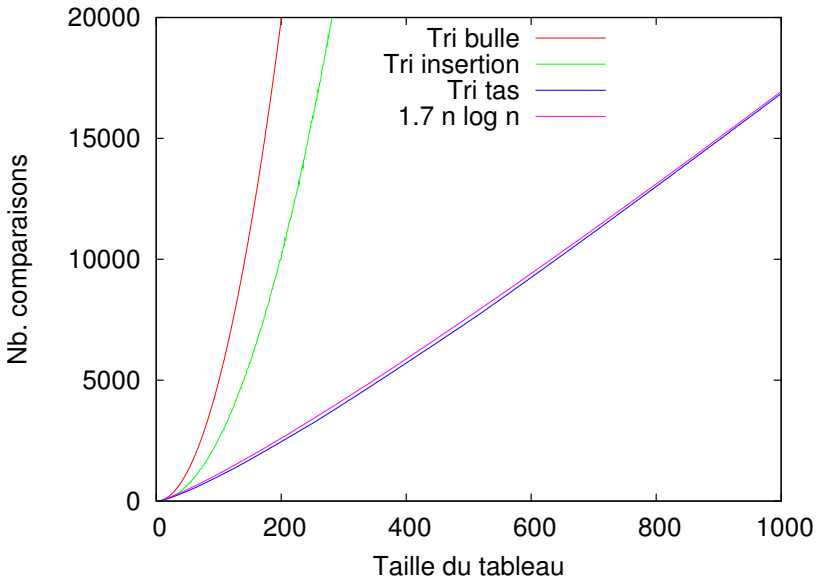
```
procedure inserer (var t : TABLEAU; const n : CARDINAL; var nbcmp : CARDINAL; var
var
   i : CARDINAL;
begin
   i := n;
   while (i > low(t)) and (t[i-1] > t[i]) do begin
      inc(nbcmp);
      inc(nbech);
      echanger(t,i,i-1);
      dec(i);
   end {while};
   if (i > low(t)) and (t[i-1] <= t[i]) then inc(nbcmp);
end {inserer};

procedure triInsertion (var t : TABLEAU; var nbcmp : CARDINAL; var nbech : CARDIN
var
   i : CARDINAL;
begin
   for i := low(t) + 1 to high(t) do begin
      inserer (t,i,nbcmp,nbech);
   end {for};
end {trier};
```

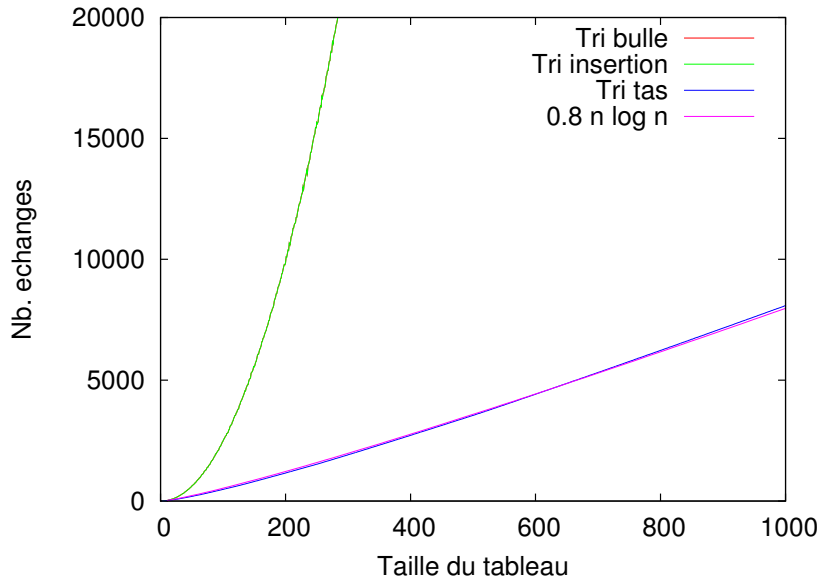
Temps de calcul - tri insersion





Temps de calcul - tri tas





Implémentation

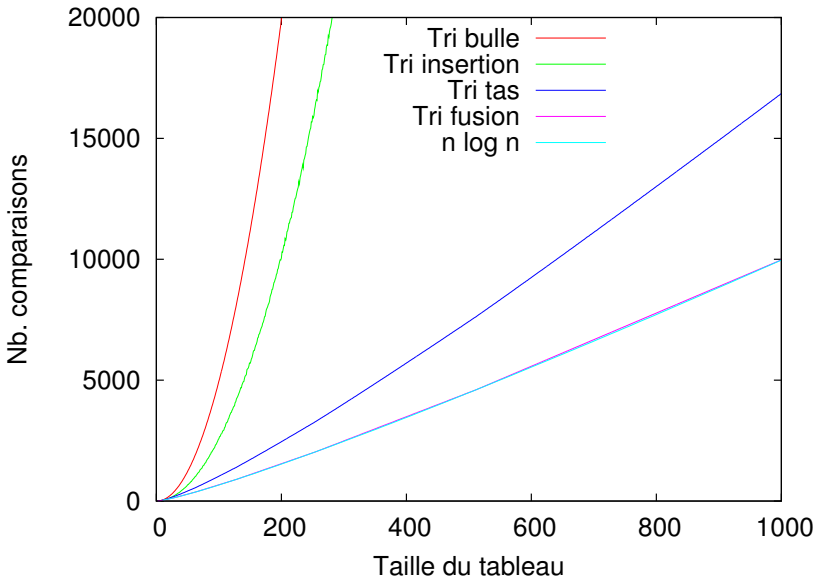
```
procedure triFusion (var t : TABLEAU; const g, d: INTEGER; var nbcmp : CARDINAL;
var
   m : INTEGER;
begin
   if g < d then begin
      m := (g + d) div 2;
      // tri recursif des parties gauche et droite du tableau
      triFusion (t, g, m, nbcmp, nbech);
      triFusion (t, m + 1, d, nbcmp, nbech);
      // fusion des parties triees
      fusionner(t,g,d,m);
   end {if};
end {triFusion};
```

L'appel initial est : `triFusion(t,low(t),high(t),cmp,ech);`

Implémentation

```
procedure fusionner (var t : TABLEAU; const g,d,m : INTEGER);
var
   u : TABLEAU;
   i, j, k : INTEGER;
begin
   setlength(u,length(t));
   for i := m downto g do u[i] := t[i];
   for j := m+1 to d do u[d+m+1-j] := t[j];
   i := g; j := d;
   for k := g to d do begin
      inc(nbcmp);
      if u[i] < u[j] then begin
         t[k] := u[i];
         i := i + 1
      end else begin
         t[k] := u[j];
         j := j - 1
      end {if};
   end {for};
end {fusionner};
```

Temps de calcul - tris fusion



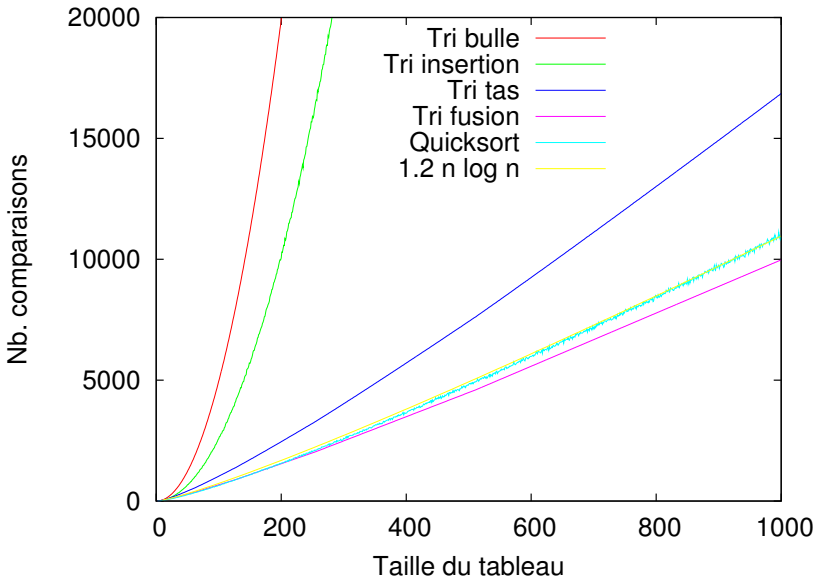
Implémentation

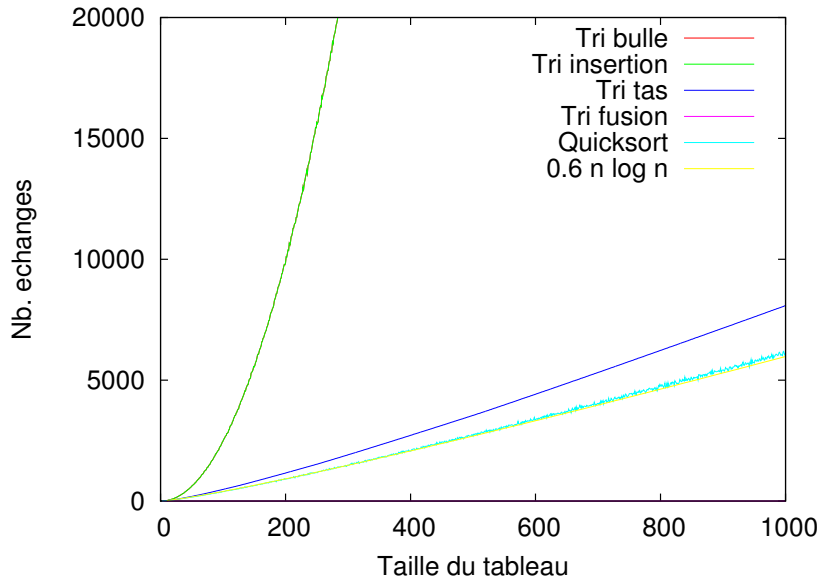
```
procedure QSort (var t : TABLEAU; const g, d: INTEGER; var nbcmp, nbech : CARDINA
var
   m : INTEGER; { position du pivot }
   v : ELEMENT; { valeur du pivot }
begin
   if g < d then begin
      v := t[g];
      m := g;
      partitionner(t,v,g,d,m,nbcmp,nbech);
      QSort (t, g, m-1, nbcmp, nbech);
      QSort (t, m+1, d, nbcmp, nbech);
   end {if};
end {Qsort};
```

Implémentation

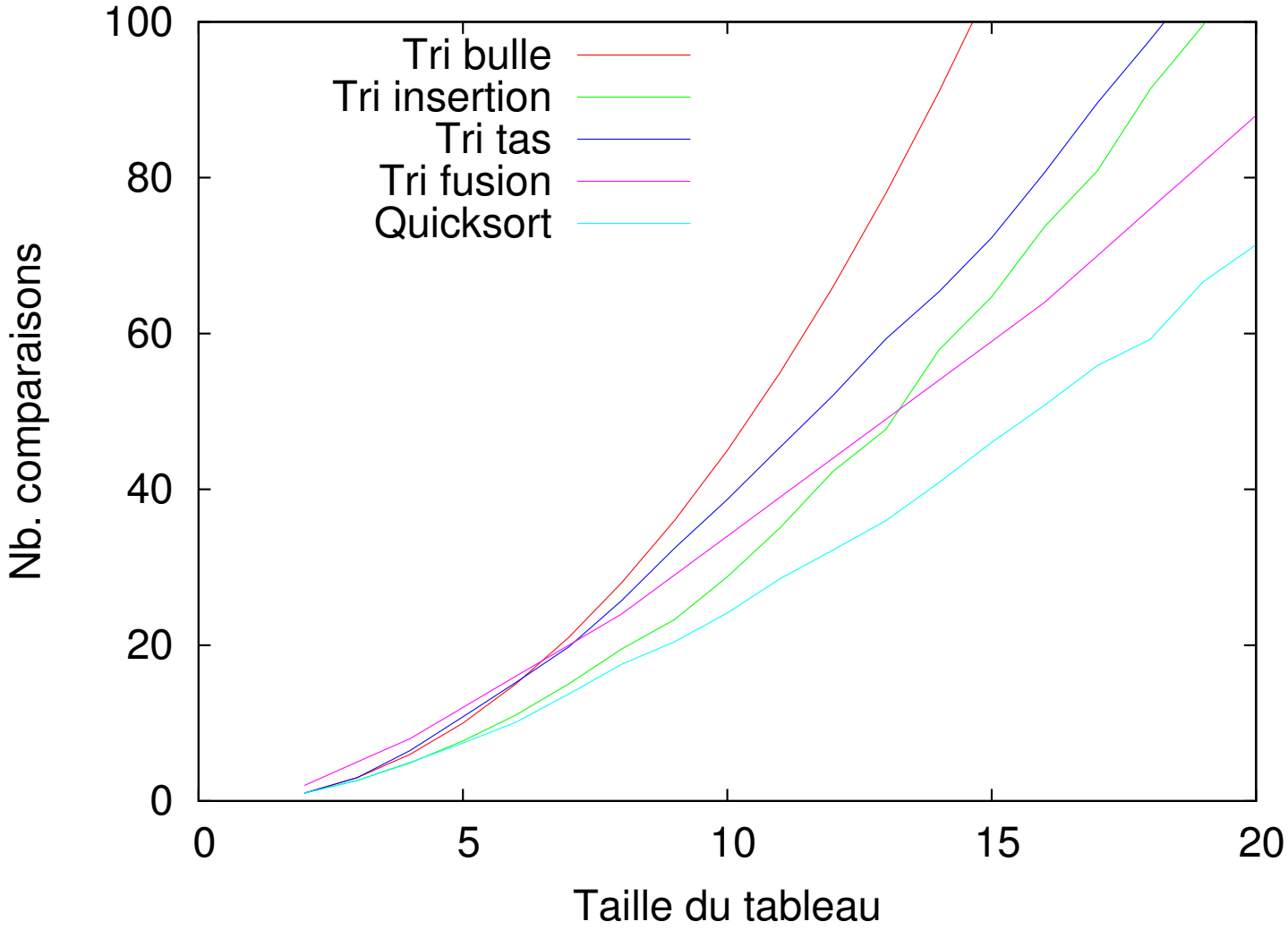
```
procedure partitionner (var t : TABLEAU;
  const v : ELEMENT; const g, d : CARDINAL; var m : CARDINAL;
  var nbcmp, nbech : CARDINAL);
var
   i, CARDINAL;
begin
   for i := g + 1 to d do begin
      inc(nbcmp);
      if t[i] < v then begin
         m := m + 1;
         inc(nbech);
         echanger(t,m,i);
      end {if};
   end {for};
   inc(nbech);
   echanger(t,m,g);
end {partitionner};
```

Temps de calcul - tris rapide





Temps de calcul sur de petites instantes



Complexité

	Temps (comparaisons)				Espace
	Meilleur des cas	Pire des cas	Moyenne	Constante mesurée	
Tri bulle	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	$\frac{1}{2}$	$\Theta(1)$
Tri insertion	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n^2)$	$\frac{1}{4}$	$\Theta(1)$
Tri par tas	$\Theta(n)$	$\mathcal{O}(n \log n)$	$\mathcal{O}(n \log n)$	1.7	$\Theta(1)$
Tri fusion	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	1	$\Theta(n)$
Tri rapide	$\Theta(n \log n)$	$\Theta(n^2)$	$\mathcal{O}(n \log n)$	1.2	$\Theta(1)$

Borne inférieure pour les tris

Théorème

Tout algorithme de tri par comparaisons exige $\Omega(n \log n)$ comparaisons dans le pire des cas.