Homework 6

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Exercise 6.5

Prove that for most Boolean function: $f : (0, 1^n)^k \to 0, 1$, $D(f) = \Omega(n)$.

Use the hint: Count the number of protocols of a given cost.
At first, there are at most $2^{2^{nk}}$ Boolean function in the exercise. Now we count the number of different protocol trees with the deep less than l. There are $k2^{2^{(k-1)n}}$ cylinders and the protocol tree's deep less than l at most 2^l cylinders. So, we could calculate the upper bound $k^{2^l} 2^{2^{(k-1)n} 2^l} = k^{2^l} 2^{2^{(k-1)n+l}}$ which exist one-to-one match between protocols and Boolean functions.
Hence we compare at $2^{2^{nk}}$ and $k^{2^l} 2^{2^{(k-1)n+l}}$, while $l = n - c$ (constant $c \geq 1$ is enough). the ratio of two values is exponential small enough.

Exercise 6.25

Let A be an n-bit string, and $1 \leq j, i \leq n$.
Define the 3-argument function $\mathrm{SUM-INDEX}(A, j, i) = A[j \oplus i]$, where $\oplus$ denotes bitwise xor. Prove that $D^{||}(\mathrm{SUM-INDEX}) = \Omega(\sqrt{n})$.

Use the hint: Reduction from INDEX.
This exercise is a special case of **Example 6.22** where k = 3. Let $A_\oplus$ be a 2 dimensional array of bits, where each dimension has n entries from string A, and satisfied the equation $A_\oplus[i_1, i_2] = A[i_1] \oplus A[i_2]$. For every j $(1 \leq j \leq 2)$, let i_j be an integer $1 \leq i_j \leq n$. Thus $A_\oplus$ is represented by $N = n^2$ bits and each i_j by $\log n$ bits. The function $\mathrm{SUM-INDEX}(\mathrm{i}_1, \mathrm{i}_2, \mathrm{A}_\oplus)$ (replace i, j to i_1, i_2) is defined to be the (i_1, i_2)-th entry of $A_\oplus$, that is $A_\oplus[i_1, i_2]$.
Hence SUM-INDEX problem is reduced to INDEX problem. and $D^{||}(\mathrm{SUM-INDEX}) = \Omega(\sqrt{n})$.