Homework 5

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Exercise 5.11

Prove that $C^D(R) \geq n^2$ is the best lower that can be proven by using **Lemma 5.9**

To use **Lemma 5.9** for the relation $R_{\oplus}$, we take X to be the set of all strings whose parity is 1 and Y to be the set of all strings whose parity is 0. In this case, the relation R defined in Lemma 5.9 is exactly R_{plus}. In addition, note that $|X| = |Y| = 2^{n-1}$, whereas $|C| = n2^{n-1}$(because for every $x \in X$ each of the n strings in Hamming distance 1 from x is in Y). Hence, $C^D(R_{\oplus}) \geq n^2$, which implies $D_{R_{\oplus}} \geq 2\log n$. Now we show that $C_D(R) \geq n^2$ is a lower bound can be proven by using **Lemma 5.9**.

Thus, we will show that it's the best lower bound. By definition

$$C = \{(x, y) : x \in X, y \in Y, d(x, y) = 1\}$$

We notice that there are at most n elements in Y with fixed $x \in X$, which means the size of $C \leq \min(n|X|, n|Y|)$. Without loss of generality, assume that $|Y|$ is always less than $|X|$, set $C \leq n|Y|$.

So, the inequality $C^D(R) \geq \frac{|C|^2}{|X||Y|} \geq \frac{n^2|Y|}{|X|} = n^2\frac{|Y|}{|X|} \geq n^2$.

Exercise 5.21

Let FORK$'$ be the relation consisting of all triples (x, y, i) such that $x, y \in \sum^l$ and i is such that $x_i = y_i$ and either $x_{i+1} \neq y_{i+1}$ or $x_{i-1} \neq y_{i-1}$. Prove that $D(\text{FORK}') = \Omega(\log l \log w)$

We will follow the prove of **Corollary 5.20** to proof it.

At first, let's consider that If there exists a c-bit (α, l) protocol for the relation FORK$'$, then there is also a $c-1$-bit $(\frac{\alpha}{2}, l)$ protocol for FORK$'$. The prove is like **Lemma 5.17** which means this lemma can suit at FORK$'$.

And we prove the lemma like **Lemma 5.18**, let $\alpha \geq \frac{\lambda}{w}$ for a large enough constant λ. If there exists a c-bit (α, l) photocol for FORK$'$, then there is also a c-bit $(\frac{\sqrt{\alpha}}{2}, \frac{l}{2})$ protocol for it. There is a row, corresponding to some string u, whose density is at least $\sqrt{\frac{\alpha}{2}}$. The new protocol works as follows, Alice and Bob use the original c-bit protocol on the lenght-l strings ux and uy(and subtract $\frac{l}{2}$ from the output). Because the same string u is concatenated to both x and y, then the output of the protocol is guaranteed to be in the second half of the string. The protocol is a $(\frac{\sqrt{\alpha}}{2}, \frac{l}{2})$. hence there are $\sqrt{\frac{\alpha}{2}}n$row whose density at least $\frac{\alpha}{2}$, we just control that there are more than $\frac{\sqrt{\alpha}}{2}$inputs and to be outputs directly.

Finally, we can prove $D(\text{FORK}') = \Omega(\log l \log w)$ easily to use lemma above like what we did in **Corollary 5.20**. Clearly, $c(1, l) \geq c(\frac{1}{w^{1/3}, l}) \geq \Omega(\log w) + c(\frac{1}{w^{2/3}}, l) \geq \Omega(\log w) + c(\frac{1}{w^{1/3}}, l/2)$ and inductively $\theta(\log l)$ times, we prove it.