Homework 6

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Problem 1

Prove L = $\{ww^Rw \mid w \in (a+b)^*\}$ not a CFL.

We use pumping lemma to solve it.

$\forall n = 3i, i \geq 0$, we found $uvwxy = a^nb^nb^na^na^nb^n \in L$. To show the difference of position, we numbered them :

$$a^nb^nb^na^na^nb^n = a_1^nb_1^nb_2^na_2^na_3^nb_3^n$$

and because of $|vwx| \leq n, |vx| > 0$, for any choice of v and x, we can't increase a_1, a_2, a_3 or b_1, b_2, b_3 at the same time.

Furthermore, for each $i > 1$, $uv^iwx^iy \notin L$. l is not a CFL.

Problem 2

We have noticed that $i+k = j+l \Rightarrow i-j = l-k$. So, we discussion this problem of two topic : $i \geq j$ and $i \leq j$. In the first case, we have the string $a^ma^jb^jc^kd^kd^m (m = i-j \geq 0)$. In the other case, we have the string $a^ib^ib^nc^nc^ld^l (n = j-i \geq 0)$. So, our answer is the union of these cases. Let we build a CFG $G = (V, a, b, c, d, P, S)$ and the elements of P are listed below.

$$S \longrightarrow S_1 \mid S_2$$

$$S_1 \longrightarrow aS_1d \mid S_{ab}S_{cd}$$

$$S_2 \longrightarrow S_{ab}S_{bc}S_{cd}$$

$$S_{ab} \longrightarrow aS_{ab}b \mid \varepsilon$$

$$S_{bc} \longrightarrow bS_{bc}c \mid \varepsilon$$

$$s_{cd} \longrightarrow cS_{cd}d \mid \varepsilon$$

Problem 3

We use interweaving to solve it.

Now we create a set $L(G) = \{10^n 10^{n+1} \mid n \geq 1\}^*$ and I would show that it is a CFL below.

we build a CFG $G = (V, \{1, 0\}, P, S_t)$ and the elements of P are listed below:

$$S_t \longrightarrow S_0 S \mid \varepsilon$$

$$S_0 \longrightarrow 1 S_1 0$$

$$S_1 \longrightarrow 0 S_1 0 \mid 010$$

And we let $L(G_1) = L(G)1$ and $L(G_2) = 10L(G)1$. So, $L(G_1) \cap L(G_2)$ is the answer we want by using interweaving.

Here are two CFGs to show G_1 and G_2.

we let $G_1 = (V, \{1, 0\}, P \cup Q_1, S)$ and the elements of Q_1 are listed below:

$$S \longrightarrow S_t 1$$

At the same way, we let $G_2 = (V, \{1, 0\}, P \cup Q, S)$ and the elements of Q_1 are listed below:

$$S \longrightarrow 10 S_t 1$$