Homework 1

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2014.12.11

Problem 1

When I realized the difference between countable and computable. I'm upset about it. So, I thought that a high school student would not understand it clearly.

We enumerate Turing computable functions by enumerating Turing Machine. And we know that different Turing Machines can reflect different positive integers. We list these integers by using binary digital below.

	1	2	3	4	5	...
f_1	$f_1(1)$	$f_1(2)$	$f_1(3)$	$f_1(4)$	$f_1(5)$	...
f_2	$f_2(1)$	$f_2(2)$	$f_2(3)$	$f_2(4)$	$f_2(5)$	...
f_3	$f_3(1)$	$f_3(2)$	$f_3(3)$	$f_3(4)$	$f_3(5)$	...
f_4	$f_4(1)$	$f_4(2)$	$f_4(3)$	$f_4(4)$	$f_4(5)$	...
f_5	$f_5(1)$	$f_5(2)$	$f_5(3)$	$f_5(4)$	$f_5(5)$	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

Now, we consider the sequence of diagonal

$$f_1(1), f_2(2), f_3(3), f_4(4), f_5(5)\cdots$$

Obviously, this sequence determines the Turing Machine.

Now, we create the opposite sequence of diagonal f' :

$$1 - f_1(1), 1 - f_2(2), 1 - f_3(3), 1 - f_4(4), 1 - f_5(5)\cdots$$

So, this sequence couldn't be same as any f_i in the list. Because $\forall f_i,\ f_i(i) \neq f'(i)$.

Therefore, The Turing Machine M which reflects f' would not be enumerated in turing computable functions.

So, there exists a Turing Machine M can't be listed. There exists a function can't be computable.

Problem 2

Sure we can.

I'll introduce a way to list all sets of T integers. obviously, this problem is a special case of $T = 5$.

Before of this, it's easy to know there are 2^d conditions in d-dimension. So if we can list one condition, we will list all condition together by using some combinations. For example, if there are $(1,2)$ in 2-dimension in the list of i-th position. we could also extend it to $(1,2),(1,-2),(-1,2),(-1,-2)$.And if it exists $x_i == x_j$ in the vector, we could delete this vector from the list.

So, we just discuss about positive condition with the border of it.

In case of $T = 1$, we list it : $(0, 1, -1, 2, -2, ...)$ and so on.

In case of $T = 2$, we list it by a order function :

$$f(x_1, x_2) = \frac{(x_1 + x_2) * (x_1 + x_2 - 1)}{2} + x_1 + 1$$

this is a common function to list two natural integers.

In case of $T = 3$, we list it by a order function :

$$f(x_1, x_2, x_3) = f(f(x_1, x_2), x_3)$$

(you can image that 2-dimension has been listed by using $f(x_1, x_2)$ So, for every triple (x_1, x_2, x_3), it just have less than $2^3 * f(x_1, x_2, x_3)$ triples in front of it. It's a useful way to list three natural integers.

So, we defined that there are a order function in case T:

$$f(x_1, x_2, ..., x_T) = f(f(x_1, x_2, ..., x_{T-1}, x_T))$$

By using this function, every vector $(x_1, x_2, ..., x_T)$, it just have less than $2^T * f(x_1, x_2, ..., x_T)$ vector in front of it.

By using this function, we can list sets of five natural integers, and we extended it.So, we list these integers successfully.

Problem 3

Sure we can.

In the Problem 2, we list any finite sets of integers. So, in this problem, we will union these lists to just one list.

furthermore, we define a pair: (x, y) which means the set in y-th position in the list of x-dimension($x \geq 1, y \geq 1$).

So, we should get the order of each (x, y). we define that $g(x, y)$ is the position of itself in new list. and we use a common way like we used in Problem 2:

$$g(x, y) = \frac{(x + y - 1)(x + y - 2)}{2} + x$$

So we list them all.

Problem 4

Sincerely, I don't know the meaning about the first '1'. But, It's no matter to solve it.

By using interweaving notation. we can get the set is equal to the set below.

$$(1\{01^i01^{2i} \mid i \geq 1\}^*01^{2n} \cap 101^2\{01^i01^{2i} \mid i \geq 1\}^*) \cup (1\{01^i01^{2i} \mid i \geq 1\}^* \cap 101^2\{01^i01^{2i} \mid i \geq 1\}^*01^{2n})$$