Homework 7

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Problem 1

Let we consider that $[pXq] \xrightarrow{*} w$ which p and q are both a state in many state PDA. So, it means that we can move from p to q which A is topped.

And we define that $w = aw_1w_2 \cdots w_k$. so we image each step of this move below:

$$[pXq] \Rightarrow a[r_0Y_1r_1][r_1Y_1r_2] \cdots [r_{k-1}Y_kr_k] \xrightarrow{*} w$$

In this case, $p = r_0, q = r_k$. And we know that $\delta(q, a, X)$ contains $(r_0, Y_1Y_2 \ldots Y_k)$.so, we allow a step move :$(r_{i-1}, w_i, Y_i) \rightarrow (r_i, \varepsilon, \varepsilon)$.

We use the symbol $\vdash$ represent 'contains'.

And we extend this move below:

$$(r_{i-1}, w_iw_{i+1} \cdots w_j, Y_iY_{i+1} \cdots Y_j) \vdash^* (r_i, w_{i+1}w_{i+2} \cdots w_j, Y_{i+1}Y_{i+2} \cdots Y_j)$$

$$(q, aw_1w_2 \cdots w_k) \vdash^* (p, \varepsilon, \varepsilon)$$

According to above, we can find that each many state move just can be understood by an one state move.

Problem 2

In the beginning, we should let the CFG be written to the Chomsky normal form. So, we can analysis it by a more easy way. Obviously, there are moves of $X \rightarrow x, Y \rightarrow y$.

And then, we consider that each variable would be either two variables($A \rightarrow BC$) or one constant($A \rightarrow a$). So, we find each string $S_iS_jS_k$ which A could be by exactly two moves.Which we means exist such grammars below:

$$A \longrightarrow S_iS_r$$

$$S_r \longrightarrow S_jS_k$$

or

$$A \longrightarrow S_lS_k$$

$$S_l \longrightarrow S_iS_j$$

Furthermore, there are the derivation $A \xrightarrow{*} xAy$ iff $\exists S_i, S_j, S_k \, s.t. \, S_i = X, S_j = A, S_k = Y$ satisfy the definition of moves above.

Problem 3

By observing these grammars, we notice that they satisfy Chomsky normal form among. So, we build the dynamic program function below:

$$f(AB) = f(BA) = TRUE$$

$$if\ f(XAY) = TRUE\ then\ f(XBAY) = TRUE, f(XaY) = TRUE$$

$$if\ f(XBY) = TRUE\ then\ f(XCCY) = TRUE, f(XbY) = TRUE$$

$$if\ f(XCY) = TRUE\ then\ f(XABY) = TRUE, f(XaY) = TRUE$$

which $X, Y = \{A, B, C, a, b, c\}^*$.The partial order of this dynamic program is the length of variable. so, if the string *bbaaba* is in the CFG define by the grammar iff $f(bbaaba) = TRUE$.

Now, we give two ways to make $f(bbaaba)$ be $TRUE$.

- $S \to AB \to BAB \to BBAB \to BBACC \to BBAABC$
- $S \to BC \to ACC \to AABC \to BAABC \to BBAABC$

So, this string can be generated in more than one way.