Homework 5

5130309059 Taring Lee

taringlee@sjtu.edu.cn

2014.12.15

Problem 1

Let we build a CFG $G = (V, T, P, S)$ for each lowercase letter is in T, and set of P has each element below:

$$S \longrightarrow aS_1dd$$

$$S_1 \longrightarrow aS_1dd \mid S_2$$

$$S_2 \longrightarrow bS_3 \mid S_4c$$

$$S_3 \longrightarrow bS_3 \mid bS_3c \mid \varepsilon$$

$$S_4 \longrightarrow S_4c \mid bS_4c \mid \varepsilon$$

Problem 2

Let we build a CFG $G = (P_t, A, P, E)$. for P_t is the set of symbol which we would be used in P; T is the set of symbol which would appear in arithmetic expressions for primary school students, so $A = \{x, y, z, 0, 1, 2, +, -, *, \div, (,)\}$. And we define some basic elements of P below:

$$V \longrightarrow x \mid y \mid z$$

$$C \longrightarrow 0 \mid 1 \mid 2$$

$$L \longrightarrow V \mid C$$

$$T_s \longrightarrow + \mid -$$

$$F_s \longrightarrow * \mid \div$$

And then, we add other complicated elements to P below:

$$E \longrightarrow T \mid F$$

$$T \longrightarrow TT_sT \mid TT_sF \mid FT_sT \mid FT_sF$$

$$F \longrightarrow (T)F_s(T) \mid (T)F_sF \mid FF_s(T) \mid FF_sF \mid L$$

Notably, we can generate arithmetic expressions without redundant parentheses by using this CFG.

Problem 3

Let we write a CFG $G = (V, \{a, b, c\}, P, S)$ to solve this problem.

In the beginning, we define some basic elements of of P below:

$$W \longrightarrow a \mid b$$

$$T_2 \longrightarrow WT_2 \mid \varepsilon$$

$$T_3 \longrightarrow WT_3 \mid cT_3 \mid \varepsilon$$

And then, we consider the situation that each string could be legit without just two symbol of $'c'$ called G_1. So, the elements of P are inserted below:

$$S_1 \longrightarrow WS_1 \mid cF_1 \mid \varepsilon$$

$$F_1 \longrightarrow WF_1 \mid cF_2 \mid \varepsilon$$

$$F_2 \longrightarrow WF_2 \mid cF_3$$

$$F_3 \longrightarrow WF_3 \mid cF_3 \mid \varepsilon$$

Next, we consider the situation of each string with just two symbol of c. We suppose that a string $w_1cw_2cw_3(w_i \in \{a, b\}^*)$. Then we consider that G_2 is allowable if $len(w_1) \neq len(w_2)$ or $len(w_2) \neq len(w_3)$(the function $len(w)$ will return the length of w). So, we add other elements for G_2 to P below:

$$S_2 \longrightarrow WPcT_2 \mid QWcT_2 \mid T_2cWP \mid T_2cQW$$

$$P \longrightarrow WP \mid WPW \mid c$$

$$Q \longrightarrow QW \mid WQW \mid c$$

However, we should also consider that it exists integer i such that the i-th position of w_1 and w_2 is different whether $len(w_1) = len(w_2)$, and it is similar to the situation of $len(w_2) = len(w_3)$.And we call it G_3, so we add other elements for G_3 to P below:

$$S_3 \longrightarrow H_abT_3 \mid H_baT_3 \mid T_2cH_abT_3 \mid T_2cH_baT_3$$

$$H_a \longrightarrow WH_aW \mid aT_2c$$

$$H_b \longrightarrow WH_bW \mid bT_2c$$

Finally, we have comprehend that the union operation of CFG would also receive a CFG. So we let the following element in P:

$$S \longrightarrow S_1 \mid S_2 \mid S_3$$

So, we did it.