Homework 4

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Problem 1

We consider a basic homomorphism h:

$$\begin{cases} h(0) = & 0 \\ h(\hat{0}) = & 001 \\ h(1) = & 1 \end{cases}$$

and we can get $h^{-1}(R)$ which is similar of the answer. $h^{-1}(R)$ is regular.

But, the element of regular set $h^{-1}(R)$ should not own substring 001. Because this substring 001 should be $\hat{0}$. So, we minus these wrong answer. Therefore, we have

$$h^{-1}(R) - (\hat{0} + 0 + 1)^*001(\hat{0} + 0 + 1)^* = R_A$$

It's easy to realize that R_A is regular.

Finally, we define a simple homomorphism g:

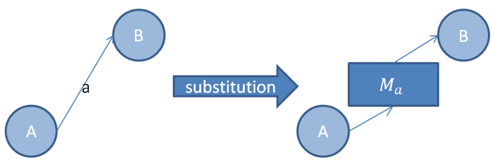
$$\begin{cases} g(0) = & 0 \\ g(\hat{0}) = & 0 \\ g(1) = & 1 \end{cases}$$

Then, the answer : $R_B = g(R_A)$ is what we want. Obviously, R_B is regular.

Problem 2

We define R is the initial regular sets. And R can be written to a ε-NFA M directly which means there are at least one state in M represent for each symbol in R.

And thinking about the substitution $s(a) = R_a$. We define M_a is the ε-NFA of R_a. So, if we change $\delta(x, a)$ in M to pass M_a. Which means if we need via $\delta(x, a)$, we become to pass R_a after substitution. Here are a picture below to show this idea.



And we can also do similar operation of $s(b) = R_b$. So, the new FA named M_s is also a ε-NFA. And the regular sets R_s of M_s is also regular.

Therefore, we have been proved that regular sets are closed under substitution of regular sets.

Problem 3

We use pumping lemma to proof they are not regular.

1. $L = \{a^i b^j c^k \mid either\ i = j\ or\ i = k\ or\ j = k\}$

$\forall n \geq 1$, we found $w = a^n b^n c^{n-1}$. And because of $w = xyz \mid y \neq \varepsilon, |xy| \leq n$.So, y would be a^+. We define that $y = a^t (t > 0)$.

So, there exists $\forall k > 1$ to make $xy^k z = a^{n+(k-1)t} b^n c^{n-1} \notin L$.

2. $L = \{(0+1)^n 1^n \mid n \geq 1\}$

$\forall n \geq 1$, we found $w = 1^n 0^n 1^{2n}$. And because of $w = xyz \mid y \neq \varepsilon, |xy| \leq n$.So, y would be 1^+. We define that $y = 1^t (t > 0)$.

So, there exists $\forall k > 1$ to make $xy^k z = 1^{n+(k-1)t} 0^n 1^{2n} \notin L$.

(we can't maintain "1^n" in $k > 1$)

3. $L = \{(0)^i 1^j \mid i\ and\ j\ are\ relatively\ prime\}$

$\forall n \geq 1$, we found $w = 0^n 1^m$. which m is the minimum prime number of $m > n$. And because of $w = xyz \mid y \neq \varepsilon, |xy| \leq n$.So, y would be 0^+. We define that $y = 0^t (t > 0)$.

So, there exists $k > 1$ to make $xy^k z = 0^{n+(k-1)t} 1^m \notin L$. Here is the reason:

Because m is prime number. So, we found that

$$(k-1)t \equiv m - n (mod m)$$

Obviously, k must be exist.