

Homework 8

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2014.12.22

Problem 1

Hint: we should think $A \rightarrow C$ instead of $A \rightarrow c$.

We found that variable A, B, C would transfer to other variable or ε . So, We extend the first sentence of these and we have:

$$S \longrightarrow aa \mid bb \mid aAa \mid bBb \mid SS \mid \varepsilon$$

Sorting out this grammar, so we have the following grammar with Chomsky Normal Form:

$$S \longrightarrow SS \mid AS_a \mid BS_b \mid \varepsilon$$

$$S_a \longrightarrow SA$$

$$S_b \longrightarrow SB$$

$$A \longrightarrow a$$

$$B \longrightarrow b$$

Easily, we know this grammar characterizes lots of relationships of each pair aSa or bSb . So, we use stack in PDA to achieve them. The following a nondeterministic pushdown automata (by empty store) would accept the same language:

$$\delta(q, \varepsilon, Z_0) = \{(q, \varepsilon)\}$$

$$\delta(q, a, Z_0) = \{(q, AZ_0)\}$$

$$\delta(q, b, Z_0) = \{(q, BZ_0)\}$$

$$\delta(q, a, A) = \{(q, AA), (q, \varepsilon)\}$$

$$\delta(q, b, B) = \{(q, BB), (q, \varepsilon)\}$$

$$\delta(q, a, B) = \{(q, AB)\}$$

$$\delta(q, b, A) = \{(q, BA)\}$$

Problem 2

Consider that there are two states q and p , so, what p should be used in stack is different from q 's. So, we use different symbol to represent it. Here is the answer, a one state pda **with the same order**:

$$\delta(q, a, Z_0) = (q, A\bar{Z}_0)$$

$$\delta(q, b, Z_0) = (q, B\bar{Z}_0)$$

$$\delta(q, a, A) = (q, A\bar{A})$$

$$\delta(q, \varepsilon, A) = (q, \bar{A})$$

$$\delta(q, b, B) = (q, B\bar{B})$$

$$\delta(q, \varepsilon, B) = (q, \bar{B})$$

$$\delta(q, a, B) = (q, A\bar{B})$$

$$\delta(q, b, A) = (q, B\bar{A})$$

$$\delta(q, a, \bar{A}) = (q, \varepsilon)$$

$$\delta(q, a, \bar{B}) = (q, \varepsilon)$$

$$\delta(q, a, \bar{Z}_0) = (q, \varepsilon)$$

Problem 3

Here is a context-free grammar **with the same order** below.

$$S \longrightarrow aA\bar{Z}_0 \mid bB\bar{Z}_0$$

$$A \longrightarrow aA\bar{A} \mid \bar{A} \mid bB\bar{A}$$

$$B \longrightarrow bB\bar{B} \mid \bar{B} \mid aA\bar{B}$$

$$\bar{A} \longrightarrow a$$

$$\bar{B} \longrightarrow b$$

$$\bar{Z}_0 \longrightarrow \varepsilon$$

Obviously, Here is a context-free grammar which be simplified.

$$S \longrightarrow aTa \mid bTb$$

$$T \longrightarrow aTa \mid bTb \mid \varepsilon$$