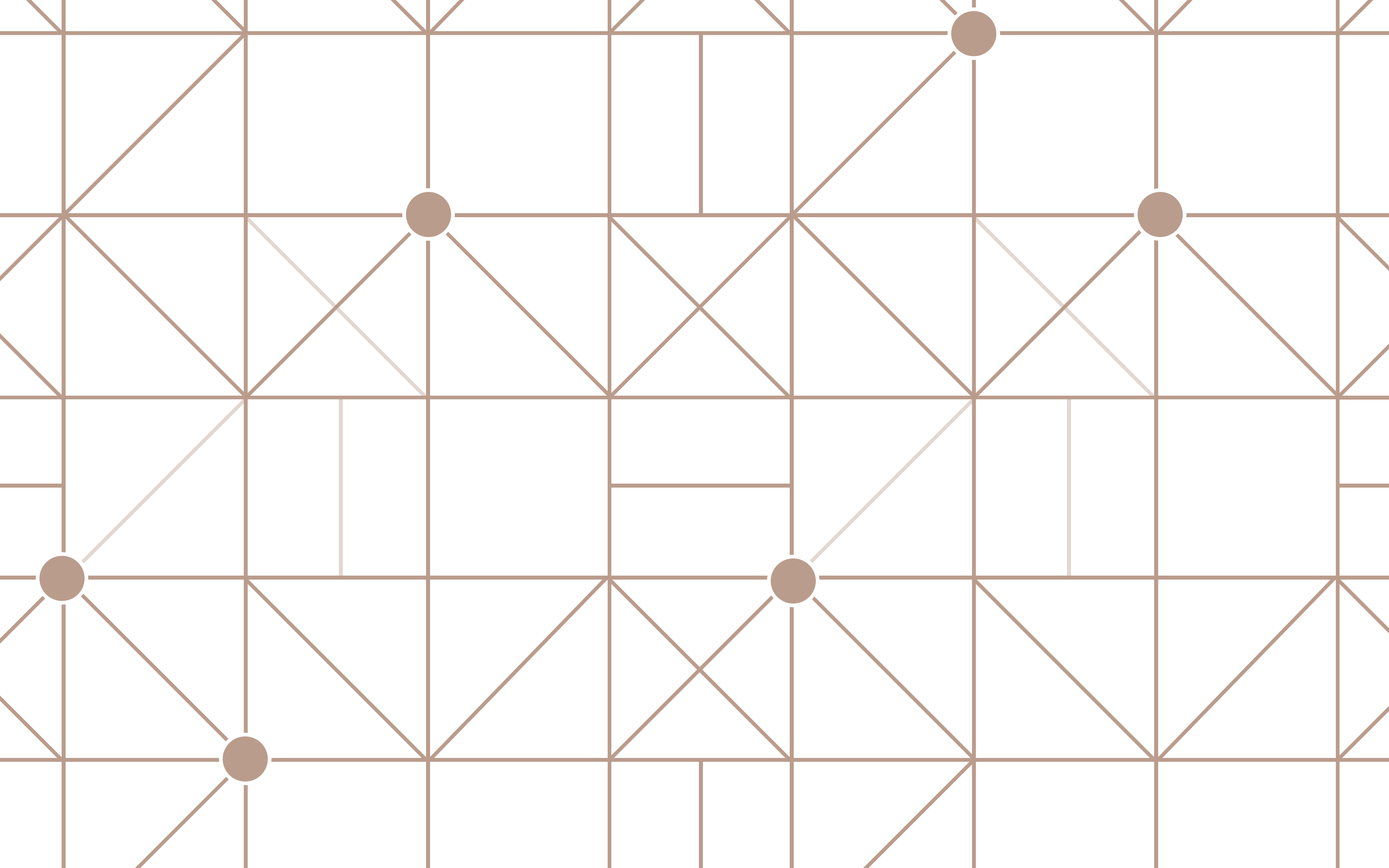


SCQuantum Bootcamp

Recap:
- vectors
- matrices
- eigenvectors & eigenvalues
- braket notation

① $AB \neq BA$ not necessary!

$$[A,B] = AB - BA$$ commutator

XY don't commute!

② Hermitian $H = H^\dagger$

Unitary $A^\dagger A = \mathbb{1}$ identity!

How do you turn a Hermitian to an Unitary?

$$e^{iH} = U$$

Hermitian

$$U^\dagger = (e^{iH})^\dagger = e^{-iH^\dagger} = e^{-iH}$$

$$UU^{\dagger} = e^{iH} e^{-iH} = \mathbb{1}$$

$$U^{\dagger}U = e^{-iH} e^{iH} = \mathbb{1}$$

$H \rightarrow$ measurements

$\hookrightarrow$ have real eigenvalues

$U \rightarrow$ evolution of the system

Quantum Mechanics

$|\psi\rangle$ = wave function $\rightarrow$ describes state of the system

operators = Hermitian, physical properties of the system

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

$\hat{H}$ $\downarrow$ Hamiltonian operator

E $\downarrow$ energy

$|\psi\rangle$ $\hookrightarrow$ state

Challenge

$$H = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad U(t) = e^{-iHt}$$

Basics of Quantum Computing

Classical $\rightarrow$ bit $\rightarrow$ 0 or 1

Quantum $\rightarrow$ qubit $\rightarrow$ 0, 1, or any superposition

linear combination of sols to Schrö. eq are also solutions

$|\psi_1\rangle$ ✓

$|\psi_2\rangle$ ✓

$\alpha|\psi_1\rangle + \beta|\psi_2\rangle$ ✓

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

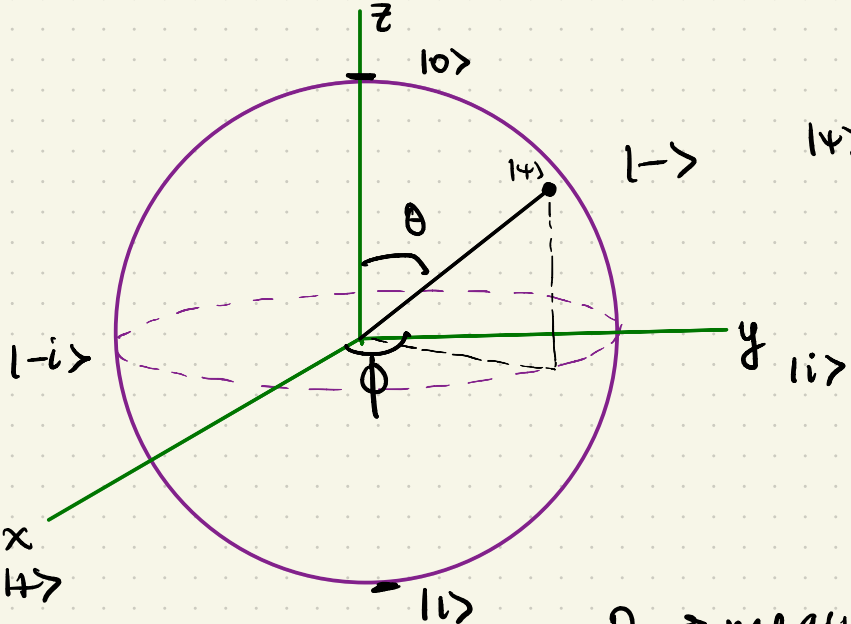
$\uparrow \quad \uparrow$

$|0\rangle, \quad |1\rangle$

$$\underbrace{|\psi\rangle}_{\text{qubit}} = \alpha|0\rangle + \beta|1\rangle$$

prob of measuring 1

$$\underbrace{|\alpha|^2}_{\text{prob of measuring 0}} + |\beta|^2 = 1$$



$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$= \underbrace{\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}\, e^{i\phi}|1\rangle}_{}$

most general qubit state

$\theta \rightarrow$ measure of superpos.

$\phi \rightarrow$ phase

$\theta = 0$

$|\psi\rangle = \cos 0\,|0\rangle + \sin 0\, e^{i\phi}|1\rangle$

$= |0\rangle$

$\theta = \pi$

$|\psi\rangle = \cos\frac{\pi}{2}|0\rangle + \sin\frac{\pi}{2}\, e^{i\phi}|1\rangle$

$(\phi = 0)$

$= |1\rangle$

Rotations of the Bloch sphere $\rightarrow$ don't change norm

$\hookrightarrow$ unitary matrices

Single Qubit Gates

Gates → linear operators acting on a qubit

$\sigma_x \quad \sigma_y \quad \sigma_z$ –

$X \quad Y \quad Z$ –

$X \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$0 \leftrightarrow 1$

bit-flip

$Y \rightarrow \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

bit + phase

$Z \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

phase-shift to $|1\rangle$

$|\psi\rangle = |0\rangle$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$X|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

Most general single qubit gate:

$$\underbrace{U(\theta, \phi,}_{\text{coords on sphere}} \lambda) = \begin{pmatrix} \cos \theta/2 & -e^{i\lambda} \sin \theta/2 \\ e^{i\phi} \sin \theta/2 & e^{i(\lambda+\phi)} \cos \theta/2 \end{pmatrix}$$

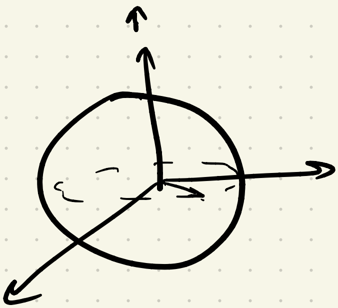
λ → global phase

$e^{i\lambda} [\,|\psi\rangle\,] \rightarrow$ not affect measurement

relative phase is imp!!

Hadamard Gate:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$ superposition of $|0\rangle$ & $|1\rangle$



$H|0\rangle = ?$ $\quad \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = ?$ $\quad \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Measurement!

collapses qubit to one of the poles

θ

$\Phi \rightarrow$ phase (rotates wrt to z axis)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$P_0 = |\langle 0|\psi\rangle|^2 = |\alpha|^2$$

$$P_1 = |\langle 1|\psi\rangle|^2 = |\beta|^2$$

$$P_i = \langle\psi|M_i^{+} M_i|\psi\rangle^2$$

$\hookrightarrow$ measurement operator for i_{th} state

$$\langle 0|\psi\rangle^{*}\langle 0|\psi\rangle$$

$$= \langle\psi|0\rangle\langle 0|\psi\rangle$$

$$\text{State} = \frac{M_i|\psi\rangle}{\sqrt{p_i}}$$

Calculate the probability of measuring $|0\rangle$ & $|1\rangle$ after applying H on $|0\rangle$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\frac{1}{2}, \frac{1}{2}$$

$$M_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$