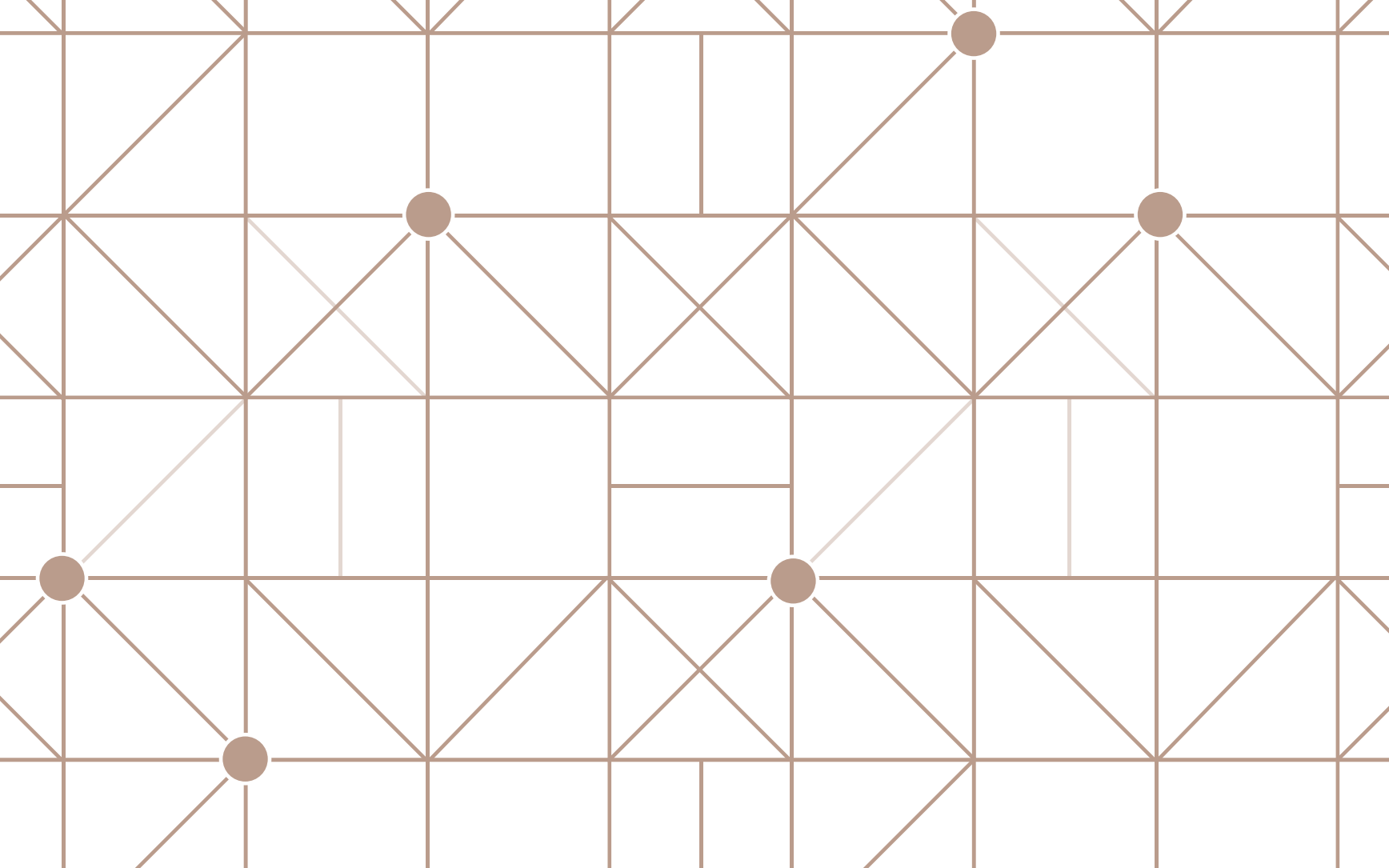
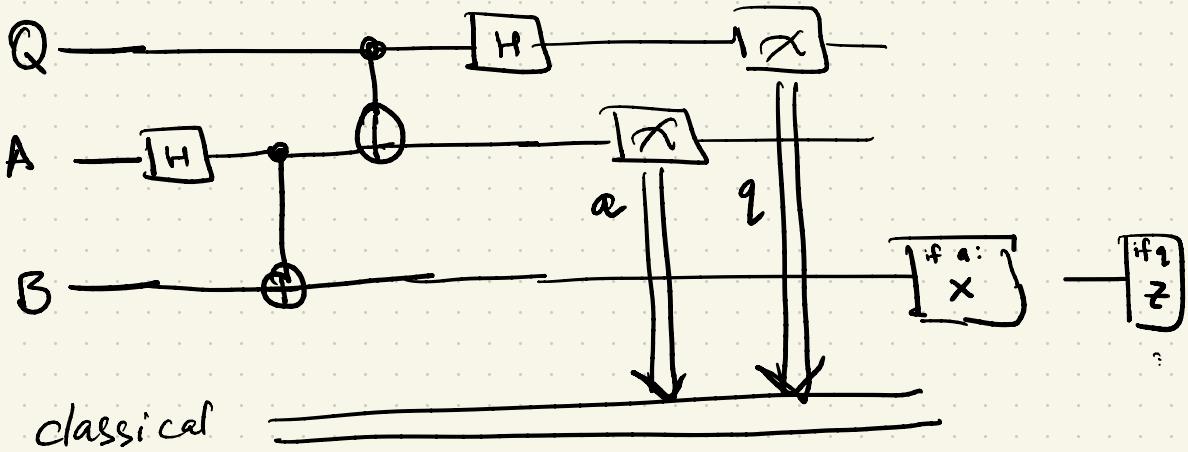


Alice → information → Bob

↳ message qubit Q

- no cloning theorem



$$Q = \alpha|0\rangle + \beta|1\rangle$$

A, B → entangled

↳ after teleportation protocol:

$B \rightarrow \quad \alpha|0\rangle + \beta|1\rangle$

$qq = 00 \rightarrow$ Bob does nothing

$qq = 01 \rightarrow$ Bob applies Z

$10 \rightarrow \quad X$

$11 \rightarrow \quad XZ$

Grover's Algorithm

-unstructured search $\quad \Sigma = \{0,1\}$

$$f : \Sigma^n \rightarrow \Sigma$$

$f(01101) = \begin{cases} 0 \\ 1 \end{cases}$ if string is correct!

- quadratic $N \rightarrow \sqrt{N}$

$x \in \Sigma^n$ $f(x) = 1$

could be many sols → any 1 works

no sols → say that!

f: no specific structure

$N = 2^n$ N strings in Σ^n

worst case scenario → how many ops?

N operations

↳ deterministic

probabilistic → $O(N)$ evals

random

Grovers → $O(\sqrt{N})$

↳ evals f on superposition of inputs

Phase query gate:

$$Z_f |x\rangle = (-1)^{f(x)} |x\rangle$$

$$OR(x) = \begin{cases} 0 & x = 0^n \\ 1 & x \neq 0^n \end{cases}$$

($x = 0000$; $x \neq 0^n$ → $\neq 0000$)

$$Z_{OR} |x\rangle = \begin{cases} |x\rangle & x = 0^n \\ -|x\rangle & x \neq 0^n \end{cases}$$

Grovers Alg

1. Initializing n-qubit register Q to all $|0^n\rangle$

↳ Apply a H to each qubit

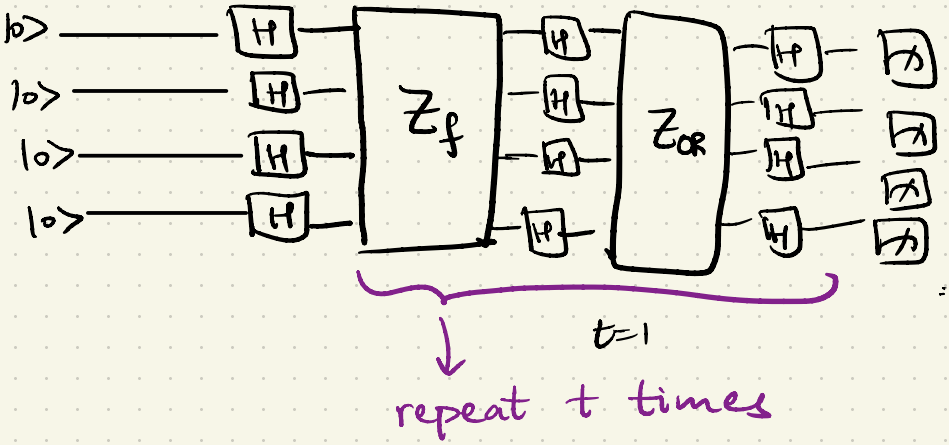
2. $G = H^{\otimes n} Z_{OR} H^{\otimes n} Z_f$ to Q

Grover's operation

Apply this t times

↳ # iterations

3. Measure Q



$t = O(\sqrt{N})$ high prob of finding solution

$A_0 = \{ x \in \Sigma^n \quad f(x) = 0 \}$ non-solutions

$A_1 = \{ x \in \Sigma^n \quad f(x) = 1 \}$ solutions

$$|A_0\rangle = \frac{1}{\sqrt{|A_0|}} \sum_{x \in A_0} |x\rangle$$

$$|A_1\rangle = \frac{1}{\sqrt{|A_1|}} \sum_{x \in A_1} |x\rangle$$

$$|\psi\rangle = \sqrt{\frac{|A_0|}{N}}\,|A_0\rangle + \sqrt{\frac{|A_1|}{N}}\,|A_1\rangle$$

↓

Uniform state → expressed as l.c of A_0, A_1

$$Z_f|A_0\rangle = |A_0\rangle \qquad Z_f|A_1\rangle = -|A_1\rangle$$

$$G|A_0\rangle = \frac{|A_0| - |A_1|}{N}|A_0\rangle + \frac{\sqrt{|A_0||A_1|}}{N}|A_1\rangle$$

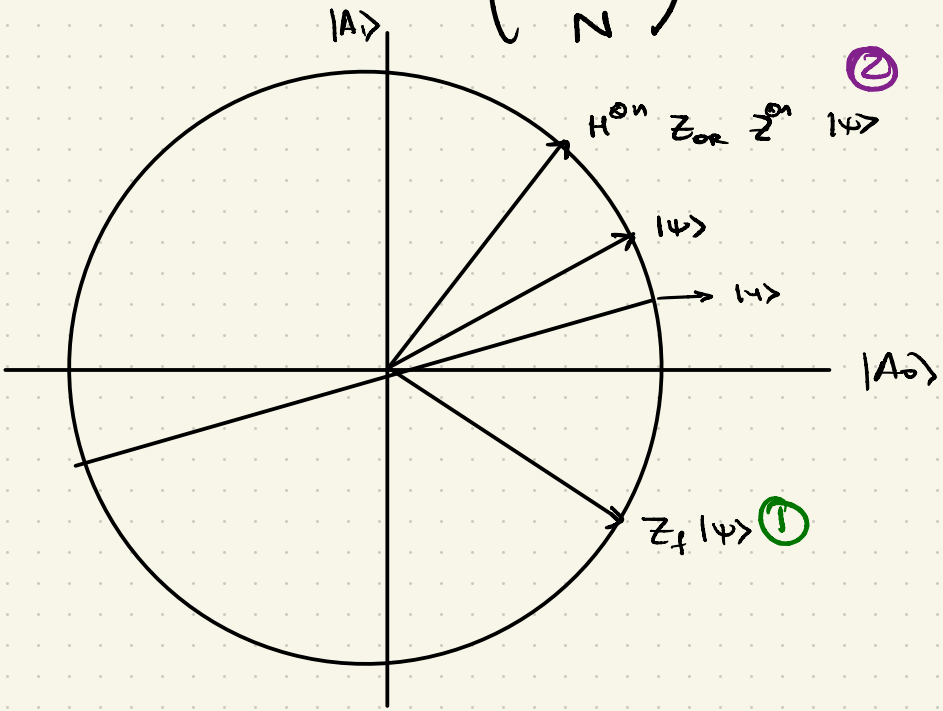
$$G|A_1\rangle = \frac{-2\sqrt{|A_0||A_1|}}{N}|A_0\rangle + \frac{|A_0| - |A_1|}{N}|A_1\rangle$$

$$Z_{OR} = 2|0^n\rangle\langle 0^n| - \mathbb{1}$$

$$G = \begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} A_0 \\ A_1 \end{pmatrix}$$

$$M = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{|A_0|}}{N}\right)$$



$$G = \underbrace{H^{\otimes n}\, Z_{OR}\, H^{\otimes n}}_{2}\ \underbrace{Z_f}_{1}$$

combine — reflection about A_0
reflection about $|4\rangle$

rotation by 2θ

Grover operation → combine refl. to
1 2θ rotation

t = # iterations , N

$$t = \left\lfloor \frac{\pi}{4} \sqrt{N} \right\rfloor$$ unique

$$t = \left\lfloor \frac{\pi}{4\theta} \right\rfloor$$ multiple

Amplitude amplification