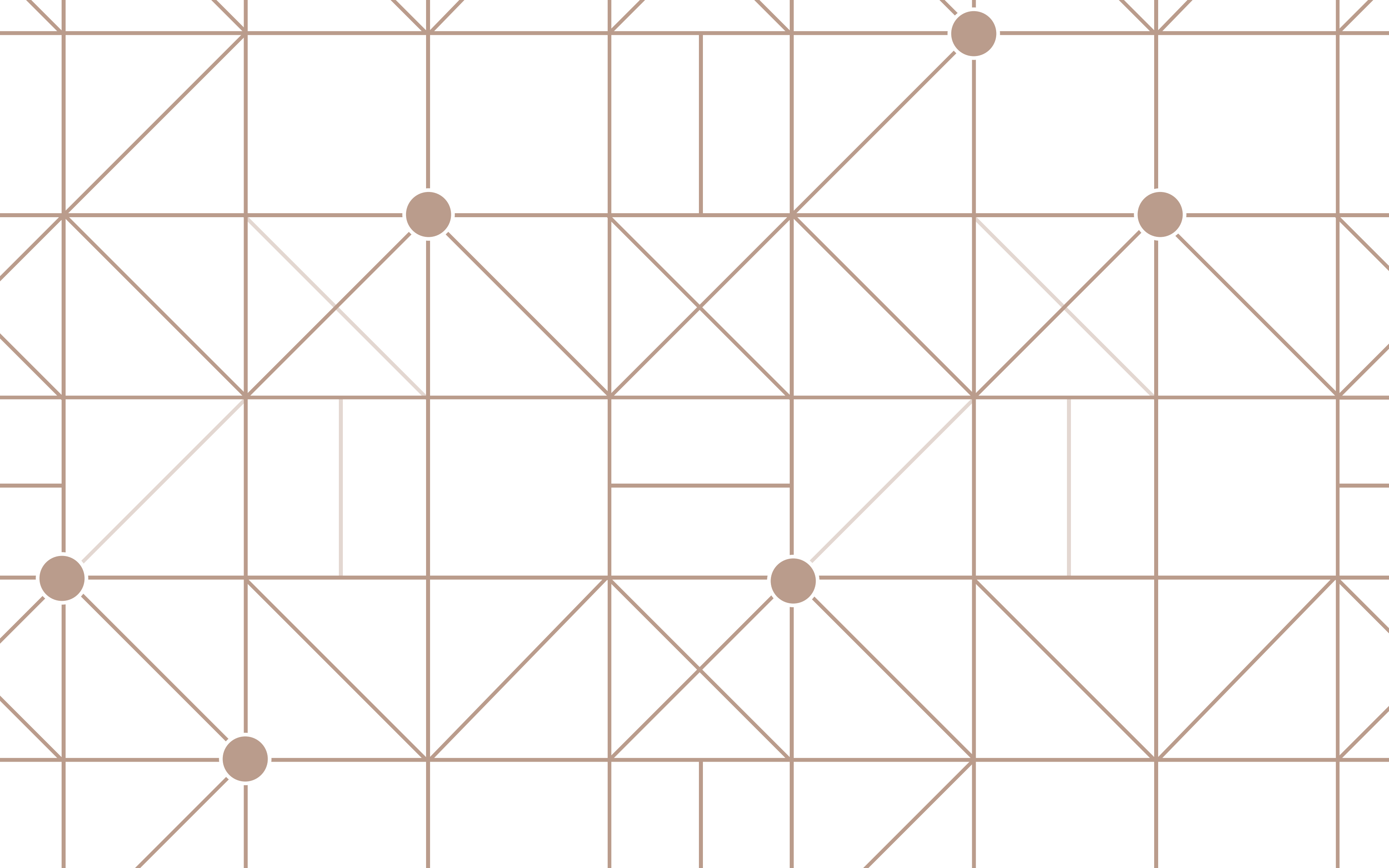


Quantum Error Correction

- WHY
  - decoherence
  - gate fidelity
  - state prep!
  - readout!
  - measurement!

$\rightarrow$ SPAM

100%

$X|0\rangle \rightarrow |1\rangle$

96% $\rightarrow$ 4 shots out of 100

$\downarrow$

$|0\rangle$

99% $\rightarrow$ 1

99.9% $\rightarrow$

...9

00

Classical

Alice $\rightarrow$ 0 to $\longrightarrow$ Bob

$0 \mapsto 000 \longrightarrow$ Bob

$\downarrow$

010

majority rule

Repetition codes!

Quantum

$0 \not\mapsto 000$

$\boxed{\text{no cloning theorem}}$

no operator U

$$U_{clone}(|\psi\rangle \otimes |0\rangle) \rightarrow |\psi\rangle \otimes |\psi\rangle$$

Types of noise

- Bit flip $\rightarrow$ X

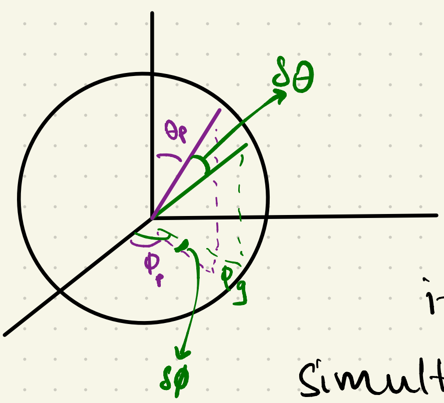
$$X|\psi\rangle = X(\alpha|0\rangle + \beta|1\rangle) = \alpha|1\rangle + \beta|0\rangle$$

- Phase flip $\rightarrow$ Z

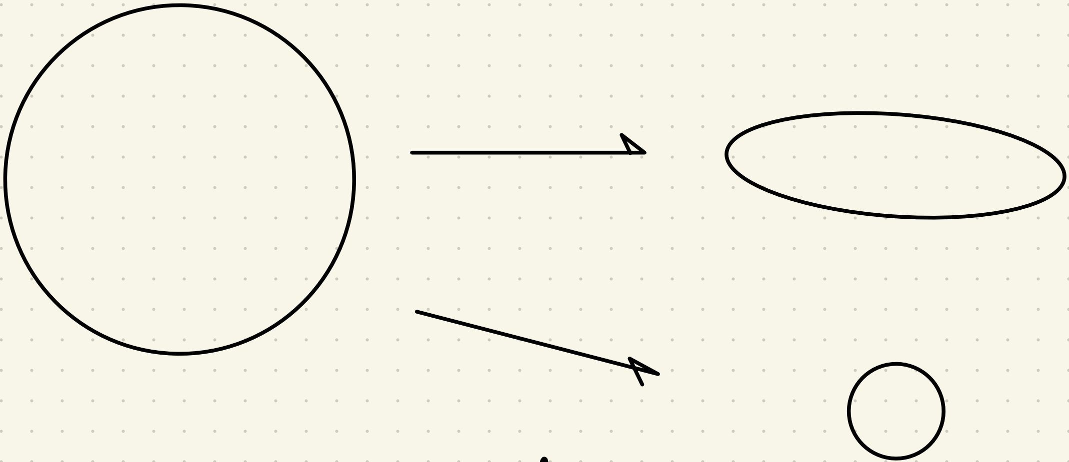
$$Z|\psi\rangle = Z(\alpha|0\rangle + \beta|1\rangle) = \alpha|0\rangle - \beta|1\rangle$$

$$Y = iXZ$$

$$U(\delta\theta, \delta\phi)|\psi\rangle = \alpha_I \mathbb{1}|\psi\rangle + \alpha_X X|\psi\rangle + \alpha_Z Z|\psi\rangle + \alpha_{XZ} \underline{XZ}|\psi\rangle$$



if you can fix X & Z simultaneously $\rightarrow$ you can do Y.



Depolarizing channel:

↳ set of possible errors

$$D_p(\rho) = (1-p)\rho + \frac{p}{3} X\rho X^\dagger + \frac{p}{3} Y\rho Y^\dagger + \frac{p}{3} Z\rho Z^\dagger$$

Bit flip error → 3 Qubit code

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\downarrow$$

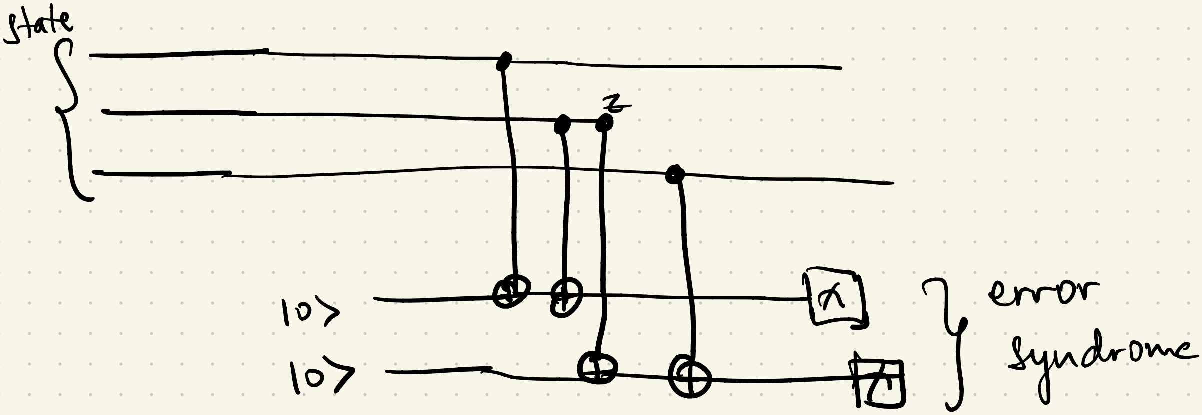
$$\alpha|000\rangle + \beta|111\rangle$$

$$X_2(\alpha|000\rangle + \beta|111\rangle) = \alpha|010\rangle + \beta|101\rangle$$

- don't measure data!

measure the error!

syndrome = measurement result from the ancilla qubits



00 → do nothing

10 → X to 1st qubit $X_1|\psi\rangle$

01 → X to 2nd qubit $X_2|\psi\rangle$

11 → X to 3rd qubit $X_3|\psi\rangle$

$X_1|\psi\rangle = \alpha|100\rangle + \beta|011\rangle$

$$\begin{array}{cc} \downarrow & \downarrow \\ 1 & 0 \end{array}$$

Quantum Error Correction: An Intro Guide

by Joschka Roffe

3-Qubit code → able to detect bit flip error

How do we fix phase flip error?

$\alpha|000\rangle + \beta|111\rangle$

$|0\rangle|1\rangle \longrightarrow |+\rangle/|-\rangle$

X or Z

X & Z error?

9 qubit code → Shor Code

- $|0\rangle \mapsto \frac{1}{2\sqrt{2}} (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$

1 qubit $\longrightarrow$ 9 qubits

- $|1\rangle \mapsto \frac{1}{2\sqrt{2}} (000 - 111)(000 - 111)(000 - 111)$

Stabilizers

$$\underbrace{Z_1 Z_2}_{\text{stabilizers}} |\psi\rangle_L = (+1) |\psi\rangle_L$$

$$Z_1 Z_2 |\psi_{err}\rangle = (-1) |\psi_{err}\rangle$$

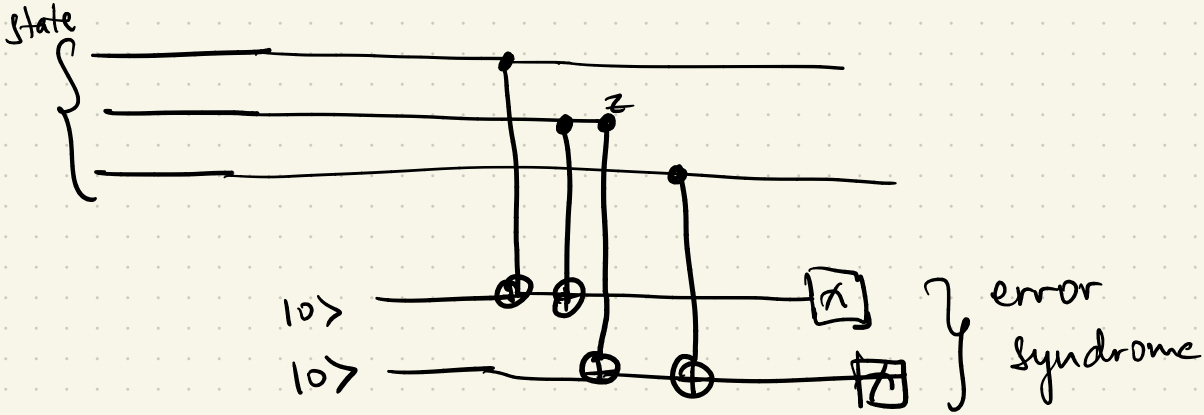
- all stabs commute w/ each other
  - ↳ allows for simultaneous measurement

- $|\psi\rangle_L \rightarrow$ logical states

$$S |\psi\rangle_L = (+1) |\psi_L\rangle$$

Logical operators

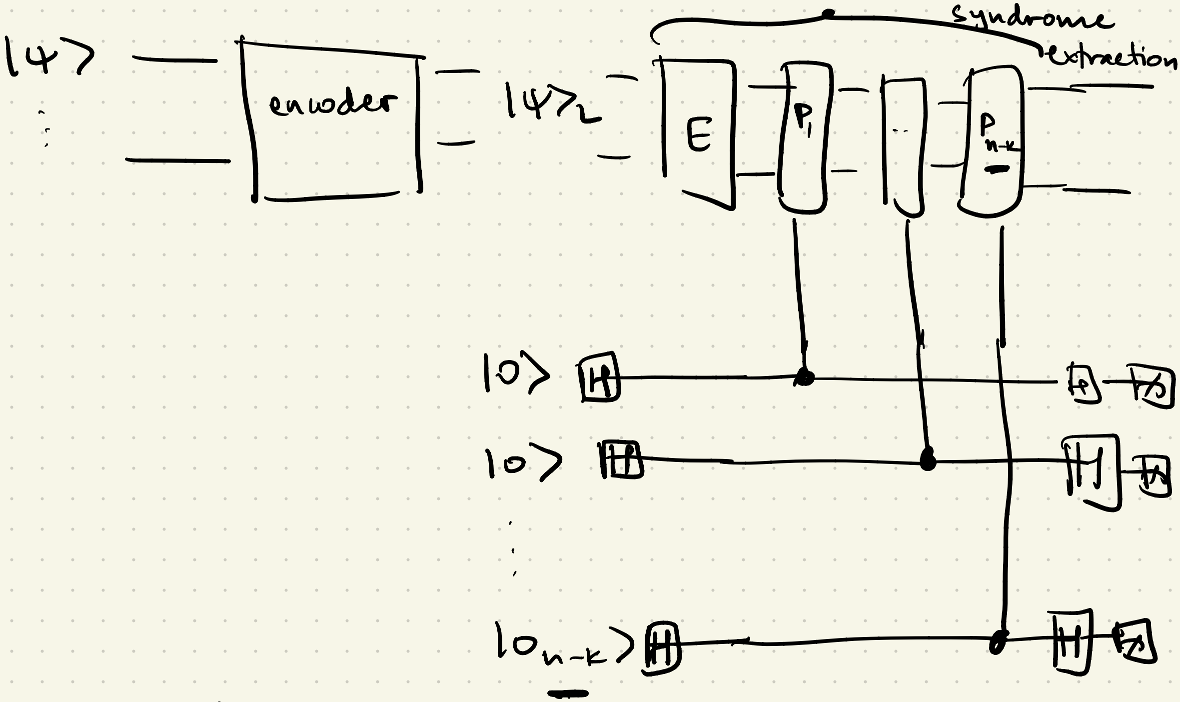
- X_i Z_i
  - commute w/ all stabilizers
  - anti commute w/ each other



$$M_1 = Z_1 Z_2$$

$$M_2 = Z_2 Z_3$$

$$X_2 (\alpha|000\rangle \quad + \beta|111\rangle)$$



encoder

error

$$\mathcal{D}\left(E\, U|\psi\rangle\right)\langle\psi|U^{\dagger}E^{\dagger}\big) = c\left(E, |\psi\rangle\right)|\psi\rangle\langle\psi|$$

decoder

QECC $=(U, \mathcal{E})$ corrects E

if $E \in \mathcal{E}$

$[[n, k, d]]$

n physical qubits

encoded into

k logical qubits

distance d

minimum size error that will go undetected d code correct floor $\lfloor \frac{d-1}{2} \rfloor$ errors

$[[9, 1, 3]]$

generator $\rightarrow$ minimum set that will get us all our stabilizers

$Z_1 Z_2$, $Z_2 Z_3$, $\underline{\underline{Z_3 Z_1}}$

$|+\rangle, |-\rangle$ $X_1 X_2$; $X_2 X_3$; $X_1 X_3$

$$X_1 X_3 = X_1 X_2 \cdot X_2 \cdot X_3$$

Shor code

$n-k$: $9-1 = \underline{8}$!

Flip error (X) in 1st block : $Z_1 Z_2$ $Z_2 Z_3$

X 2nd : $Z_4 Z_5$, $Z_5 Z_6$

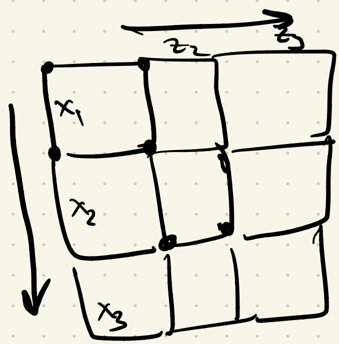
X 3rd : $Z_7 Z_8$, $Z_8 Z_9$

Z across 1& 2nd : $X_1 X_2 X_3 X_4 X_5 X_6$

Z 2nd & 3rd : $X_4 X_5 X_6 X_7 X_8 X_9$

Surface Code

↳ topological codes



Z stabilizers

X stab