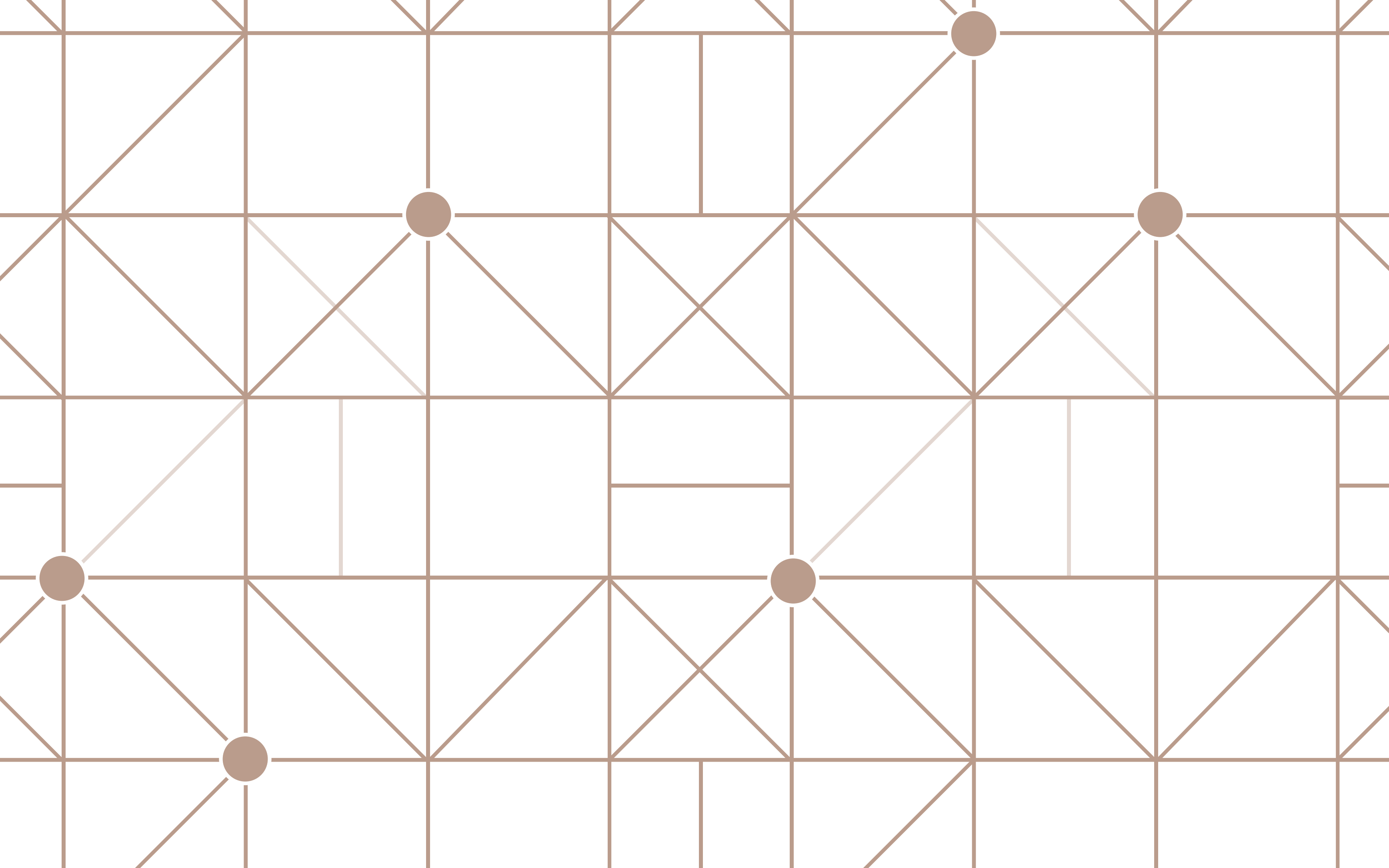


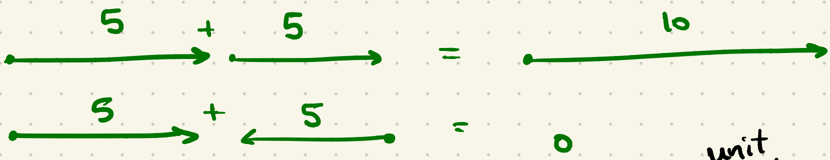
SC Quantum Bootcamp

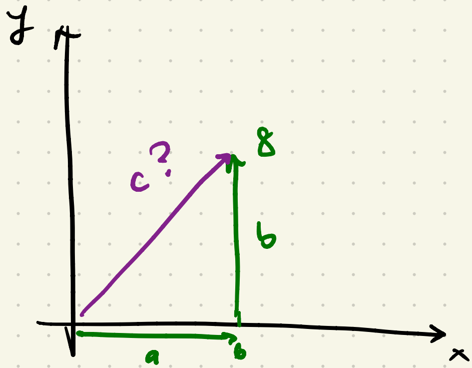
Math: - Linear Algebra
- Complex Numbers
- Probability & Measurements

Vectors
Matrices
Dirac notation
Basis sets
Eigenvals + vecs

Vectors

- magnitude
- direction





unit vectors

$$\vec{V} = V_x\hat{x} + V_y\hat{y}$$

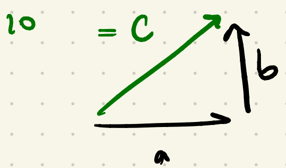
$$= \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

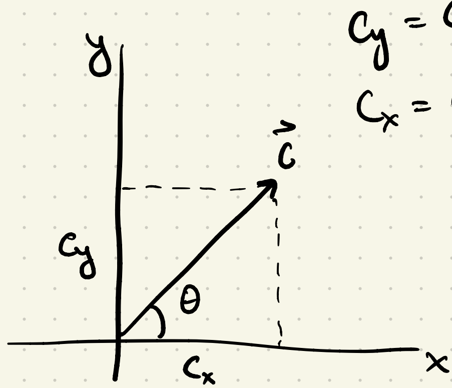
$$|\vec{V}| = \sqrt{V_x^2 + V_y^2}$$

$$\vec{a} = \underline{6}\hat{x} + 0\hat{y} \qquad \vec{b} = 0\hat{x} + \underline{8}\hat{y}$$

$$\vec{c} = 6\hat{x} + 8\hat{y}$$

$$|\vec{c}| = \sqrt{6^2 + 8^2} = 10$$





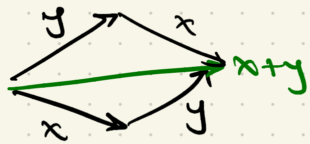
$$c_y = c \sin\theta$$

$$c_x = c \cos\theta$$

Properties of vector addition:

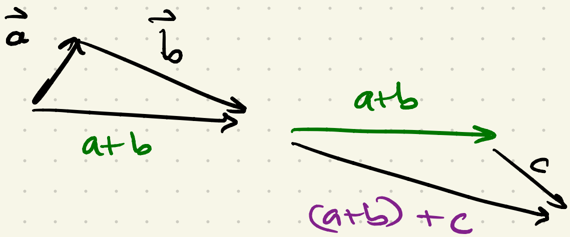
- commutative

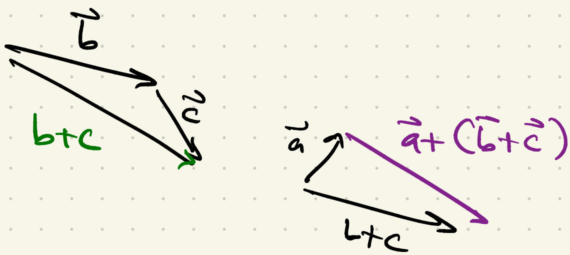
$$\vec{x} + \vec{y} = \vec{y} + \vec{x}$$

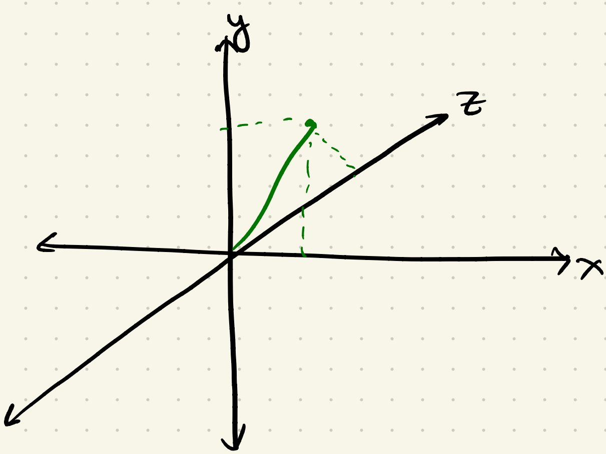


- associative

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$







$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

Column vector $\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$ Row vector $(v_1 \, v_2 \cdots v_n)$

Dirac Bra-ket notation

$$|v\rangle \equiv \text{ket} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$$

$$|v\rangle\langle v|$$

$$\langle v|v\rangle$$

$$\langle v| \equiv \text{bra} = (v_1 \cdots v_n)$$

$$\langle v| = \underbrace{(|v\rangle^*)^T}_{\text{conjugate transpose}} = |v\rangle^\dagger$$

Hermitian

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}^T = (1 \;\; 2)$$

$i = \sqrt{-1}$ $\quad z = 1 + 3i \quad z^* = 1 - 3i$

$$|a\rangle = \begin{pmatrix} 2+3i \\ 4 \end{pmatrix} \qquad (|a\rangle)^T = \begin{pmatrix} 2+3i & 4 \end{pmatrix}$$

$$|a\rangle^\dagger = \begin{pmatrix} 2-3i & 4 \end{pmatrix}$$

Multiplication

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad b = \text{scalar}$$

$$b\vec{v} = \begin{pmatrix} bv_1 \\ bv_2 \end{pmatrix} \qquad \text{num} \cdot \text{vec} = \text{vec}$$

Inner product (dot product)

$$\vec{v}_1 = \begin{pmatrix} v_{1x} \\ v_{1y} \end{pmatrix} \qquad \vec{v}_2 = \begin{pmatrix} v_{2x} \\ v_{2y} \end{pmatrix}$$

$$\vec{v}_1 \cdot \vec{v}_2 = (v_{1x} \cdot v_{2x}) + (v_{1y} v_{2y})$$

$$= \text{number}$$

Cross product

$$\vec{v_1} \times \vec{v_2} = \text{vector}$$

$$\vec{v_1} = \begin{pmatrix} v_1x \\ v_1y \\ v_1z \end{pmatrix} \qquad \vec{v_2} = \begin{pmatrix} v_2x \\ v_2y \\ v_2z \end{pmatrix}$$

$$\vec{v_1} \times \vec{v_2} = \begin{array}{ccc} x & y & z \\ \end{array}$$
$$\begin{vmatrix} v_1x & v_1y & v_1z \\ v_2x & v_2y & v_2z \end{vmatrix}$$

$$= (v_1y\, v_2z - v_1z\, v_2y)\,\hat{x}$$

$$- (v_1x\, v_2z - v_1z\, v_2x)\,\hat{y}$$

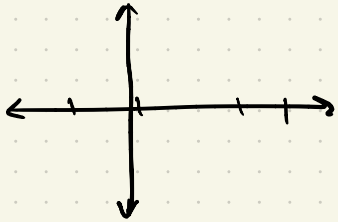
vector!!

$$+ (v_1x\, v_2y - v_1y\, v_2x)\,\hat{z}$$

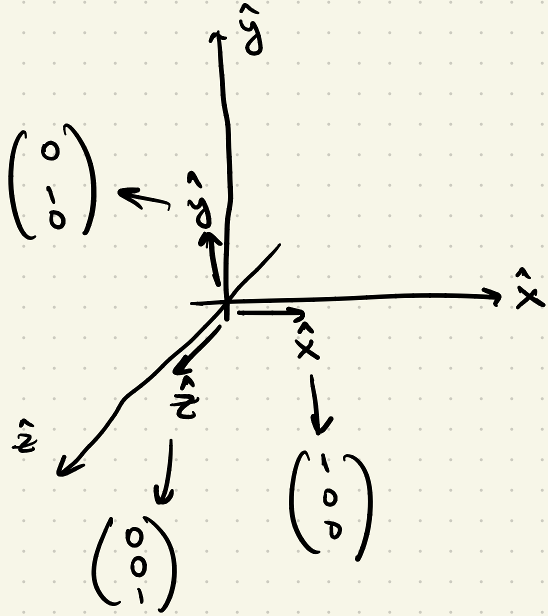
$$\vec{v_1} = \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix} \qquad \vec{v_2} = \begin{pmatrix} 3i \\ 2i+1 \\ 2-4i \end{pmatrix}$$

$$\vec{v_1} \times \vec{v_2}$$

Basis Sets



$\hat{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\hat{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$0 = \hat{x} \cdot \hat{y}, \quad \hat{y} \cdot \hat{z}, \quad \hat{z} \cdot \hat{x}$??

$1 = \hat{x} \cdot \hat{x}; \quad \hat{y} \cdot \hat{y}; \quad \hat{z} \cdot \hat{z}$??

orthonormal basis

↳ unit magnitude

orthogonal - perpendicular

$\vec{x} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ $\vec{y} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$ $\vec{z} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$

Normalization

$$|\vec{x}| = \sqrt{2^2+0^2+0^2} = 2$$

$$\frac{\vec{x}}{|\vec{x}|} = \frac{1}{2}\begin{pmatrix}2\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

Matrices

$$M = \begin{pmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{pmatrix} \quad \begin{matrix} 2 \times 2 \\ \text{rows} \quad \text{cols} \end{matrix}$$

$\uparrow$ indices to describe

scalar multiplication

$a =$ scalar

$$aM = \begin{pmatrix} am_{11} & am_{21} \\ am_{12} & am_{22} \end{pmatrix}$$

vector multiplication

$$\begin{pmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} m_{11}v_1 + m_{12}v_2 \\ m_{12}v_1 + m_{22}v_2 \end{pmatrix}$$

$$2 \times \boxed{2 \qquad 2} \times 1 \quad = \quad 2 \times 1$$

cols of 1st = rows of 2nd

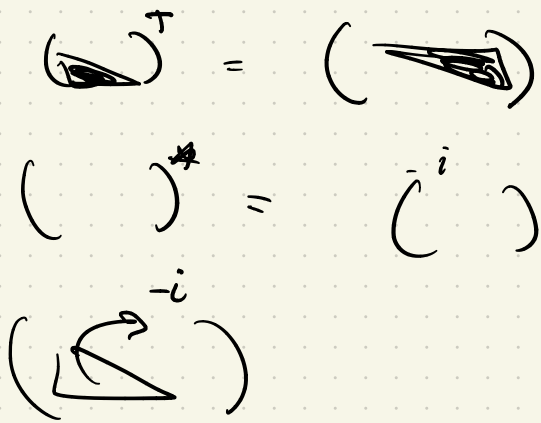
matrix - matrix

$$\underset{A}{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}} \underset{B}{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{22}b_{12} + a_{22}b_{22} \end{pmatrix}$$

matrix transpose

conjugate

$*T$



Hermitian matrix !!

$$A^{+} = A$$

Unitary matrix !!

$$A^{+}A = \mathbb{1}$$

inner product $\quad \vec{v}_1 \cdot \vec{v}_2 = \text{scalar}$

$$\langle v_1 | v_2 \rangle$$

cross product $\quad \vec{v}_1 \times \vec{v}_2 = \text{vector}$

outer product = vector

$$\underbrace{|v_2\rangle}_{3\times 1}\underbrace{\langle v_1|}_{1\times 3} \Rightarrow \text{3x3 matrix}$$

$$|\vec{v}_1\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \qquad \langle v_2| = \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 \end{pmatrix}$$

$$|v_1\rangle\langle v_2| = \begin{pmatrix} \psi_1\phi_1 & \psi_1\phi_2 & \psi_1\phi_3 \\ & \boxed{\psi_2\phi_2} & \\ & & \end{pmatrix}$$

Eigenvectors + Eigenvalues

$$\underbrace{A|\psi\rangle = \lambda|\psi\rangle}$$

λ ↑ eigenvalue

$|\psi\rangle$ ↪ eigenve

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$(A - \lambda I)|\psi\rangle = 0$$

↪ assuming non-zero vector

$$\Downarrow$$

$$\det(A - \lambda I) = 0$$

↪ sols only exist if $\det(A) \neq 0$ $\Updownarrow$ A can be inverted

$$\det\left[\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right] = 0$$

$$\det\begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix}$$

$$(2-\lambda)^2 - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$(\lambda-1)(\lambda-3) = 0$$

$$\boxed{\lambda = 1, 3}$$

$$A|\psi\rangle = 1|\psi\rangle$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \qquad |\psi\rangle = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\psi_1 = -\psi_2$$

$$A|\psi\rangle = 3|\psi\rangle$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} 3\psi_1 \\ 3\psi_2 \end{pmatrix} \qquad |\psi\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\psi_1 = \psi_2$$

$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ it has eigenvals: 1, 3

eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$