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Matkul : Aljabar Vektor dan Matriks.

UTS Aljabar Vektor dan Matriks

1) a. $2C^T + 3A$

Jawab:

$$2C^T = \begin{pmatrix} 4 & 12 \\ 12 & 4 \\ 8 & 6 \\ 19 & 18 \\ 30 & 13 \\ 2 & 9 \end{pmatrix} + 3A = \begin{pmatrix} 15 & 6 \\ 18 & 9 \\ -6 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 12 \\ 12 & 4 \\ 8 & 6 \\ 19 & 18 \\ 30 & 13 \\ 2 & 9 \end{pmatrix} //$$

b) $C + \frac{1}{2}B$

Jawab:

Tidak bisa dijumlah karena matriks tersebut memiliki ordo yang berbeda.

c) $A \cdot B$

$$A = \begin{pmatrix} 5 & 2 \\ 6 & 3 \\ -2 & 1 \end{pmatrix} \cdot B = \begin{pmatrix} 8 & -4 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 44 & -28 \\ 24 & 15 \\ 14 & -3 \end{pmatrix} //$$

2) a. $\begin{cases} X_1 + X_2 + 2X_3 = 8 \\ -X_1 + 2X_2 + 3X_3 = 1 \\ 3X_1 - 7X_2 + 4X_3 = 10 \end{cases}$

Jawab

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -2 & 3 \\ 3 & -7 & 4 \end{pmatrix} \quad \det(A) = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -2 & 3 \\ 3 & -7 & 4 \end{vmatrix} = 24$$

$$(X_1) = \begin{pmatrix} 8 & 1 & 2 \\ 1 & -2 & 3 \\ 10 & -7 & 4 \end{pmatrix} \quad \det(X_1) = \begin{vmatrix} 8 & 1 & 2 & 8 & 1 \\ 1 & -2 & 3 & 1 & -2 \\ 10 & -7 & 4 & 10 & -7 \end{vmatrix} = 152$$

$$X_2 = \begin{pmatrix} 1 & 8 & 2 \\ -1 & 1 & 3 \\ 3 & 10 & 4 \end{pmatrix} \quad \det(X_2) = \begin{vmatrix} 1 & 8 & 2 & 1 & 8 \\ -1 & 1 & 3 & -1 & 1 \\ 3 & 10 & 4 & 3 & 10 \end{vmatrix} = 52$$

$$X_3 = \begin{pmatrix} 1 & 1 & 8 \\ -1 & -2 & 1 \\ 3 & -7 & 10 \end{pmatrix} \quad \det(X_3) = \begin{vmatrix} 1 & 1 & 8 & 1 & 1 \\ -1 & -2 & 1 & -1 & -2 \\ 3 & -7 & 10 & 3 & -7 \end{vmatrix} = 114$$

$$X_1 = \frac{\det(X_1)}{\det(A)} = \frac{152}{24} = 6,3\bar{3}$$

$$X_2 = \frac{\det(X_2)}{\det(A)} = \frac{52}{24} = 2,1\bar{6}$$

$$X_3 = \frac{\det(X_3)}{\det(A)} = \frac{114}{24} = 4,75$$

$$b) \begin{cases} X_1 + X_2 + 2X_3 = 9 \\ 2X_1 + 4X_2 - 3X_3 = 1 \\ 3X_1 + 6X_2 - 5X_3 = 0 \end{cases}$$

Jawab

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{pmatrix} \quad \det(A) = \begin{vmatrix} 1 & 1 & 2 & 1 & 1 \\ 2 & 4 & -3 & 2 & 4 \\ 3 & 6 & -5 & 3 & 6 \end{vmatrix} = -1$$

$$X_1 = \begin{pmatrix} 9 & 1 & 2 \\ 1 & 4 & -3 \\ 0 & 6 & -5 \end{pmatrix} \quad \det(X_1) = \begin{vmatrix} 9 & 1 & 2 & 9 & 1 \\ 1 & 4 & -3 & 1 & 4 \\ 0 & 6 & -5 & 0 & 6 \end{vmatrix} = 1$$

$$X_2 = \begin{pmatrix} 1 & 9 & 2 \\ 2 & 1 & -3 \\ 3 & 0 & -5 \end{pmatrix} \quad \det(X_2) = \begin{vmatrix} 1 & 9 & 2 & 1 & 9 \\ 2 & 1 & -3 & 2 & 1 \\ 3 & 0 & -5 & 3 & 0 \end{vmatrix} = -2$$

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$$X_3 \begin{pmatrix} 1 & 1 & 9 \\ 2 & 4 & 1 \\ 3 & 6 & 0 \end{pmatrix} \det(X_3) \begin{vmatrix} 1 & 1 & 9 & 1 & 1 \\ 2 & 4 & 1 & 2 & 4 \\ 3 & 6 & 0 & 3 & 6 \end{vmatrix} = -3$$

$$X_1 = \frac{\det(X_1)}{\det(A)} = \frac{1}{-1} = -1$$

$$X_2 = \frac{\det(X_2)}{\det(A)} = \frac{-2}{-1} = 2$$

$$X_3 = \frac{\det(X_3)}{\det(A)} = \frac{-3}{-1} = 3$$

3) a) Tentukan determinan matriks D.

Jawab.

$$D \begin{pmatrix} 2 & 4 & 2 \\ 5 & 3 & 4 \\ 3 & 1 & 2 \end{pmatrix} \det(D) \begin{vmatrix} 2 & 4 & 2 & 2 & 4 \\ 5 & 3 & 4 & 5 & 3 \\ 3 & 1 & 2 & 3 & 1 \end{vmatrix}$$

$$\det(D) = 12 + 48 + 10 - 18 - 8 - 40 = 4 //$$

b) Tentukan invers

Jawab

$$\text{kof}(D) \begin{pmatrix} \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 5 & 4 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 5 & 3 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 4 & 2 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 4 & 2 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 5 & 3 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 2 & 2 & -4 \\ -6 & -2 & 10 \\ 10 & 2 & -14 \end{pmatrix}$$

$$\text{kof}(D)^T = \begin{pmatrix} 2 & -6 & 10 \\ 2 & -2 & 2 \\ -4 & 10 & -14 \end{pmatrix}$$

Jadi:

$$D^{-1} = \frac{1}{\det(D)} \cdot \text{adj}(D)$$

$$= \frac{1}{4} \cdot \begin{pmatrix} 2 & -6 & 10 \\ 2 & -2 & 2 \\ -4 & 10 & -14 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{4} & \frac{-6}{4} & \frac{10}{4} \\ \frac{2}{4} & \frac{-2}{4} & \frac{2}{4} \\ -1 & \frac{10}{4} & \frac{-14}{4} \end{pmatrix} //$$