MATHS IN FINANCE

FORMULAS

I. SIMPLE INTEREST RATE

Here, computations are easy: interest are proportional to time.

I.1 Short-term borrowing « Exact/360 »

The duration of the loan is less than 1 year. There is no payment between the start of the transaction (date of the loan) and the end of the transaction (date of repayment).

S : Borrowed amount
J_1 : Borrowing day
J_2 : Repayment day
r : interest rate (warning $x\% = \frac{x}{100}$)
The number of days taken into account is the following:

$$d = J_2 - J_1$$

Interests:

The value of due interests is I, where

$$I = \frac{d}{360}.r.S$$

Case of postcounted interest (also referred to as arrears payment):

Interest is paid at the end of the period, on the day J_2.

The amount paid back on day J_2 Is

$$S_{remb} = S.(1+\frac{d}{360}r)$$

Case of discounted interest:

Interest is paid at the beginning of the period, at day J_1.

The amount received by the borrower at day J_1 is

$$S_{init} = S.(1-\frac{d}{360}r).$$

The amount paid back by the borrower on the day J_2 is

$$S_{remb} = S$$

I.2 Bill discounting « Exact/360 »

The principle of calculation is that of discounted interest.

I.3 Bonds « Exact/365 », « coupon » computation

N : Nominal amount
n : Full length in years
r : interest rate (warning $x\% = \frac{x}{100}$)
J : Anniversary date of issuance of bond

Coupon paid at each anniversary date is:

$$C = r.N$$

At any other time in the year, let d be the number of days since the last coupon payment. The "Accrued coupon" Cc, corresponds to the interest already acquired to date by the holder of the obligation and is worth (be careful with leap years):

$$Cc = r.\frac{d}{365}.N \text{ or } r.\frac{d}{366}.N$$

The last payment, after n years is worth:

$$N + C = (1 + r).N$$

II. COMPOUND INTEREST RATE

II.1 Amount paid back in one time

S : Borrowed amount
n : Full length of the loan, in years or fractions of years. The fraction of the year is to be taken with a year of 365 or 366 days.
i : interest rate (warning $x\% = \frac{x}{100}$)

The amount paid back at the end is :

$$S_{remb} = S.(1+i)^n$$

The value of due interests is I, where :

$$I = S.((1+i)^n - 1)$$

II.2 Borrowing with constant repayments

S: Borrowed amount
n: Full length of the loan, in number of periods corresponding to the refunds made (often it is months).
m: amount of payments made at the end of each period (example: monthly payments)
i: interest rate for the period above mentioned (warning $x\% = \frac{x}{100}$)

We begin to compute:

$$v = \frac{1}{1+i}$$

And:

$$a_n = \frac{1}{i}(1 - v^n)$$

Then:

$$S = a_n.m.$$

II.3 Yield to maturity, internal rate of return

Let i be this rate and n a date (considered from today in years or fractions of years).

Point of vue #1: The payment of the amount F at date n has an « actual value » of

$$V = \frac{F}{(1+i)^n}$$

Point of vue #2: If we decide to borrow today the amount V with interest rate i, we have to pay back F at n, with

$$F = V.(1+i)^n$$

More generally, for a given cash flow of F_1, …, F_n at respective dates D_1, …, D_n given in years or fraction of years, (the amounts can be positive or negative), the actual value of this cash flow with internal rate of return i is :

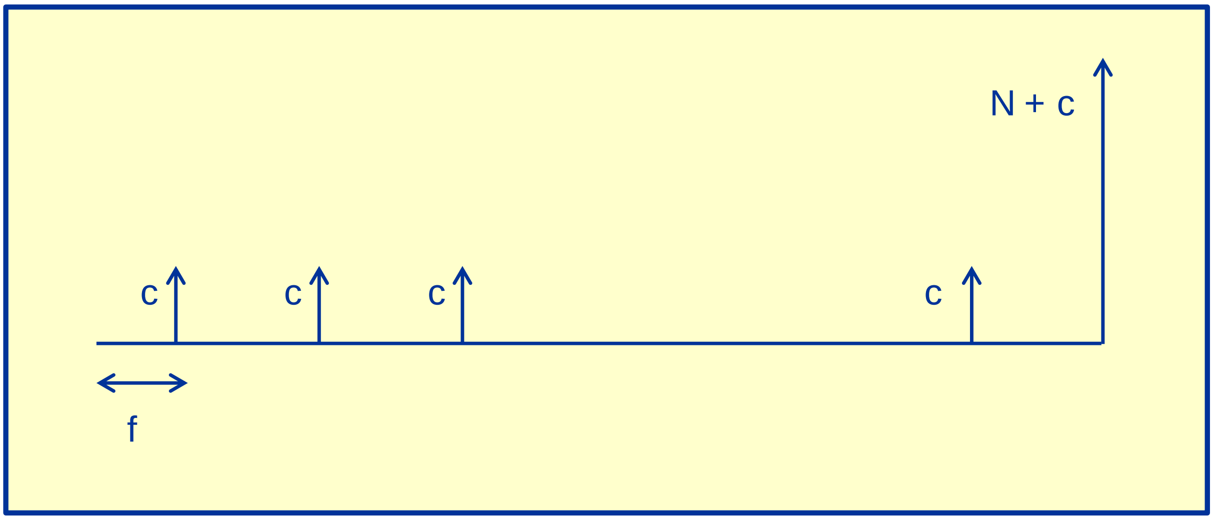
$$V = \frac{F_1}{(1+i)^{D_1}} + \ldots + \frac{F_n}{(1+i)^{D_n}}$$

II.4 Case of the bonds, actual value of the cash flow with respect to the yield to maturity

The prices of the bonds are given in economic news, this is the "clean price". The yield to maturity is the internal rate of return of the bond. One has:

Actual value of future cash flows = Clean price + Accrued coupon

To explain the value of bonds, one must examine future cash flows:



In this figure, c represents the coupons, N the nominal value and f the fraction of year (d/365 or d/366) between time 0 and date of payment of first coupon.

The yield to maturity r depends on the rates prevailing on the market at the time of the valuation. The actual value of future cash flows is then the following (be careful with the number n, there are n dates coupon payments + the maturity, that is n+1 dates in all):

$$V = \sum_{k=0}^{n-1} \frac{c}{\left(1+r\right)^{f+k}} + \frac{N}{\left(1+r\right)^{f+n}}$$

The duration is the mean value of the dates of the cash flows – length of time to receipt, weighted by means of the « actual value » of these cash flows, as shown by the following formula :

$$D = \frac{1}{V} \left\{ \sum_{k=0}^{n-1} \left(f+k\right) \frac{c}{\left(1+r\right)^{f+k}} + \left(f+n\right) \frac{N}{\left(1+r\right)^{f+n}} \right\}.$$

Duration measures the sensitivity of the value V to a change of the yield to maturity r:

$$\frac{1}{V} \frac{\partial V}{\partial r} = -\frac{D}{1+r}.$$

When f=0 (and therefore no more coupon payment at time 0 – that is n dates in all in the formulas instead n+1), with the following notations:

$$v = \frac{1}{1+r}$$
$$a_n = \frac{1}{r}\left(1 - v^n\right)$$
$$b_n = \frac{1}{rv}\left(a_n - nv^{n+1}\right)$$

We have – in this precise case where f = 0 and just after the coupon payment at t = 0:

$$V = a_n c + v^n N$$
$$D = \frac{1}{V}\left(b_n c + nv^n N\right).$$