Answer Sheet 2

Topic: Camera Models, Optimization, Calibration

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Part 2: Optimization

The difference between these curve fitting examples is as follows:

- Curve fitting: it can fit data with certain noise, but is still sensitive to outliers. It uses ceres::CostFunction.
- Robust curve fitting: it is robust to outliers. Apart from ceres::CostFunction, it also uses CauchyLoss(0.5) in order to associate the loss function with a residual block. The argument 0.5 specifies the scale of Cauchy loss.

Part 3: Camera calibration

In this file it uses CLI::app to parse args in the command line. Via the app.add_option() function, it introduces show_gui, dataset_path and cam_model as command line parameters. The result of two camera is also closed and in a reasonable range.
Please see the calibration.cpp on the merge request.
The outputs of four camera models are attached below. From the result we can see that the transform matrices and camera intrinsic parameters have been well estimated. The convergence speed is very fast (8-20s), which shows the high efficiency of Ceres.
The result of calibration (Figure 1) shows that the red circle and magneta circle almost overlap.
Pinhole camera:

```
Solver Summary (v 2.0.0-eigen-(3.3.8)-lapack-suitesparse-(5.7.1)-cxsparse-(3.2.0)-
eigensparse -no_openmp)

 Original                  Reduced
Parameter blocks                        56                      55
Parameters                              394                     387
```

Fig. 1 Calibration Result.

Effective parameters		340	334
Residual blocks		12442	12442
Residuals		24884	24884
Minimizer		TRUST_REGION	
Sparse linear algebra library		SUITE_SPARSE	
Trust region strategy		LEVENBERG_MARQUARDT	
Given		Used	
Linear solver		SPARSE_NORMAL_CHOLESKY	SPARSE_NORMAL_CHOLESKY
Threads		12	12
Linear solver ordering		AUTOMATIC	55
Cost:			
Initial		1.795667e+07	
Final		1.565735e+05	
Change		1.780009e+07	
Minimizer iterations		16	
Successful steps		16	
Unsuccessful steps		0	
Time (in seconds):			
Preprocessor		0.019879	
Residual only evaluation		0.534973 (16)	
Jacobian & residual evaluation		18.543207 (16)	

```
Linear solver                    0.062087 (16)
Minimizer                        19.191715

Postprocessor                    0.001010
Total                            19.212604

Termination:                     CONVERGENCE (Function tolerance reached.
|cost_change|/cost: 2.158064e-13 <= 1.000000e-12)

{
        "value0": {
                "cam.T_i_c": [
                {
                        "px": 0.0,
                        "py": 0.0,
                        "pz": 0.0,
                        "qx": 0.0,
                        "qy": 0.0,
                        "qz": 0.0,
                        "qw": 1.0
                },
                {
                        "px": 0.10960220177714075,
                        "py": -0.00010490052806695571,
                        "pz": 0.0037400002055087749,
                        "qx": 0.005389357928739904,
                        "qy": -0.03818008498755914,
                        "qz": 0.003278934586413792,
                        "qw": 0.9992509617305538
                }
                ],
                "cam.intrinsics": [
                {
                        "cam_type": "pinhole",
                        "fx": 546.2147378586979,
                        "fy": 551.5811503832607,
                        "cx": 399.0884109494394,
```

```
                        "cy": 208.37160821354002,
                        "p1": 0.0,
                        "p2": 0.0,
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                },
                {
                        "cam_type": "pinhole",
                        "fx": 547.2084958014361,
                        "fy": 550.3778903534294,
                        "cx": 368.86284114057539,
                        "cy": 215.16198122823935,
                        "p1": 0.0,
                        "p2": 0.0,
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                }
                ]
        }
}
```


Extended Unified Camera Model:

```
Solver Summary (v 2.0.0-eigen-(3.3.8)-lapack-suitesparse-(5.7.1)-cxsparse-(3.2.0)-
eigensparse -no_openmp)

 Original                  Reduced
Parameter blocks                        56                      55
Parameters                              394                     387
 Effective parameters                   340                     334
Residual blocks                         12442                   12442
Residuals                               24884                   24884

Minimizer                        TRUST_REGION
```

```
Sparse linear algebra library      SUITE_SPARSE
Trust region strategy      LEVENBERG_MARQUARDT

Given                      Used
Linear solver              SPARSE_NORMAL_CHOLESKY SPARSE_NORMAL_CHOLESKY
Threads                                12                     12
Linear solver ordering                 AUTOMATIC              55

Cost:
 Initial                          5.353182e+06
Final                             1.627604e+02
Change                            5.353019e+06

Minimizer iterations                        7
Successful steps                            7
Unsuccessful steps                          0

Time (in seconds):
Preprocessor                          0.008855

Residual only evaluation              0.232922 (7)
Jacobian & residual evaluation        8.479236 (7)
Linear solver                         0.028666 (7)
Minimizer                             8.763984

Postprocessor                         0.000933
Total                                 8.773773

Termination:                      CONVERGENCE (Parameter tolerance reached.
Relative step_norm: 2.527948e-09 <= 1.000000e-08.)

{
        "value0": {
                "cam.T_i_c": [
                {
                        "px": 0.0,
```

```
                "py": 0.0,
                "pz": 0.0,
                "qx": 0.0,
                "qy": 0.0,
                "qz": 0.0,
                "qw": 1.0
            },
            {
                "px": 0.11002631815223632,
                "py": −0.00028908964378220297,
                "pz": 0.000246181144400705,
                "qx": 0.007123595350702433,
                "qy": 0.0006300007991471633,
                "qz": 0.0010774305748141569,
                "qw": 0.9999738479738503
            }
            ],
            "cam.intrinsics": [
            {
                "cam_type": "eucm",
                "fx": 460.9794986620865,
                "fy": 459.62650958263319,
                "cx": 365.8867492896418,
                "cy": 249.346486044204,
                "p1": 0.591664809640774,
                "p2": 1.1241125399441403,
                "p3": 0.0,
                "p4": 0.0,
                "width": 752,
                "height": 480
            },
            {
                "cam_type": "eucm",
                "fx": 459.79606434876208,
                "fy": 458.40630958578478,
                "cx": 379.35442505013966,
                "cy": 256.0400994980228,
```

```
                        "p1": 0.6060839010938546,
                        "p2": 1.088350919730381,
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                }
                ]
        }

}
```


Kannala-Brandt Camera Model:

```
Solver Summary (v 2.0.0-eigen-(3.3.8)-lapack-suitesparse-(5.7.1)-cxsparse-(3.2.0)-
eigensparse -no_openmp)

                                     Original                  Reduced
Parameter blocks                           56                       55
Parameters                                394                      387
Effective parameters                      340                      334
Residual blocks                         12442                    12442
Residuals                               24884                    24884

Minimizer                        TRUST_REGION

Sparse linear algebra library    SUITE_SPARSE
Trust region strategy            LEVENBERG_MARQUARDT

                                     Given                     Used
Linear solver           SPARSE_NORMAL_CHOLESKY    SPARSE_NORMAL_CHOLESKY
Threads                                    12                       12
Linear solver ordering                AUTOMATIC                    55

Cost:
Initial                          5.788049e+06
Final                            1.619844e+02
Change                           5.787887e+06
```

```
Minimizer iterations                8
Successful steps                    8
Unsuccessful steps                  0

Time (in seconds):
Preprocessor                        0.009358

Residual only evaluation            0.272357 (8)
Jacobian & residual evaluation      9.929500 (8)
Linear solver                       0.032390 (8)
Minimizer                           10.260839

Postprocessor                       0.000936
Total                               10.271133

Termination:                        CONVERGENCE (Parameter tolerance reached.
Relative step_norm: 1.775292e-10 <= 1.000000e-08.)

{
        "value0": {
                "cam.T_i_c": [
                {
                        "px": 0.0,
                        "py": 0.0,
                        "pz": 0.0,
                        "qx": 0.0,
                        "qy": 0.0,
                        "qz": 0.0,
                        "qw": 1.0
                },
                {
                        "px": 0.11002263076486109,
                        "py": -0.0002883117629451379,
                        "pz": 0.00025193738175583363,
                        "qx": 0.007129648722668718,
                        "qy": 0.0006308484199464747,
```

```
                        "qz": 0.0010771978182017289,
                        "qw": 0.9999738045490108
                }
                ],
                "cam.intrinsics": [
                {
                        "cam_type": "kb4",
                        "fx": 461.1581901632604,
                        "fy": 459.8080285947971,
                        "cx": 365.89723869011746,
                        "cy": 249.35492318319005,
                        "p1": −0.004593052532205222,
                        "p2": 0.02753243203702901,
                        "p3": −0.036977671770998209,
                        "p4": 0.018610217245568693,
                        "width": 752,
                        "height": 480
                },
                {
                        "cam_type": "kb4",
                        "fx": 459.74480686519805,
                        "fy": 458.3535053463388,
                        "cx": 379.36396149546416,
                        "cy": 256.0435254270624,
                        "p1": 0.004549588612680235,
                        "p2": 0.002485490491602717,
                        "p3": −0.010601348784718934,
                        "p4": 0.010781041140887182,
                        "width": 752,
                        "height": 480
                }
                ]
        }

}
```


Double Sphere Model:

```
Solver Summary (v 2.0.0-eigen-(3.3.8)-lapack-suitesparse-(5.7.1)-cxsparse-(3.2.0)-
eigensparse -no_openmp)

                                         Original                  Reduced
Parameter blocks                            56                        55
Parameters                                  394                       387
Effective parameters                        340                       334
Residual blocks                             12442                     12442
Residuals                                   24884                     24884

Minimizer                        TRUST_REGION

Sparse linear algebra library    SUITE_SPARSE
Trust region strategy            LEVENBERG_MARQUARDT

Given                            Used
Linear solver                    SPARSE_NORMAL_CHOLESKY SPARSE_NORMAL_CHOLESKY
Threads                                     12                        12
Linear solver ordering                      AUTOMATIC                 55

Cost:
Initial                          5.353182e+06
Final                            1.627482e+02
Change                           5.353019e+06

Minimizer iterations                        15
Successful steps                            13
Unsuccessful steps                          2

Time (in seconds):
Preprocessor                     0.008344

Residual only evaluation         0.488787 (15)
Jacobian & residual evaluation   15.653587 (13)
Linear solver                    0.055071 (15)
Minimizer                        16.240517
```

```
Postprocessor                          0.000917
Total                                  16.249778

Termination:                           CONVERGENCE (Parameter tolerance reached.
Relative step_norm: 6.648845e−09 <= 1.000000e−08.)

{
        "value0": {
                "cam.T_i_c": [
                {
                        "px": 0.0,
                        "py": 0.0,
                        "pz": 0.0,
                        "qx": 0.0,
                        "qy": 0.0,
                        "qz": 0.0,
                        "qw": 1.0
                },
                {
                        "px": 0.11002674958788142,
                        "py": −0.00028913779866585087,
                        "pz": 0.000246625049919901,
                        "qx": 0.0071236589880659538,
                        "qy": 0.0006289220700000381,
                        "qz": 0.001077495211590831,
                        "qw": 0.9999738481299002
                }
                ],
                "cam.intrinsics": [
                {
                        "cam_type": "ds",
                        "fx": 351.03728321686966,
                        "fy": 350.0074555977382,
                        "cx": 365.88809735482138,
                        "cy": 249.34573836993608,
                        "p1": −0.2385312817269929,
                        "p2": 0.567869484529095,
```

```
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                },
                {
                        "cam_type": "ds",
                        "fx": 362.95328870306289,
                        "fy": 361.85685537441079,
                        "cx": 379.35501913798887,
                        "cy": 256.03924167771847,
                        "p1": −0.2106378372305548,
                        "p2": 0.5776109411992818,
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                }
                ]
        }

}
```