Answer Sheet 2

Topic: Camera Models, Optimization, Calibration

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Part 2: Optimization

The difference between these curve fitting examples is as follows:

- Curve fitting: it can fit data with certain noise, but is still sensitive to outliers. It uses ceres::CostFunction.
- Robust curve fitting: it is robust to outliers. Apart from ceres::CostFunction, it also uses CauchyLoss(0.5) in order to associate the loss function with a residual block. The argument 0.5 specifies the scale of Cauchy loss.

Part 3: Camera calibration

In this file it uses CLI::app to parse args in the command line. Via the app.add_option() function, it introduces show_gui, dataset_path and cam_model as command line parameters. The result of two camera is also closed and in a reasonable range.
Please see the calibration.cpp on the merge request.
The outputs of four camera models are attached below. From the result we can see that the convergence speed is very fast (8-20s), which shows the high efficiency of Ceres.
The result of calibration (Figure 1) shows that the red circle and magneta circle almost overlap.
Pinhole camera:

```
Solver Summary (v 2.0.0-eigen-(3.3.8)-lapack-suitesparse-(5.7.1)-cxsparse-(3.2.0)-
eigensparse -no_openmp)

 Original                  Reduced
Parameter blocks                        56                      55
Parameters                              394                     387
 Effective parameters                   340                     334
```

图 1: Calibration Result.

Residual blocks		12442	12442
Residuals		24884	24884
Minimizer		TRUST_REGION	
Sparse linear algebra library		SUITE_SPARSE	
Trust region strategy		LEVENBERG_MARQUARDT	
Given		Used	
Linear solver		SPARSE_NORMAL_CHOLESKY	SPARSE_NORMAL_CHOLESKY
Threads		12	12
Linear solver ordering		AUTOMATIC	55
Cost:			
Initial		1.795667e+07	
Final		1.565735e+05	
Change		1.780009e+07	
Minimizer iterations		16	
Successful steps		16	
Unsuccessful steps		0	
Time (in seconds):			
Preprocessor		0.019879	
Residual only evaluation		0.534973 (16)	
Jacobian & residual evaluation		18.543207 (16)	
Linear solver		0.062087 (16)	

```
Minimizer                                19.191715

Postprocessor                             0.001010
Total                                    19.212604

Termination:                             CONVERGENCE (Function tolerance reached.
|cost_change|/cost: 2.158064e−13 <= 1.000000e−12)

{
        "value0": {
                "cam.T_i_c": [
                {
                        "px": 0.0,
                        "py": 0.0,
                        "pz": 0.0,
                        "qx": 0.0,
                        "qy": 0.0,
                        "qz": 0.0,
                        "qw": 1.0
                },
                {
                        "px": 0.10960220177714075,
                        "py": −0.00010490052806695571,
                        "pz": 0.0037400002055087749,
                        "qx": 0.005389357928739904,
                        "qy": −0.03818008498755914,
                        "qz": 0.003278934586413792,
                        "qw": 0.9992509617305538
                }
                ],
                "cam.intrinsics": [
                {
                        "cam_type": "pinhole",
                        "fx": 546.2147378586979,
                        "fy": 551.5811503832607,
                        "cx": 399.0884109494394,
                        "cy": 208.37160821354002,
```

```
                        "p1": 0.0,
                        "p2": 0.0,
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                },
                {
                        "cam_type": "pinhole",
                        "fx": 547.2084958014361,
                        "fy": 550.3778903534294,
                        "cx": 368.86284114057539,
                        "cy": 215.16198122823935,
                        "p1": 0.0,
                        "p2": 0.0,
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                }
                ]
        }
}
```


Extended Unified Camera Model:

```
Solver Summary (v 2.0.0-eigen-(3.3.8)-lapack-suitesparse-(5.7.1)-cxsparse-(3.2.0)-
eigensparse -no_openmp)

 Original                  Reduced
Parameter blocks                        56                      55
Parameters                              394                     387
 Effective parameters                   340                     334
Residual blocks                         12442                   12442
Residuals                               24884                   24884

Minimizer                       TRUST_REGION

```

```
Sparse linear algebra library      SUITE_SPARSE
Trust region strategy      LEVENBERG_MARQUARDT

Given                      Used
Linear solver              SPARSE_NORMAL_CHOLESKY SPARSE_NORMAL_CHOLESKY
Threads                                        12                     12
Linear solver ordering                 AUTOMATIC                     55

Cost:
 Initial                          5.353182e+06
Final                             1.627604e+02
Change                            5.353019e+06

Minimizer iterations                         7
Successful steps                             7
Unsuccessful steps                           0

Time (in seconds):
Preprocessor                          0.008855

Residual only evaluation            0.232922 (7)
Jacobian & residual evaluation      8.479236 (7)
Linear solver                       0.028666 (7)
Minimizer                             8.763984

Postprocessor                         0.000933
Total                                 8.773773

Termination:                      CONVERGENCE (Parameter tolerance reached.
Relative step_norm: 2.527948e-09 <= 1.000000e-08.)

{
        "value0": {
                "cam.T_i_c": [
                {
                        "px": 0.0,
                        "py": 0.0,
```

```
                "pz": 0.0,
                "qx": 0.0,
                "qy": 0.0,
                "qz": 0.0,
                "qw": 1.0
            },
            {
                "px": 0.11002631815223632,
                "py": −0.00028908964378220297,
                "pz": 0.000246181144400705,
                "qx": 0.007123595350702433,
                "qy": 0.0006300007991471633,
                "qz": 0.0010774305748141569,
                "qw": 0.9999738479738503
            }
            ],
            "cam.intrinsics": [
            {
                "cam_type": "eucm",
                "fx": 460.9794986620865,
                "fy": 459.62650958263319,
                "cx": 365.8867492896418,
                "cy": 249.346486044204,
                "p1": 0.591664809640774,
                "p2": 1.1241125399441403,
                "p3": 0.0,
                "p4": 0.0,
                "width": 752,
                "height": 480
            },
            {
                "cam_type": "eucm",
                "fx": 459.79606434876208,
                "fy": 458.40630958578478,
                "cx": 379.35442505013966,
                "cy": 256.0400994980228,
                "p1": 0.6060839010938546,
```

```
                        "p2": 1.088350919730381,
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                }
                ]
        }

}
```


Kannala-Brandt Camera Model:

```
Solver Summary (v 2.0.0-eigen-(3.3.8)-lapack-suitesparse-(5.7.1)-cxsparse-(3.2.0)-
eigensparse -no_openmp)

                                     Original                  Reduced
Parameter blocks                           56                       55
Parameters                                394                      387
Effective parameters                      340                      334
Residual blocks                         12442                    12442
Residuals                               24884                    24884

Minimizer                        TRUST_REGION

Sparse linear algebra library    SUITE_SPARSE
Trust region strategy            LEVENBERG_MARQUARDT

                                     Given                     Used
Linear solver           SPARSE_NORMAL_CHOLESKY    SPARSE_NORMAL_CHOLESKY
Threads                                    12                        12
Linear solver ordering                AUTOMATIC                      55

Cost:
Initial                          5.788049e+06
Final                            1.619844e+02
Change                           5.787887e+06

```

```
Minimizer iterations                8
Successful steps                    8
Unsuccessful steps                  0

Time (in seconds):
Preprocessor                 0.009358

Residual only evaluation     0.272357 (8)
Jacobian & residual evaluation 9.929500 (8)
Linear solver                0.032390 (8)
Minimizer                   10.260839

Postprocessor                0.000936
Total                       10.271133

Termination:                CONVERGENCE (Parameter tolerance reached.
Relative step_norm: 1.775292e-10 <= 1.000000e-08.)

{
        "value0": {
                "cam.T_i_c": [
                {
                        "px": 0.0,
                        "py": 0.0,
                        "pz": 0.0,
                        "qx": 0.0,
                        "qy": 0.0,
                        "qz": 0.0,
                        "qw": 1.0
                },
                {
                        "px": 0.11002263076486109,
                        "py": -0.0002883117629451379,
                        "pz": 0.00025193738175583363,
                        "qx": 0.007129648722668718,
                        "qy": 0.0006308484199464747,
                        "qz": 0.0010771978182017289,
```

```
                        "qw": 0.9999738045490108
                }
                ],
                "cam.intrinsics": [
                {
                        "cam_type": "kb4",
                        "fx": 461.1581901632604,
                        "fy": 459.8080285947971,
                        "cx": 365.89723869011746,
                        "cy": 249.35492318319005,
                        "p1": −0.004593052532205222,
                        "p2": 0.02753243203702901,
                        "p3": −0.036977671770998209,
                        "p4": 0.018610217245568693,
                        "width": 752,
                        "height": 480
                },
                {
                        "cam_type": "kb4",
                        "fx": 459.74480686519805,
                        "fy": 458.3535053463388,
                        "cx": 379.36396149546416,
                        "cy": 256.0435254270624,
                        "p1": 0.004549588612680235,
                        "p2": 0.002485490491602717,
                        "p3": −0.010601348784718934,
                        "p4": 0.010781041140887182,
                        "width": 752,
                        "height": 480
                }
                ]
        }

}
```


Double Sphere Model:


```
Solver Summary (v 2.0.0−eigen−(3.3.8)−lapack−suitesparse−(5.7.1)−cxsparse−(3.2.0)−
```

eigensparse −no_openmp)

	Original	Reduced
Parameter blocks	56	55
Parameters	394	387
Effective parameters	340	334
Residual blocks	12442	12442
Residuals	24884	24884

Minimizer	TRUST_REGION	
Sparse linear algebra library	SUITE_SPARSE	
Trust region strategy	LEVENBERG_MARQUARDT	
Given	Used	
Linear solver	SPARSE_NORMAL_CHOLESKY	SPARSE_NORMAL_CHOLESKY
Threads	12	12
Linear solver ordering	AUTOMATIC	55

Cost:

Initial	5.353182e+06
Final	1.627482e+02
Change	5.353019e+06

Minimizer iterations	15
Successful steps	13
Unsuccessful steps	2

Time (in seconds):

Preprocessor	0.008344
Residual only evaluation	0.488787 (15)
Jacobian & residual evaluation	15.653587 (13)
Linear solver	0.055071 (15)
Minimizer	16.240517
Postprocessor	0.000917

```
Total                                   16.249778

Termination:                            CONVERGENCE (Parameter tolerance reached.
Relative  step_norm: 6.648845e−09 <= 1.000000e−08.)

{
        "value0": {
                "cam.T_i_c": [
                {
                        "px": 0.0,
                        "py": 0.0,
                        "pz": 0.0,
                        "qx": 0.0,
                        "qy": 0.0,
                        "qz": 0.0,
                        "qw": 1.0
                },
                {
                        "px": 0.11002674958788142,
                        "py": −0.00028913779866585087,
                        "pz": 0.000246625049919901,
                        "qx": 0.0071236589880659538,
                        "qy": 0.0006289220700000381,
                        "qz": 0.001077495211590831,
                        "qw": 0.9999738481299002
                }
                ],
                "cam.intrinsics": [
                {
                        "cam_type": "ds",
                        "fx": 351.03728321686966,
                        "fy": 350.0074555977382,
                        "cx": 365.88809735482138,
                        "cy": 249.34573836993608,
                        "p1": −0.2385312817269929,
                        "p2": 0.567869484529095,
                        "p3": 0.0,
```

```
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                },
                {
                        "cam_type": "ds",
                        "fx": 362.95328870306289,
                        "fy": 361.85685537441079,
                        "cx": 379.35501913798887,
                        "cy": 256.03924167771847,
                        "p1": −0.2106378372305548,
                        "p2": 0.5776109411992818,
                        "p3": 0.0,
                        "p4": 0.0,
                        "width": 752,
                        "height": 480
                }
                ]
        }

}
```