Midterm Exam

M4320–Math Modeling Fall 2017 Tarleton State Dr. Scott Cook Assigned 2017-10-19

1) For the following system, determine (a) linear (L) or non-linear (NL) (b) order (c) autonomous (A) or non-autonomous (NA). Convert to first order, autonomous if it is not already.

$$x_t = x_{t-1} + x_{t-2}y_{t-1}$$
$$y_t = y_{t-1} + 5tx_{t-2}$$

2) Consider the Lotka-Volterra model. Describe what each parameter means (ie. what aspect of the predator-prey system it encodes). (Suggestion: Uses phrases like "If we increase ..., then ... will happen". Use the terms "rabbits" and "foxes" if it helps you.)

$$x_t = x_{t-1} + rx_{t-1}\left(1 - \frac{x_{t-1}}{K}\right) - \left(1 - \frac{1}{by_{t-1}+1}\right)x_{t-1}$$
$$y_t = y_{t-1} - dy_{t-1} + cx_{t-1}y_{t-1}$$

3) Define the terms eigenvector and eigenvalue for a square matrix A.

4) Find all equilibrium points for the system below and determine stability of at least one of them.

$$x_t = 2x_{t-1} - y_{t-1} - x_{t-1}^2 + x_{t-1}y_{t-1}$$
$$y_t = -x_{t-1}^2$$

5) When I searched "good dynamical systems exam questions", Google got the network below.

5.1) What is the adjacency matrix A?

5.2) What is the transition matrix T for "naive" PageRank (without random jumps - users can only follow links).

5.3) What flaw(s) do you see?

5.4) We fix the flaw(s) by introducing a random jump. Let q be the random jump probability in "wise" PageRank. Let $p = 1 - q$. How do you modify the T above? (You don't need to actually compute the new T, just describe what you would ask the computer to do).

5.5) np.linalg.eig(T) gives the result below for the T above. In what order will Google display the search results?

$$\begin{bmatrix} 1.0+0.j & -0.9+0.j & 0.0+0.j & 0.0-0.j \end{bmatrix}, \begin{bmatrix} 0.03-0.j & -0.00+0.j & 0.71+0.j & 0.71-0.j \\ 0.02-0.j & 0.00+0.j & -0.00+0.j & -0.00-0.j \\ 0.48-0.j & 0.71+0.j & 0.00-0.j & 0.00+0.j \\ 0.46-0.j & -0.71+0.j & -0.71-0.j & -0.71+0.j \end{bmatrix}$$

6) Do **TWO** of following

6.1) Draw the first 6 steps of the cobweb plot for $x_t = 3 - (x_{t-1} - 2)^2$, $x_0 = 1$.

6.2) define chaos (A dynamical system is chaotic if ...)

6.3) define stochastic (A square matrix is stochastic if ...)

6.4) describe variable rescaling (What you DO to rescale and why you would want to do it. You do not need specific

details - broad but coherent descriptions are fine)

6.5) State Occam's Razor.