1/4) Excercise 4 Adv.Alg 2011

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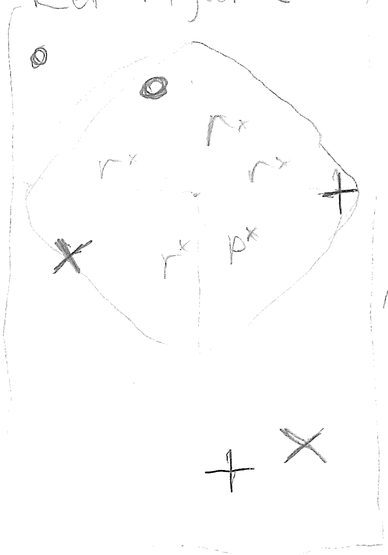
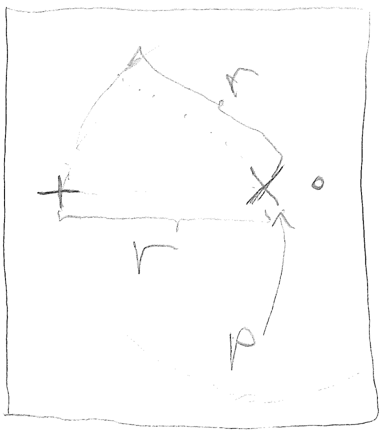
Let figures +, X, o be diffrent colors, "c_1=X, c_2=+, ..."



Just an example of how it can look like. (Not part of proof)
p^x is optimal point with radius r^x.
Note: p^x need not be a point among the set of colored points as in the figure.

now fixate p to be the colored point that the greedy algorithm found gives minimum radius. r is the best radius as described by greedy.



The approximation ratio, r/r^x, should be shown to be ≤ 2.

Lets show $r/r^* \leq 2$:

let $p_1 \ldots p_k$ be k color - diverse points

that the optimal solution covers.

p_i has color i,

there might be multiple points

that the optimal solution covers

with the same color i, then

p_i can be any such point.

We also know that at least one

point with color i is covered

by the optimal solution,

since every solution must

do that.

the optimal solution

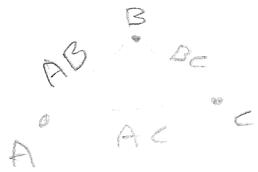
Since sol covered each p_i:

$$\forall i. \quad |\overrightarrow{p^x p_i}| \leq r^x$$

And now I claim that for any fixated p_i (i constant):

$$\forall j. \quad |\overrightarrow{p_i p_j}| \leq 2r^x \quad \text{(claim, to be shown)}$$

Being on the plane means triangle inequality holds. $\left(|\overrightarrow{AC}| \leq |\overrightarrow{AB}| + |\overrightarrow{BC}|\right)$



We can see p^x as an intermediete point

$\overrightarrow{p_i p_j} = \overrightarrow{p_i p^x} + \overrightarrow{p^x p_j}$ thus

$$\forall j. \quad |\overrightarrow{p_i p_j}| \overset{\text{T.I.}}{\leq} |\overrightarrow{p_i p^x}| + |\overrightarrow{p^x p_j}| \leq r^x + r^x = 2r^x$$

Our claim is proven.

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The greedy algorithm tries $p = p_1$, since in fact it tries with all colored points.

The attempt $p = p_1$

will not

~~pick~~ worse than picking

$p_2 \cdots p_k$. And that have covered all colors.

More Formally, distance for p and picked point of color c is less or equal than $|\overrightarrow{pp_c}|$. (greedy assures this)

So that means

$$r \leq \max_j |\overrightarrow{pp_j}|$$

And since $\forall_j : |\overrightarrow{p_i p_j}| \leq 2r^*$ we get the desired

$$r \leq 2r^* \Rightarrow r/r^* \leq 2 \quad \square$$