Probability Distributions

Binomial, Poisson, Normal

Probability and Statistics

Probability is the chance of an outcome in an experiment (also called event).

Event: Tossing a fair coin
Outcome: Head, Tail

Probability deals with **predicting** the likelihood of **future** events.

Statistics involves the **analysis** of the **frequency** of **past** events

A random variable is a rule that assigns a numerical value to an outcome of interest.

A probability distribution is a definition of probabilities of the values of random variable.

DISTRIBUTION

- Frequency Distribution: It is a listing of observed / actual frequencies of all the outcomes of an experiment that actually occurred when experiment was done.
- Probability Distribution: It is a listing of the probabilities of all the possible outcomes that could occur if the experiment was done.
  - It can be described as:
    - A diagram (Probability Tree)
    - A table
    - A mathematical formula

- The relationship between the values of a random variable and the probability of their occurrence summarized by means of a device called a probability distribution.
- It may be expressed in the form of a table, a graph or a formula.
- Knowledge of probability distribution of a random variable provides the researchers with a powerful tool for summarizing and describing a set of data and for reaching conclusion about a population on the basis of sample.

Types of Probability Distributions

Discrete probability distributions

- **Binomial distribution**
- Multinomial distribution
- **Poisson distribution**
- Hypergeometric distribution

Continuous probability distributions

- **Normal distribution**
- Standard normal distribution
- Gamma distribution
- Exponential distribution
- Chi square distribution
- Lognormal distribution
- Weibull distribution

Types of Probability Distribution [PD]

- Probability Distribution
  - Discrete PD
    - Binomial Distribution
    - Poisson Distribution
  - Continuous PD
    - Normal Distribution

Discrete Probability Distributions

- A discrete random variable is a variable that can assume only a countable number of values

Many possible outcomes:

- number of complaints per day
- number of TV’s in a household
- number of rings before the phone is answered

Only two possible outcomes:

- gender: male or female
- defective: yes or no
- spreads peanut butter first vs. spreads jelly first

Continuous Probability Distributions

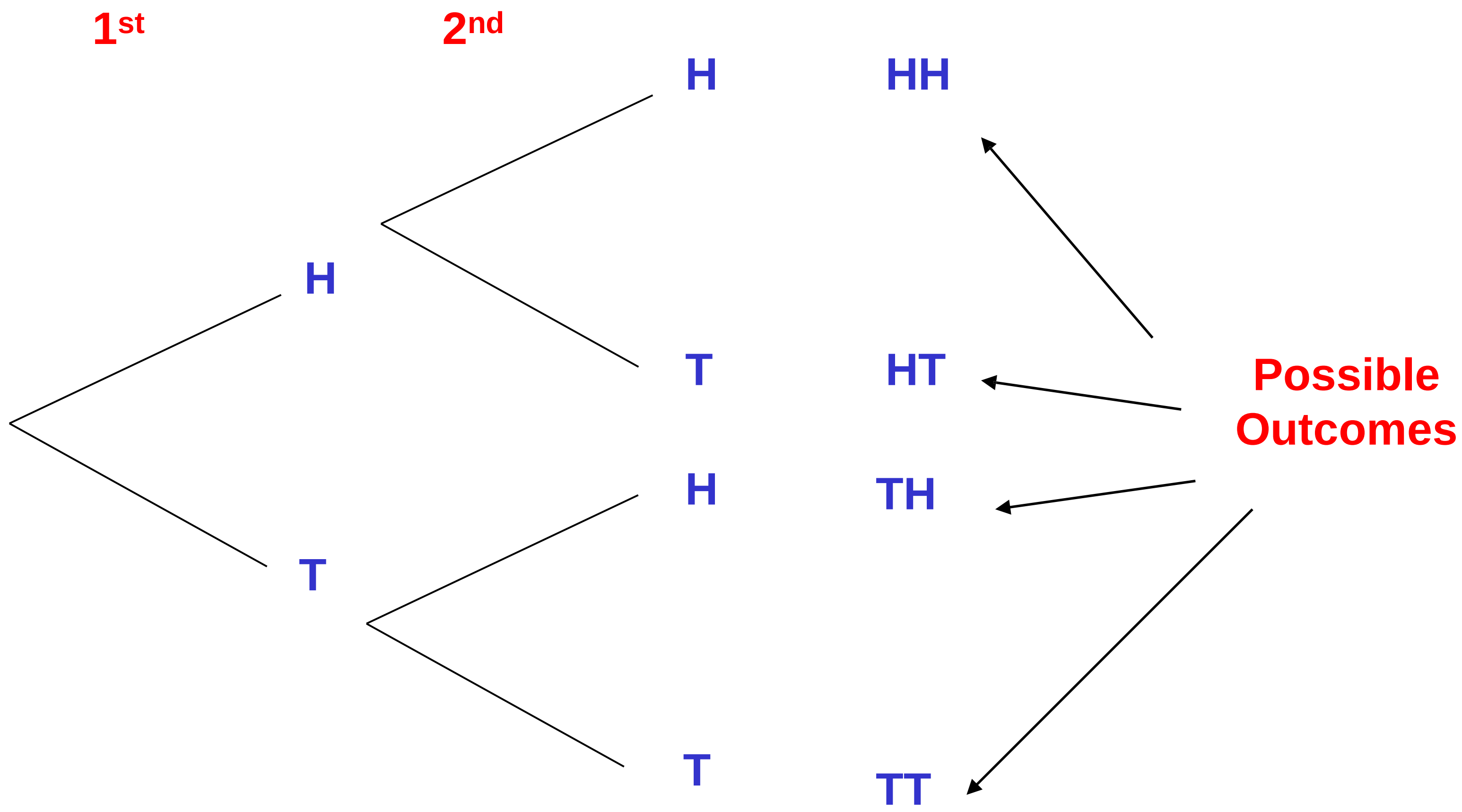
- A continuous random variable is a variable that can assume any value on a continuum (can assume an uncountable number of values)
  - thickness of an item
  - time required to complete a task
  - temperature of a solution
  - height, in inches
- These can potentially take on any value, depending only on the ability to measure accurately.

Probability Distribution

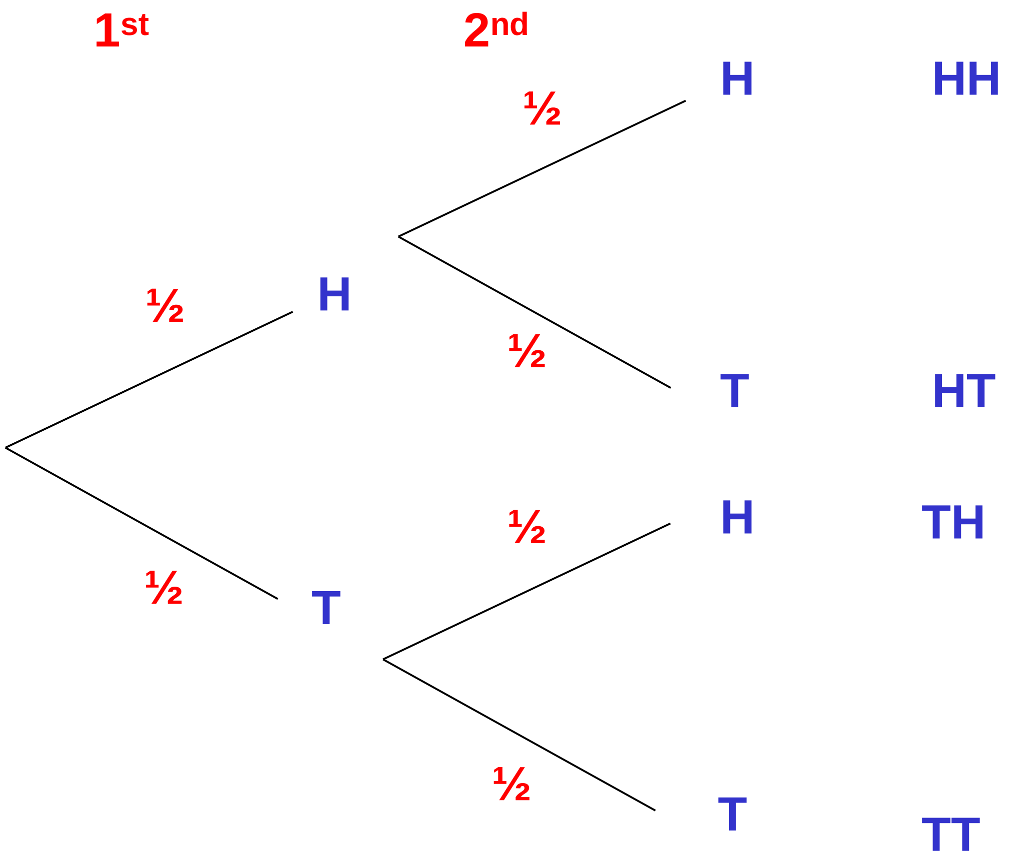
- Discrete Distribution: Random Variable can take only limited number of values. Ex: No. of heads in two tosses.

- Continuous Distribution: Random Variable can take any value. Ex: Height of students in the class.

Tree Diagram –
A fair coin is tossed twice



Attach probabilities



P(H,H)=½x½=¼

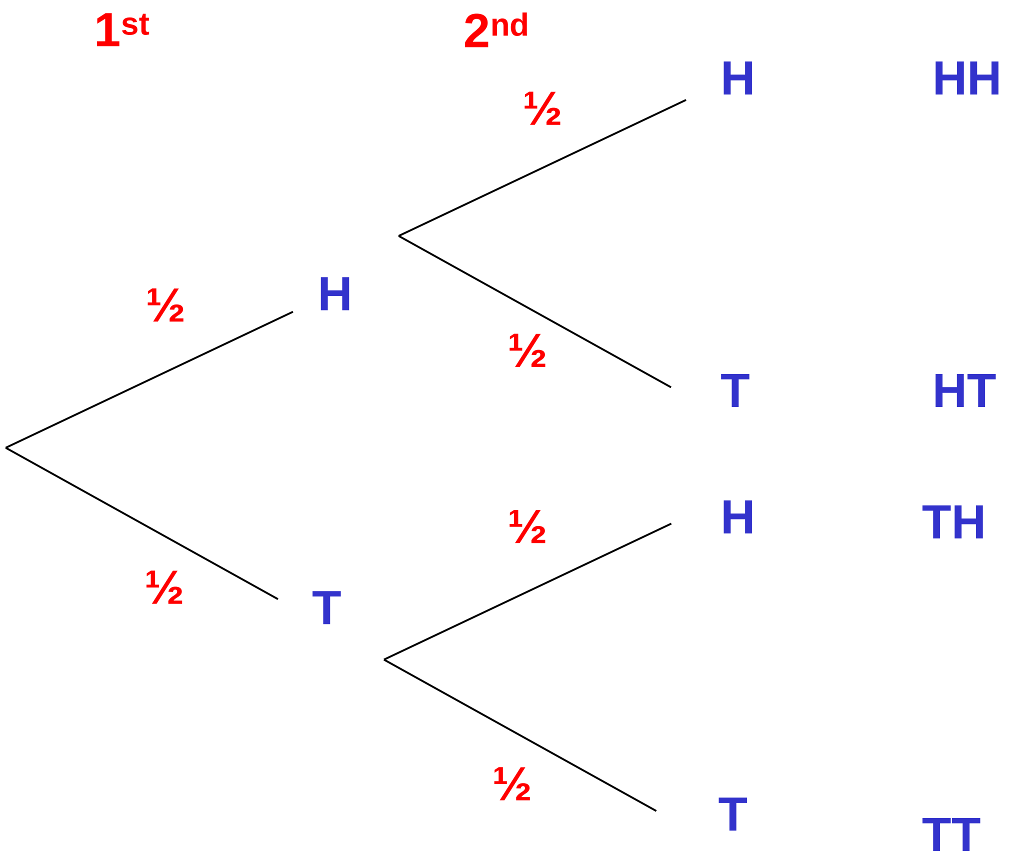
P(H,T)=½x½=¼

P(T,H)=½x½=¼

P(T,T)=½x½=¼

INDEPENDENT EVENTS – 1st spin has no effect on the 2nd spin

Calculate probabilities



P(H,H)=½x½=¼ *

P(H,T)=½x½=¼ *

P(T,H)=½x½=¼ *

P(T,T)=½x½=¼

Probability of *at least* one Head?

Ans: ¼ + ¼ + ¼ = ¾

DISCRETE PD – EXAMPLE (TABLE)

- Tossing a coin three times:
  $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
  - Let X represents "No. of heads"

X	Frequency	P (X=x)
0	1	1/8
1	3	3/8
2	3	3/8
3	1	1/8

BINOMIAL DISTRIBUTION

- There are certain phenomena in nature which can be identified as **Bernoulli's processes**, in which:
  - There is a fixed number of n trials carried out
  - Each trial has only two possible outcomes say success or failure, true or false etc.
  - Probability of occurrence of any outcome remains same over successive trials
  - Trials are statistically independent

BINOMIAL DISTRIBUTION

Binomial distribution was discovered by James Bernoulli (1654-1705).

Let a random experiment be performed repeatedly and the occurrence of an event in a trial be called as success and its non-occurrence is failure.

Consider a set of n independent trails (n being finite), in which the probability p of success in any trail is constant for each trial. Then q=1-p is the probability of failure in any trail.

The Binomial Distribution
Bernoulli Random Variables

- Imagine a simple trial with only two possible outcomes
  - Success (*S*)
  - Failure (*F*)
- Examples
  - Toss of a coin (heads or tails)
  - Sex of a newborn (male or female)
  - Survival of an organism in a region (live or die)
  - Suppose that the probability of success is p
  - What is the probability of failure?
  - $q = 1 - p$

Binomial Distribution Settings

- A manufacturing plant labels items as either defective or acceptable
- A firm bidding for a contract will either get the contract or not
- A marketing research firm receives survey responses of “yes I will buy” or “no I will not”
- New job applicants either accept the offer or reject it

The Binomial Distribution
Overview

- What is the probability of obtaining *x* successes in *n* trials?

- Example
  - What is the probability of obtaining 2 heads from a coin that was tossed 5 times?

$$P(HHTTT) = (1/2)^5 = 1/32$$

Condition for Binomial distribution

We get the binomial distribution under the following experimentation conditions -

1. The number of trial n is finite
2. The trials are independent of each other.
3. The probability of success p is constant for each trial.
4. Each trial must result in a success or failure.
5. The events are discrete events.

Characteristics of a binomial random variable

- The experiment consists of n identical trials
- There are only 2 possible outcomes on each trial. We will denote one outcome by S (for Success) and the other by F (for Failure).
- The probability of S remains the same from trial to trial. This probability will be denoted by p, and the probability of F will be denoted by q ($q = 1-p$).
- The trials are independent.
- The binomial random variable x is the number of S' in n trials.

Properties

1. If p and q are equal, the given binomial distribution will be symmetrical.
If p and q are not equal, the distribution will be skewed distribution.

2. Mean = E(x) = np

3. Variance =V(x) = npq   (mean>variance)
σ_x^2 = *Variance (X) = np(1-p)*
4. Standard Deviation of BD: $\sigma = \sqrt{npq}$
σ_x =*SD (X)*= $\sqrt{np(1-p)}$

BINOMIAL DISTRIBUTION

- **Binomial distribution** is a discrete PD which expresses the probability of one set of alternatives – success (p) and failure (q)
  - **P(X = x) = $^{n}C_{r}p^{r}q^{n-r}$ (Prob. Of r successes in n trials)**
    - n = no. of trials undertaken
    - r = no. of successes desired
    - p = probability of success
    - q = probability of failure

- The shape of the binomial distribution depends on the values of p and n

The probability distribution:

$$P_x = {}^nC_x \; p^x q^{n-x} \quad (x = 0, 1, 2, \ldots, n),$$

Where,

p = probability of a success on a single trial, $q=1-p$

n = number of trials, x= number of successes in n trials

$${}^nC_x = \frac{n!}{x!(n-x)!} = \text{combination of } x \text{ from } n$$

Application

1. Quality control measures and sampling process in industries to classify items as defectives or non-defective.

2. Medical applications such as success or failure, cure or no-cure.

Example 1

Eight coins are tossed simultaneously. Find the probability of getting atleast six heads.

Solution

Here number of trials, n = 8, p denotes the probability of getting a head.

$$\therefore \; p = \frac{1}{2} \text{ and } q = \frac{1}{2}$$

If the random variable X denotes the number of heads, then the probability of a success in n trials is given by

$P(X = x) = nc_x \, p^x \, q^{n-x}, \; x = 0, 1, 2, ..., n$

$$= 8C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x} = 8C_x \left(\frac{1}{2}\right)^8$$

$$= \frac{1}{2^8} 8C_x$$

Probability of getting atleast six heads is given by

$$P(x \geq 6) = P(x = 6) + P(x = 7) + P(x = 8)$$

$$= \frac{1}{2^8} 8C_6 + \frac{1}{2^8} 8C_7 + \frac{1}{2^8} 8C_8$$

$$= \frac{1}{2^8} \left[8C_6 + 8C_7 + 8C_8 \right]$$

$$= \frac{1}{2^8} \left[28 + 8 + 1 \right] = \frac{37}{256}$$

$$P(x) = {}^nC_r\, p^r q^{n-r}$$

$$P(x \geq 6) = \frac{1}{2^8}\,{}^8C_6 + \frac{1}{2^8}\,{}^8C_7 + \frac{1}{2^8}\,{}^8C_8$$

$$= \frac{1}{2^8}\left[{}^8C_6 + {}^8C_7 + {}^8C_8\right]$$

$$= \frac{1}{2^8}\left[28 + 8 + 1\right] = \frac{37}{256}$$

$${}^nC_r = \frac{n!}{(n-r)! \times r!}$$

$${}^8C_6 = \frac{8!}{(8-6)! \times 6!}$$

$$\frac{8!}{2! \times 6!}$$

$$\frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2! \times 6!}$$

$$\frac{8 \times 7 \times \cancel{6!}}{2! \times \cancel{6!}}$$

$$\frac{\overset{4}{\cancel{8}} \times 7}{\cancel{2} \times 1} = 4 \times 7 = 28$$

$${}^8C_7 = \frac{8!}{(8-7)! \times 7!}$$

$$\frac{8!}{1! \times 7!}$$

$$\frac{8 \times \cancel{7!}}{1! \times \cancel{7!}}$$

$$= 8$$

$${}^8C_8 = \frac{8!}{(8-8)! \times 8!}$$

$$= \frac{\cancel{8!}}{0! \times \cancel{8!}} = 1$$

$\boxed{0! = 1}$ rule

Example 2 Ten coins are tossed simultaneously. Find the probability of getting (i) atleast seven heads (ii) exactly seven heads (iii) atmost seven heads

Example 3:20 wrist watches in a box of 100 are defective. If 10 watches are selected at random, find the probability that (i) 10 are defective (ii) 10 are good (iii) at least one watch is defective (iv) at most 3 are defective.

Example 2 Ten coins are tossed simultaneously. Find the probability of getting (i) atleast seven heads (ii) exactly seven heads (iii) atmost seven heads

Solution

p = Probability of getting a head = $\frac{1}{2}2$

q = Probability of not getting a head = $\frac{1}{2}$

The probability of getting x heads throwing 10 coins simultaneously is given by

$P(X = x) = nC_x\, p^x\, q^{n-x}$, x = 0, 1, 2, ..., n

$$= 10C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = 10C_x \left(\frac{1}{2}\right)^{10}$$

$$= \frac{1}{2^{10}} 10C_x$$

i) Probability of getting atleast seven heads

$P(x \geq 7) = P(x = 7) + P(x = 8) + P(x = 9) + P(x = 10)$

$$= \frac{1}{2^{10}}\left[10C_7 + 10C_8 + 10C_9 + 10C_{10}\right]$$

$$= \frac{1}{1024}\left[120 + 45 + 10 + 1\right] = \frac{176}{1024}$$

ii) Probability of getting exactly 7 heads

$$P(x = 7) = \frac{1}{2^{10}} 10C_7 = \frac{1}{2^{10}}(120) = \frac{120}{1024}$$

iii) Probability of getting almost 7 heads

$P(x \leq 7) = 1 - P(x > 7)$

$$= 1 \text{ symbol } \{P(x = 8) + P(x = 9) + P(x = 10)\}$$

$$= 1 \quad \frac{1}{2^{10}}\left[10C_8 + 10C_9 + 10C_{10}\right]$$

$$= 1 - \frac{1}{2^{10}}\left[45 + 10 + 1\right]$$

$$= 1 - \frac{56}{1024}$$

$$= \frac{968}{1024}$$

Example 3:20 wrist watches in a box of 100 are defective. If 10 watches are selected at random, find the probability that (i) 10 are defective (ii) 10 are good (iii) at least one watch is defective (iv) at most 3 are defective.

Solution

20 out of 100 wrist watches are defective

Probability of defective wrist watch, p $= \frac{20}{100} = \frac{1}{5}$

$q = 1 - p = \frac{4}{5}$

Since 10 watches are selected at random, $n = 10$

$P(X = x) = nC_x \, p^x q^{n-x}, \; x = 0, 1, 2, \ldots, 10$

$$= 10C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{10-x}$$

i) Probability of selecting 10 defective watches

$$P(x = 10) = = 10C_{10}\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^{0} = 1.\frac{1}{5^{10}}.1 = \frac{1}{5^{10}}$$

ii) Probability of selecting 10 good watches (i.e. no defective)

$$P(x = 0) = \; 10C_0\left(\frac{1}{5}\right)^{0}\left(\frac{4}{5}\right)^{10}$$

$$= 1.1\left(\frac{4}{5}\right)^{10} = \left(\frac{4}{5}\right)^{10}$$

iii) Probability of selecting at least one defective watch

$$P(x \geq 1) = 1 - P(x < 1)$$

$$= 1 - P(x = 0)$$

$$= 1 - \ 10C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10}$$

$$=1-\left(\frac{4}{5}\right)^{10}$$

iv) Probability of selecting at most 3 defective watches

$$P\ (x \text{ £ } 3) = P\ (x = 0) + P(x =1) + P(x = 2) + P(x = 3)$$

$$= 10C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10} + 10C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^9 + 10C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8 + 10C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^7$$

$$= 1.1\left(\frac{4}{5}\right)^{10} + 10\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^9 + \frac{10.9}{1.2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8 + \frac{10.9.8}{1.2.3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^7$$

$$= 1.\ (0.107) + 10\ (0.026) + 45\ (0.0062) + 120\ (0.0016)$$

$$= 0.859 \text{ (approx)}$$

PRACTICE QUESTIONS – BD

- Four coins are tossed simultaneously. What is the probability of getting:
  - No head 1/16
  - No tail 1/16
  - Two heads 3/8

- The probability of a bomb hitting a target is 1/5. Two bombs are enough to destroy a bridge. If six bombs are fired at the bridge, find the probability that the bridge is destroyed. (0.345)

- If 8 ships out of 10 ships arrive safely. Find the probability that at least one would arrive safely out of 5 ships selected at random. (0.999)

PRACTICE QUESTIONS – BD

- A pair of dice is thrown 7 times. If getting a total of 7 is considered as success, find the probability of getting:
  - No success $(5/6)^7$
  - 6 successes $35.(1/6)^7$
  - At least 6 successes $36.(1/6)^7$
- Eight-tenths of the pumps were correctly filled. Find the probability of getting exactly three of six pumps correctly filled. (0.082)

Measures of Central Tendency and Dispersion for the Binomial Distribution

- Mean of BD: $\mu = np$
- Standard Deviation of BD: $\sigma = \sqrt{npq}$

- The mean of BD is 20 and its SD is 4. Find n, p, q.

  (100, 1/5, 4/5)

- The mean of BD is 20 and its SD is 7. Comment.

Mean = 20 $\quad$ SD = 4

$\mu = np$

$\mu = 20$

$SD = \sqrt{npq}$

$4 = \sqrt{20 \times q}$

$4 = (20 \times q)^{1/2}$

$4^2 = 20 \times q$

$16 = 20q$

$\frac{16}{20} = q$

$\frac{4}{5} = q$

$p = 1 - q$

$= 1 - \frac{4}{5}$

$= \frac{5-4}{5}$

$p = \frac{1}{5}$

$n \times p \times q$

$\cancel{n \times \frac{1}{5} \times \frac{4}{5}}$

$\cancel{n \times \frac{4}{25}}$

Fitting of Binomial Distribution

- Four coins are tossed 160 times and the following results were obtained:

No. of heads	0	1	2	3	4
Frequency	17	52	54	31	6

Fit a binomial distribution under the assumption that the coins are unbiased.

- Fit a binomial distribution to the following data:

X	0	1	2	3	4
f	28	62	46	10	4

The Poisson Distribution

- When there is a large number of trials, but a small probability of success, binomial calculation becomes impractical
  - Example: Number of deaths from horse kicks in the Army in different years
- The mean number of successes from n trials is $\boldsymbol{\mu = np}$
  - Example: 64 deaths in 20 years from thousands of soldiers

POISSON DISTRIBUTION

Characteristics of the Poisson Distribution:

- The outcomes of interest are rare relative to the possible outcomes
- The average number of outcomes of interest per time or space interval is λ
- The number of outcomes of interest are random, and the occurrence of one outcome does not influence the chances of another outcome of interest
- The probability of that an outcome of interest occurs in a given segment is the same for all segments

Condition for Poisson distribution

Poisson distribution is the limiting case of binomial distribution under the following assumptions.

1. The number of trials n should be indefinitely large ie., n->8

2. The probability of success p for each trial is indefinitely small.

3. np= λ, should be finite where λ is constant.

Properties

1. Poisson distribution is defined by single parameter λ.

2. Mean = λ

3. Variance = λ.

Mean and Variance are equal.

POISSON DISTRIBUTION

- When there is a large number of trials, but a small probability of success, binomial calculation becomes impractical
- If λ = mean no. of occurrences of an event per unit interval of time/space, then probability that it will occur exactly 'x' times is given by

- **P(x)** $= \frac{\lambda^x e^{-\lambda}}{x!}$ where e is napier constant & e = 2.7182

The Poisson Distribution

- If we substitute μ/n for p, and let n tend to infinity, the binomial distribution becomes the Poisson distribution:

$$P(x) = \frac{e^{-\mu}\mu^{x}}{x!}$$

- Poisson distribution is applied where random events in space or time are expected to occur
- Deviation from Poisson distribution may indicate some degree of non-randomness in the events under study.

Application

1. It is used in quality control statistics to count the number of defects of an item.
2. In biology, to count the number of bacteria.
3. In determining the number of deaths in a district in a given period, by rare disease.
4. The number of error per page in typed material.
5. The number of plants infected with a particular disease in a plot of field.
6. Number of weeds in particular species in different plots of a field.

Example 4: Suppose on an average 1 house in 1000 in a certain district has a fire during a year. If there are 2000 houses in that district, what is the probability that exactly 5 houses will have a fire during the year? [given that $e^{-2} = 0.13534$]

Solution:

Mean, $\bar{x} = np$, $n = 2000$ and $p = \frac{1}{1000}$

$$= 2000 \times \frac{1}{1000}$$

$$\lambda = 2$$

The Poisson distribution is

$$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$P(X = 5) = \frac{e^{-2} 2^{5}}{5!}$$

$$= \frac{(0.13534) \times 32}{120}$$

$$= \mathbf{0.036}$$

Example 5

If 2% of electric bulbs manufactured by a certain company are defective. Find the probability that in a sample of 200 bulbs i) less than 2 bulbs ii) more than 3 bulbs are defective.[e-4 = 0.0183]

Solution

The probability of a defective bulb $= p = \frac{2}{100} = 0.02$

Given that n = 200 since p is small and n is large

We use the Poisson distribution

mean, $m = np = 200 \times 0.02 = 4$

Now, Poisson Probability function, $P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

i) Probability of less than 2 bulbs are defective

$= P(X<2)$

$= P(x = 0) + P(x = 1)$

$= e^{-4} + e^{-4} (4)$

$= e- 4 (1 + 4) = 0.0183 \times 5$

$= 0.0915$

ii) Probability of getting more than 3 defective bulbs

$P(x > 3) = 1- P(x \leq 3)$

$= 1- \{P(x = 0) + P(x =1) + P(x=2) + P(x=3)\}$

$$= 1 - e^{-4}\left\{1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!}\right\}$$

$= 1- \{0.0183 \times (1 + 4 + 8 + 10.67)\}$

$= 0.567$

PRACTICE PROBLEMS – POISSON DISTRIBUTION

- On a road crossing, records show that on an average, 5 accidents occur per month. What is the probability that 0, 1, 2, 3, 4, 5 accidents occur in a month? (0.0067, 0.0335, 0.08425, 0.14042, 0.17552, 0.17552)

- In case, probability of greater than 3 accidents per month exceeds 0.7, then road must be widened. Should the road be widened? (Yes)

- If on an average 2 calls arrive at a telephone switchboard per minute, what is the probability that exactly 5 calls will arrive during a randomly selected 3 minute interval? (0.1606)

- It is given that 2% of the screws are defective. Use PD to find the probability that a packet of 100 screws contains:
  - No defective screws (0.135)
  - One defective screw (0.270)
  - Two or more defective screw (0.595)

Characteristics of Poisson Distribution

- It is a discrete distribution
- Occurrences are statistically independent
- Mean no. of occurrences in a unit of time is proportional to size of unit (if 5 in one year, 10 in 2 years etc.)
- Mean of PD is $\lambda = np$
- Standard Deviation of PD is $\sqrt{\lambda} = \sqrt{np}$
- It is always right skewed.
- PD is a good approximation to BD when n > or = 20 and p< or = 0.05

THE Gaussian or NORMAL DISTRIBUTION

- Discovered in 1733 by de Moivre as an approximation to the binomial distribution when the number of trails is large

- Derived in 1809 by Gauss

- Importance lies in the Central Limit Theorem, which states that the sum of a large number of independent random variables (binomial, Poisson, etc.) will approximate a normal distribution

  - Example: Human height is determined by a large number of factors, both genetic and environmental, which are additive in their effects. Thus, it follows a normal distribution.

Normal distribution

Continuous Probability distribution is normal distribution.

It is also known as error law or Normal law or Laplacian law or Gaussian distribution. Many of the sampling distribution like student-t, f distribution and $\chi 2$ distribution.

Definition

A continuous random variable x is said to be a normal distribution with parameters μ and σ^2 , if the density function is given by the probability law .

The mean m and standard deviation s are called the parameters of Normal distribution.

The normal distribution is expressed by $X \sim N(m, \sigma^2)$

Condition of Normal Distribution

i) **Normal distribution is a limiting form of the binomial distribution under the following conditions.**

a) n, the number of trials is indefinitely large and

b) Neither p nor q is very small.

iii) Constants of normal distribution are mean = m, variation =s_2, Standard deviation = s.

Properties of normal distribution

1. The normal curve is bell shaped and is symmetric at x = m.
2. Mean, median, and mode of the distribution are coincide

   i.e., Mean = Median = Mode = m
3. It has only one mode at x = m (i.e., unimodal)
4. The points of inflection are at x = m ± s
5. The maximum ordinate occurs at x = m and its value is =
6. Area Property P(m - s < ´ < m + s) = 0.6826

   P(m - 2s < ´ < m + 2s) = 0.9544

   P(m - 3s < ´ < m + 3s) = 0.9973

The Normal Distribution

‘Bell Shaped’

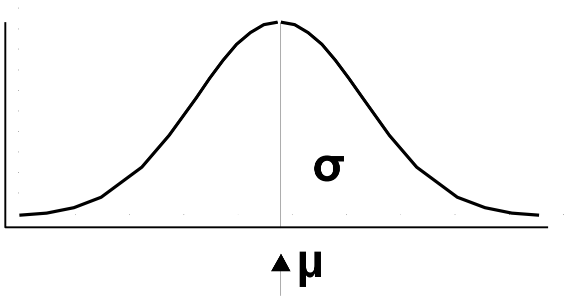
Symmetrical

Mean, Median and Mode are Equal

Location is determined by the mean, μ

Spread is determined by the standard deviation, σ

The random variable has an infinite theoretical range: + ∞ to – ∞



Mean = Median = Mode

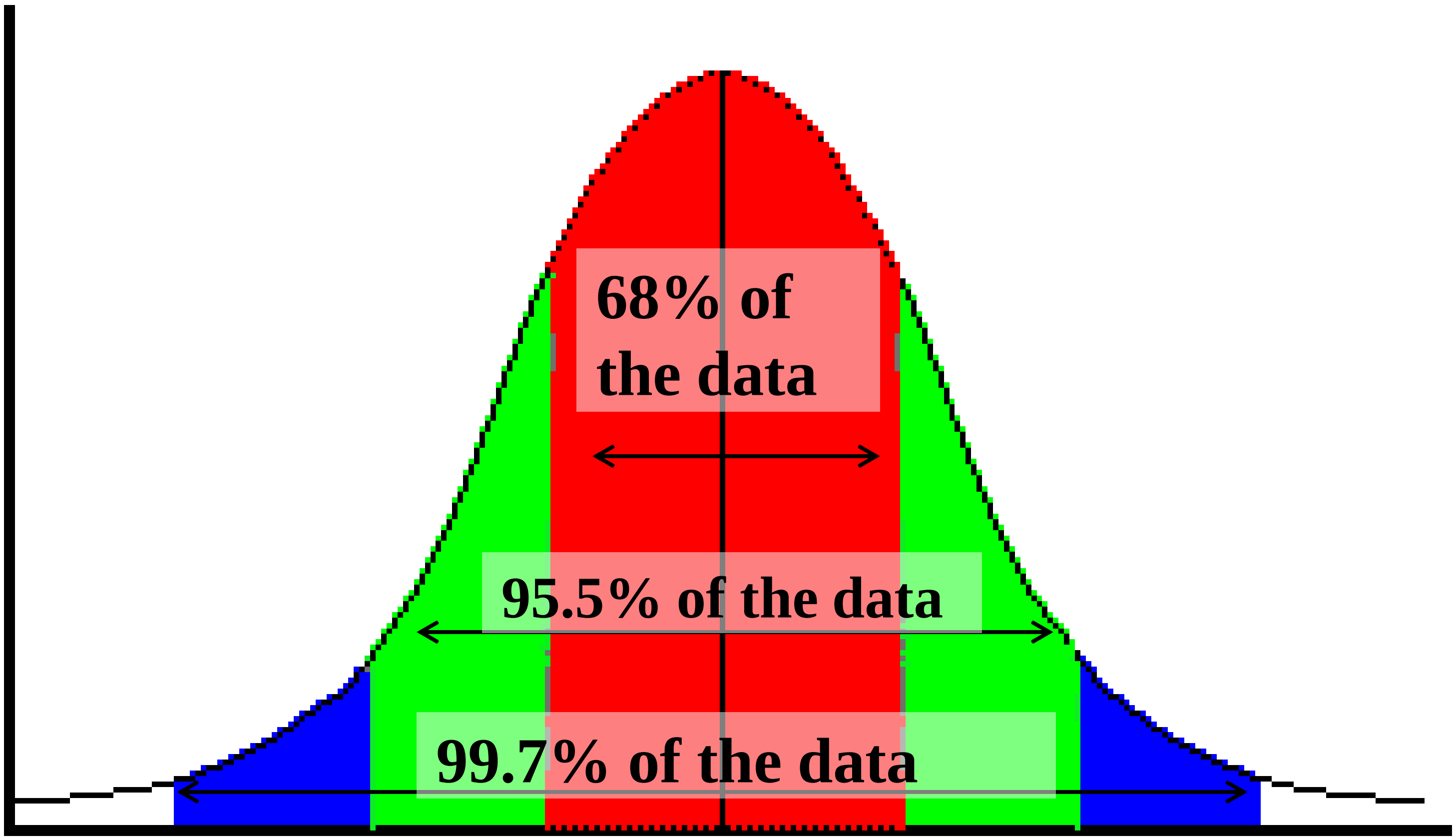
NORMAL DISTRIBUTION

- It is a continuous PD i.e. random variable can take on any value within a given range. Ex: Height, Weight, Marks etc.
- Developed by eighteenth century mathematician – astronomer Karl Gauss, **so also called Gaussian Distribution.**
- It is symmetrical, unimodal (one peak).
- Since it is symmetrical, its mean, median and mode all coincides i.e. all three are same.
- The tails are asymptotic to horizontal axis i.e. curve goes to infinity without touching horizontal axis.
- X axis represents random variable like height, weight etc.
- Y axis represents its probability density function.
- Area under the curve tells the probability.
- The total area under the curve is 1 (or 100%)
- Mean = μ, SD = σ

DEFINING A NORMAL DISTRIBUTION

- Only two parameters are considered: **Mean & Standard Deviation**
  - Same Mean, Different Standard Deviations
  - Same SD, Different Means
  - Different Mean & Different Standard Deviations

68-95-99.7 RULE



Area Under The Curve

- The mean ± 1 standard deviation covers approx. 68% of the area under the curve
- The mean ± 2 standard deviation covers approx. 95.5% of the area under the curve
- The mean ± 3 standard deviation covers 99.7% of the area under the curve

STANDARD NORMAL PD

- In standard Normal PD, Mean = 0, SD = 1
- $Z = \frac{x - \mu}{\sigma}$
  - Z = No. of std. deviations from x to mean. Also called Z Score

Let X be random variable which follows normal distribution with mean m and variance s2 .The standard normal variate is defined as which follows standard normal distribution with mean 0 and standard deviation 1 i.e., Z ~ N(0,1).

Example 6: In a normal distribution whose mean is 12 and standard deviation is 2. Find the probability for the interval from x = 9.6 to x = 13.8

Solution

Given that Z~ N (12, 4) 2

$$P(1.6 \leq Z \leq 13.8) = P\left(\frac{9.6-12}{2} \leq Z \leq \frac{13.8-12}{2}\right)$$

$$= P(-1.2 \leq Z \leq 0) + P(0 \leq Z \leq 0.9)$$

$$= P(0 \leq Z \leq 1.2) + P(0 \leq Z \leq 0.9) \quad \text{[by using symmetric property]}$$

$$= 0.3849 + 0.3159$$

$$= 0.7008$$

When it is converted to percentage (ie) 70% of the observations are covered between 9.6 to 13.8.

Example 7: For a normal distribution whose mean is 2 and standard deviation 3. Find the value of the variate such that the probability of the variate from the mean to the value is 0.4115

Solution:

Given that Z~ N (2, 9)

To find X_1:

We have $P(2 \leq Z \leq X_1) = 0.4115$

$$P\left(\frac{2-2}{3} \leq \frac{X-\mu}{\sigma} \leq \frac{X_1 - 2}{3}\right) = 0.4115$$

$$P(0 \leq Z \leq Z_1) = 0.4115 \text{ where } Z_1 = \frac{X_1 - 2}{3}$$

[From the normal table where 0.4115 lies is rthe value of Z_1]

Form the normal table we have $Z_1 = 1.35$

$$\therefore 1.35 = \frac{X_1 - 2}{3}$$

$$\Rightarrow 3(1.35) + 2 = X_1$$

$$= X_1 = 6.05$$

(i.e) 41 % of the observation converged between 2 and 6.05

Practice Problems – Normal Distribution

- Mean height of gurkhas is 147 cm with SD of 3 cm. What is the probability of:
  - Height being greater than 152 cm. 4.75%
  - Height between 140 and 150 cm. 83.14%

- Mean demand of an oil is 1000 ltr per month with SD of 250 ltr.
  - If 1200 ltrs are stocked, What is the satisfaction level? 78%
  - For an assurance of 95%, what stock must be kept? 1411.25 ltr

- Nancy keeps bank balance on an average at Rs. 5000 with SD of Rs. 1000. What is the probability that her account will have balance of :
  - Greater than Rs. 7000 0.0228
  - Between Rs. 5000 and Rs. 6000

Questions

1. For a Poisson distribution

(a) mean > variance (b) mean = variance

(c) mean < variance (d) mean < variance

Ans: mean = variance

2. In normal distribution, skewness is

(a) one **(b) zero**

(c) greater than one (d) less than one

Ans: zero

3. Poisson distribution is a distribution for rare events

Ans: True

4. The total area under normal probability curve is one.

Ans: True

5. Poisson distribution is for continuous variable.

Ans: False

6. In a symmetrical curve mean, median and mode will coincide.

Ans: True