Introduction to Algebraic Information Theory for Quantitative Finance
Homework 3

August 30, 2025

Timothy Tarter
James Madison University
Department of Mathematics

1. Prove that if D is an integral domain, and $D[x]$ is an polynomial integral domain,
   - $p, q \in D[x]$ implies that $\partial(p+q) \leq \max\{\partial p, \partial q\}$.
   - $\partial(pq) = \partial p + \partial q$.
2. Find $\gcd(x^3 + x - 2, x^7 - x)$.
3. Find $\gcd(x^4 + x^2 - 11, x^3 - 4)$.
4. Find $\gcd(x^2 + 17, x^4 - x^2)$.
5. Show that $\mathbb{R}[x]/\langle x^3 + x + 1\rangle$ is a field, and find what field it is isomorphic to via FHT.
6. Show that $\mathbb{Z}[x]$ is not a PID.
7. Show that $g = 7x^4 + 10x^3 - 2x^2 + 4x - 5$ is irreducible over $\mathbb{Q}$.
8. Show that $x^3 + 3x + 12$ is irreducible over $\mathbb{Q}$.
9. Show that $5x^4 - 10x^3 + 10x - 3$ is irreducible.
10. Show that $3x^4 - 7x + 5$ is irreducible.
11. Prove Gauss's lemma.
12. Prove that $\mathbb{Q}[\sqrt{2}]$ is a vector space. What is its basis? What is $[\mathbb{Q}[\sqrt{2}] : \mathbb{Q}]$?
13. Prove that if $L : K$ and $M : L$, $[M : L][L : K] = [M : K]$.
14. Show that $\mathbb{Q}$ is a subfield of $\mathbb{Q}[\sqrt{2}]$.