Introduction to Algebraic Information Theory for Quantitative Finance
Homework 2

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1. Prove that the ring $\mathbb{Z}_{82} \simeq \langle 41 \rangle \times \langle 2 \rangle$.
2. Prove that if $R = \langle I_1 \rangle \times \cdots \times \langle I_n \rangle$ then $|R| = |\langle I_1 \rangle| \times \cdots \times |\langle I_n \rangle|$ iff R is a PID.
3. For the following rings, determine if they are ID, PID, ED, or Fields:
   - The ring of upper triangular matrices over $\mathbb{C}$, with $+$ as vector addition and $\times$ as matrix multiplication / the standard inner product.
   - $\mathbb{Z}$
   - $\mathbb{Z}_{32}$
   - $\mathbb{Z}_{41}$
4. Find 787^{-1} mod 1447 and 1447^{-1} mod 787.
5. Find 1061^{-1} mod 997 and 997^{-1} mod 1061.
6. Find 619^{-1} mod 2003 and 2003^{-1} mod 619.
7. Find 1567^{-1} mod 23 and 23^{-1} mod 1567.
8. Find 6473^{-1} mod 4973 and 4973^{-1} mod 6473.
9. Find 42^{87} mod 79.
10. Find 96^{114} mod 23.
11. Find 5162^{242} mod 240.
12. Find 56^{2003} mod 2000.
13. Solve $x^2 \equiv 14 \bmod 17$, if such roots exist.
14. Find all the ideals of the ring $\mathbb{Z}_{231}$, and for each such ideal I, compute $\mathbb{Z}_{231}/I$. What is it isomorphic to by FHT?

15. For the previous problem: is $\mathbb{Z}_{231}$ an ID, PID, ED, or field? What about the quotient groups you computed?