Madison Institute for Mathematical Finance: Week Two Homework

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PLEASE NOTE:

This homework is not complete yet because I (Tim) am writing it day by day. Check DAILY for an updated list of problems on Github.

Problem 1: For each of the following lists of vectors, determine if a list is a basis using the long way (show linear independence and spanning, or lack thereof). Then, try the short way ($A^T A$ is diagonal iff the columns of A are an orthogonal basis for V). Can you conclude orthogonality or orthonormality?

0.0.1 Problem 1.1

$$\beta = \{ \begin{bmatrix} 4 \\ 5 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \\ -1 \end{bmatrix} \} \tag{1}$$

0.0.2 Problem 1.2

$$\beta = \{ \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \} \tag{2}$$

0.0.3 Problem 1.3

$$\beta = \{ \begin{bmatrix} 1 \\ 5 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \\ -5 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ -4 \\ -2 \end{bmatrix} \} \tag{3}$$

0.0.4 Problem 1.4

$$\beta = \{\begin{bmatrix} -3 \\ -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -3 \\ 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ -1 \\ 2 \end{bmatrix}\} \tag{4}$$

Problem 2: Write a short explanation, using an example from question 1, about why we can't just use the A^TA trick to check if something is a basis. What *can* we use it for?

Problem 3: Come up with an orthogonal basis for $\mathbb{R}^3$ like we did in class, then make it orthonormal.

Problem 4: Check to make sure that you can multiply the matrices below, then, if posisble, perform the matrix multiplication.

Problem 4.1

$$\begin{bmatrix} 3 & 4 \\ -1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ -1 & -2 & 5 \end{bmatrix} \tag{5}$$

Problem 4.2

$$\begin{bmatrix} -1 & 0 & 11 \\ 2 & -2 & -4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 & -1 \\ 5 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \tag{6}$$

Problem 4.3

$$\begin{bmatrix} 0 & 2 \\ -1 & 5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix} \tag{7}$$

Problem 5: Briefly explain the geometric interpretation of changing the frame of reference (via change of basis) of a vector. How does it relate to solving systems of linear equations (with Gaussian elimination)?

Problem 6: Solve the following matrix-vector equations with Gaussian Elimination. Then do it again by inverting the matrix, and use A^{-1} to check your work.

Problem 6.1

$$\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \tag{8}$$

Problem 6.2

$$\begin{bmatrix} 4 & 5 & -2 \\ 1 & -1 & 3 \\ -1 & 4 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \tag{9}$$

Problem 6.3

$$\begin{bmatrix} -1 & 4 & 2 \\ 0 & 1 & 5 \\ 1 & 7 & -3 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ -3 \\ 11 \end{bmatrix} \tag{10}$$

Problem 7: Draw a picture explaining the image and kernel of a matrix, $A : V \to W$. Write a little bit (not much) explaining what you understand about what we talked about in class.