Madison Institute for Mathematical Finance
Comprehensive Final Exam Question Bank

November 21, 2025

Timothy Tarter
James Madison University
Department of Mathematics

Any of the following questions could be pulled to be on the actual final exam which will be administered on the Monday night when we get back from break (December 1st). You are allowed one 8.5”×11” cheat sheet, front and back. It is advised that you use this cheat sheet for definitions, but you are welcome to put anything that you think might be helpful. The exam will technically be for three hours, but I'm willing to stay as late as needed for anybody who wants more time. Additionally, the exam will be collaborative in nature for sections 2 and 3. Everyone will have to turn in their own exam paper, but you are all welcome to collaborate (but not copy each other) on computational questions and proofs. I will do my best to pick problems which round out to a three-hour exam - I don't want any of you to stress too much about this! The goal is for you to have references of what you've learned this semester, and I want this to feel rewarding - not terrifying.

1 Definitions & Theorems

State the following fully, with precise mathematical notation.

1. FTC 1 & 2.
2. Green's Theorem.
3. Stokes' Theorem.
4. Orthogonal and orthonormal vectors.
5. The lemma for showing that the columns of a matrix form an orthogonal or orthonormal basis for a vector space.
6. Change of basis formula (with inner products).
7. Normal matrix.
8. Unitary matrix.
9. Hermitian / self-adjoint matrix.

10. Positive semi-definite matrices.
11. Algorithm for finding a basis for the image and kernel of a matrix.
12. Spectral theorem for finite operators (include assumptions).
13. Singular value decomposition.
14. Least upper bound property.
15. Archimedean property and density.
16. Open ball in the standard topology on $\mathbb{R}$.
17. Boundary point.
18. Limit point.
19. Sequence convergence.
20. Cauchy sequence.
21. Subsequence.
22. Sequence lemma.
23. Epsilon-delta continuity of functions (include a drawing - it helps!)
24. Topological continuity.
25. The Heine-Borel Theorem
26. The Extreme Value Theorem (in terms of compactness).
27. Series convergence.
28. The Riemann-Stieltjes integral (this starts with creating a partition of the domain).
29. Sigma-algebra.
30. Borel sigma-algebra.
31. Lebesgue outer and inner measure.
32. Darboux sums (upper and lower).
33. Criterion for a set to be Riemann measurable.
34. Criterion for a set to be Borel-measurable.
35. Borel-measurable function.
36. State the Ito integral, with a complete setup.
37. Define a Martingale process from measure-theoretical foundations.

2 Computational Problems

1. Check if the columns of a provided 3×3 matrix form a basis for $\mathbb{R}^3$.
2. Compute the SVD of a provided 3×3 matrix.
3. Compute the spectral decomposition of a provided 3×3 matrix.
4. Find a basis for the image and kernel of a provided 3×3 matrix.
5. Solve a provided non-homogeneous second order differential equation with method of undetermined coefficients.
6. Solve a provided non-homogeneous second order differential equation with the Laplace transform.
7. Given a system of linear first order ODE's, determine its behavior with the trace determinant plane.
8. Compute the Lagrange polynomial for a set of three points, (x, y).
9. Compute the probability of getting a flush in poker (texas-hold-em).
10. In a ballroom dancing class the students are divided into group A and group B. There are six people in group A and seven in group B. If four A's and four B's are chosen and paired off, how many pairings are possible?
11. If we roll two dice and take our random variable to be the product of the faces, what is the expected value of any given dice roll? What is the probability of getting $X = 7$? What is the variance of any given roll?
12. Prove that the expected value of the exponential distribution is $\frac{1}{\lambda}$.
13. Show that the sequence $\{\frac{1}{n}\}_{n=1}^{\infty} \to 0$.
14. Show that the sequence $\{(-1)^n\}_{n=1}^{\infty} \not\to m$ for any m.
15. Show that $f(x) = x^2$ is continuous on the interval $[1, 2]$ using both an epsilon-delta argument and a topological continuity argument.
16. Prove that the Dirichlet function is not Riemann measurable, but is Lebesgue measurable. Conclude that there is a functional use for the Lebesgue integral.

3 Conceptual Questions

1. Explain the Central Limit Theorem.
2. What's right / wrong with $\mathbb{Q}$? How does $\mathbb{R}$ fix $\mathbb{Q}$?
3. Briefly explain Cantor's argument for either the uncountability of $\mathbb{R}$ or the countability of $\mathbb{Q}$.
4. Prove that all convergent sequences are bounded.

5. Provide a counterexample which shows that there exist bounded sequences which are not convergent.
6. Prove that a sequence is Cauchy iff it is convergent.
7. Explain how we proved the extreme value theorem - this should take at least two pages, but does not have to be entirely rigorous. Just state the theorems used and a proof sketch of each theorem or a reference to a previous proof sketch in this exam.
8. Show that any compact set in $\mathbb{R}^1$ is a sigma-algebra. Is it Riemann measurable? How about Lebesgue measurable?
9. Derive a Brownian process is from a measure theoretical standpoint.
10. Derive the Black-Scholes Merton Equation. Reconcile our "measure-theoretically precise" definition of a Brownian process with the one used in our derivation of BSM.
11. State 3 fundamental assumptions of BSM which break in real life. Do you think that we can still reasonably use the BSM model to prices financial derivatives?