

Madison Institute for Mathematical Finance: Week Three Homework

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Problem 1: We started on some first order homogeneous method of undetermined coefficient problems in class on day one. If you didn't finish your problem in class, get it from this list and finish it now!

$$3y' - \frac{1}{2}y = e^x(4x - 1)$$

or

$$y' + 7y = 4\cos(2\pi x) - e^{2x}$$

Problem 2: Solve the following DE's with method of undetermined coefficients. (Hint: start with the homogeneous case!)

Problem 2.1

$$y' - y = 3x^2 \tag{1}$$

Problem 2.2

$$y'' - y = 5 \tag{2}$$

Problem 2.3

$$y'' + y - 3 = e^{-x} \tag{3}$$

Problem 3: Solve the following DE's with the Laplace transform.

Problem 3.1

$$y'' - y' + y = 0 \quad (4)$$

Problem 3.2

$$4y'' + 2y = e^{5x} \quad (5)$$

Problem 3.2

$$4y'' + 2y = \sin(3x) \quad (6)$$

Problem 4: Use the Trace-Determinant plane to figure out what type of DE we are dealing with, then solve for the closed-form solution. Graph the associated phase plane.

Problem 4.1

$$\frac{dx}{dt} = x - 3y \quad (7)$$

$$\frac{dy}{dt} = 2x + 4y \quad (8)$$

Problem 4.2

$$\frac{dx}{dt} = 3x - y \quad (9)$$

$$\frac{dy}{dt} = -x - 7y \quad (10)$$