MIMF Fall 2025 Syllabus

September 8, 2025

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Timothy's General Lecture Dates: Monday/Thursday

Nicholas' General Lecture Dates: Tuesday/Wednesday

Week of September 1

Week 1: Calculus

- (M) Instantaneous Rate of Change (Timothy)
  - Limit definition of a derivative
- (M) Derivatives (Timothy)
  - Visualization and intuition
  - Rules for derivatives
- (T, W) Integrals (Nicholas)
  - Visualization and intuition
  - Integration techniques
- (W, TH) Partial Derivatives (Nicholas)
- (TH) Single-Variable Optimization (Timothy)
- (TH) Gradient Vector, Lagrange Multipliers (Timothy)
- (TH) Contour and Surface Integrals (Timothy)
- (TH) Green's theorem, Divergence, and the 3D Heat Equation (Timothy)

Homework Topics:

- Programming Assignment: Set up VS Code (yes VS Code, even if you've used another IDE in the past)
- Monday: Derivative rules practice
- Tuesday: Integrals (U-sub)
- Wednesday: Integrals (IBP), Partial Derivatives
- Thursday: Optimization

Week of September 8

Week 2: Linear Algebra

- (M) What is a vector? (Timothy)
- (M) What is a vector space? Inner product space? (Timothy)
- (M) Bases: what are they good for? (Timothy)
- (M, T) Linear transformations (Timothy)
  - (M) Orthogonality goes HERE
  - (M, T) Inner products: the dot product and trigonometry
  - (T) Matrix multiplication as repeated inner products
  - (T) Change of basis
  - (T) Show that the trivial basis is the identity transformation
  - (T) Show that any basis can be inverted to the trivial basis
- (T) Is there an easy way to show if vectors are a basis? (Timothy)
  - (T) Row reduction
  - (T, W) $A^T A = diag(\cdot)$ iff orthogonal
- (W) Orthonormality (Nicholas)
- (W) Riesz Representation Formula (Nicholas)
- (W) All of the types of matrices (normal, unitary (orthogonal if $\mathbb{R}$), positive-semidefinite, symmetric, etc.) (Nicholas)
- (W) Finding a basis for the kernel and image of a matrix (Timothy)
- (W) Eigenvectors and Eigenvalues (Timothy)
- (TH) Spectral theorem for finite operators (Timothy)
- (TH) LU decomposition and SVD decomposition (Nicholas)

Homework Topics:

- Programming Assignment: use Numpy and/or SageMath to determine if a list of vectors form a basis for a vector space, compute inner products, multiply matrices, and LU decomposition.
- Monday: Are these vectors orthogonal? Inner product practice. Is it obviously a basis? Brief essay explaining topics covered. (∼200 words)
- Tuesday: Matrix multiplication practice. Change of basis practice. Row reduction practice. "Is it an orthogonal basis?" practice (the easy way).
- Wednesday: Is it an orthonormal basis? Essay on properties of normal, unitary, and pos-semi-def matrices (∼400 words). Finding a basis for the kernel and image of a matrix. Finding the eigenvectors and eigenvalues of a matrix.
- Thursday: LU decomposition practice, spectral decomposition practice, SVD practice.

Week of September 15

Week 3: Differential Equations

- (M) Linear ODEs (Timothy)
- (M) Non-linear ODEs (Timothy)
- (M, T) Integrating factor method (Timothy)
- (T) Path Integrals (Timothy)
- (T) Laplace & Fourier Transforms (Timothy)
- (W) Phase plane stuff (Nicholas)
- (W, TH) Linear Systems of ODEs (Timothy)
- (TH) Population models & introduction to Monte-Carlo simulation (Timothy)

Homework Topics:

- Programming Assignment: Use Monte-Carlo simulation to build a Lotka-Volterra model in Python.
- Monday: Basic solution practice (more calc II).
- Tuesday: Integrating factor method practice, path integral practice, Laplace solution practice.
- Wednesday: Algebraic solutions to systems of ODEs, eigenvalue solutions to systems of ODEs.
- Thursday: More systems of ODEs practice and population model practice.

Week of September 22

Week 4: Numerical Methods

- (M) Runge-Kutta ODE Solver (Timothy)
- (M) Gröbner basis (Timothy)
- (T) Lagrange Interpolation (Nicholas)
- (T) Shamir Secret Sharing (Timothy)
- (W) Newton-Raphson in depth (Nicholas)
- (W) Monte-Carlo simulation (Nicholas)
- (TH) Simplex method (Timothy)
- (TH) Network Optimization (Timothy)

Homework Topics:

- Monday: Pick either RK-ODE solver or Grobner basis and implement in Python.
- Tuesday: Implement the Shamir Secret Sharing algorithm in Python.
- Wednesday: Use Monte-Carlo simulation to figure out the probability of any occurrence of a combination of three dice.
- Thursday: Write a program that implements the Simplex method on a set of linear utility functions with linear constraints.

Week of September 29

Week 5: Probability and Stochastic Processes

- (M) Discrete Distributions (Timothy)
  - Combinatorics
  - Binomial
  - Geometric
  - Negative Binomial
  - Hyper-Geometric
  - Poisson
  - Discrete Uniform
  - Expected Values for discrete
- (T) Continuous Distributions (Timothy)
  - Uniform
  - Binomial

  - Normal (standard and otherwise)
  - Lognormal
  - Expected values for continuous
- (W) Applications (Co-Lecture)
  - Binomial asset pricing model (Nicholas)
  - Delta hedging (Timothy)
  - Black-Scholes Merton (Timothy)
  - Blackjack and Poker (Timothy)

  (Co-lecture)
- (TH) Markov chains (Timothy)
- (TH) Type 1 and 2 error as validation metrics (Timothy)

Homework Topics:

- Monday: Boilerplate computational work.
- Tuesday: Boilerplate computational work.
- Wednesday: Implement Delta-Hedging in Excel.
- Thursday: Let there be a keypad, 0-9. Put a chess knight on zero. Move randomly; find the steady-state distribution for the knight at any point in time. Find the probability that $P(K_t = 3)$ if $t = 100$. Find a markov process and implement it in Python, write a short 1-2 page report.

Week of October 6

Week 6: Real Analysis

- (M) Basic set theory stuff (definitions and concepts)
- (M) What's right with $\mathbb{Q}$ (Field Axioms) (Timothy)
- (M) What's wrong with $\mathbb{Q}$ (Timothy)
- (M) Infinity (and why it's a lot weirder than you think) (Nicholas)
- (M) Cardinality of $\mathbb{N}$ vs. $\mathbb{R}$ (Nicholas)
- (M) Least upper bound property, and how $\mathbb{R}$ fixes $\mathbb{Q}$ (Timothy)
- (M) Archimedean property, density, triangle inequality (Timothy)
- (T) Standard metric topology (of $\mathbb{R}$) (Timothy)
- (T) Compactness (Timothy)
- (W) Sequences (Timothy)

- (W) Cauchy Sequences and completeness (Timothy)
- (TH) Functions and continuity (Timothy)
- (TH) Topological continuity (Timothy)
- (TH) Compactness the Extreme Value Theorem (Timothy)
- (TH) Differentiability (Timothy)

Homework Topics:

- Monday: Prove DeMorgan's laws. Show that the Gaussian integers have the same cardinality as $\mathbb{Z}$. Explain Cantor's arguments for countable and uncountable infinities in your own words. Explain how $\mathbb{R}$ fixes $\mathbb{Q}$ IN-DEPTH (2-3 pages). Compute some boilerplate LUBs of sets. Prove that countable unions of countable sets are countable. Prove that uncountable unions of countable sets are not necessarily countable. Prove that $\mathbb{P}(\mathbb{Z})$ has the same cardinality as $\mathbb{R}$. Prove that $inf\{\frac{1}{n}|n \in \mathbb{Z}^+\}$.
- Tuesday: Borrow some questions from one of Tim's practice exams.
- Wednesday: Prove Cauchy iff Convergent. Show that $\{\frac{1}{n}\}_{n\in\mathbb{N}} \to 0$. Show that if $\{a_n\} \to a$ and $\{b_n\} \to b$, that $\{a_n b_n\} \to ab$. Summarize our proof for the sequence lemma. Show that the set of accumulation points of a sequence is closed.
- Thursday: Explain why every function $f : \mathbb{Z} \to \mathbb{R}$ is automatically continuous on $\mathbb{Z}$. Show that $f(x) = x^2$ is a continuous function on $\mathbb{R}$. Show that $f(x) = x^2$ is differentiable on $\mathbb{R}$, and use the $\varepsilon - \delta$ definition of a derivative to differentiate it. Same thing for $f(x) = \frac{sin(x)}{x}$. Use topological continuity to show that if A is closed in $\mathbb{R}$, and $f : A \to \mathbb{R}$, f is a continuous function. Prove that closed and bounded in $\mathbb{R}^n$ iff compact. Write a 5-page essay explaining what you learned this week and why you think Real Analysis is important.

Week of October 13

Week 7: Measure Theory (Timothy)

- (M) Summation and sum convergence
- (M) The Riemann-Stiltjes Integral
  - Darboux sums and left/right integration
- (T) Measure
  - Sigma-algebras
  - Induced measure
  - Lebesgue outer/inner measure
- (T) Lebesgue integration
  - The Dirichlet function

- (W) Borel Measurability
- (W) Measurable Functions

Homework Topics:

- Programming Assignment: Write a python program to compute the Lebesgue outer and inner measure of a set, and determine if it's Lebesgue measurable. Do the same for left and right Darboux sums to show Riemann integrability.
- Monday: Show that the power series for e^x converges to e^x. Prove that a series that is not absolutely convergent can converge to any real number. Practice some basic series convergence proofs. Do some Riemann-Stiltjes Integrals; do some Ito integrals too, but call them RS integrals - we'll talk about it next week.
- Tuesday: Show that any compact set in $\mathbb{R}$ is a σ-algebra. Practice determining if a set is Lebesgue measurable. Practice Lebesgue integrals.
- Wednesday: Practice showing that a set / function is Borel measurable. Write a short essay explaining why we care about Borel measurability of functions when thinking about probability theory. Write a 2-4 page essay explaining what you've learned about measure theory and how you think it plays into QFIN.
- Thursday: Catch up on any work you haven't finished.

Community Night: October 18

Week of October 27

Week 8: Stochastic Differential Equations (Timothy)

- (M) What is a random variable, really?
- (M) Probability and measure
- (M) Analogues of ODEs
- (M) Chebychev's inequality
- (M) Borel-Cantelli lemma
- (M) Gaussian processes & Brownian motion
- (T) The Ito Integral
- (T) Ito's Lemma (revisited) and fully deriving BSM
- (W) Martingales and the Martingale Representation Theorem
- (TH) Existence and Uniqueness theorem
- (TH) SDE Examples

Homework Topics:

- Programming Assignment: Implement an option-pricing model using Monte-Carlo simulation, the martingale assumption, and Brownian / Geometric Brownian Motion.
- Monday: Explain what Tim said about why we're learning measure theory. Practice simple linear stochastic ODEs. Explain how we proved Chebychev's inequality and the Borel-Cantelli lemma. Prove that $\tilde{B}_t = B_{t_0+t} - B_{t_0}$ is a Brownian motion if B_t is Brownian and $t_0 \geq 0$.
- Tuesday: Compute a ton of Ito integrals, assuming RS integrability. Use Ito integrals to price options assuming a certain prior distribution.
- Wednesday: Derive Ito's lemma. Explain how we proved the Martingale Representation theorem (2-3 pages). Explain how the MRT is analogous to the Riesz Representation formula if we assume a steady state distribution. Compute $\int_0^t B_s^2 dB_s = \frac{1}{3}B_t^3 - \int_0^t B_s ds$. Let X_t, Y_t be Ito proceses in $\mathbb{R}$. Prove that $d(X_t Y_t) = X_t dY_t + Y_t dX_t + dX_t * dY_t$. Deduce the stochastic IBP formula, $\int_0^t X_s dY_s = X_t Y_t - X_0 Y_0 = \int_0^t Y_s dX_s - \int_0^t dX_s * dY_s$.
- Thursday: Take a bunch of SDE examples from page 77 of Oksendal.

Week of November 3

Week 9: Stochastic Simulation Techniques

- (M) Bayesian inference (Timothy)
- (M) Kalman filtering (Timothy)
- (T) Integration by simulation (Nicholas)
- (W) Power-series solutions for ODEs and stochastic ODEs (Timothy)

Homework Topics:

- Monday: Implement a Kalman filter for finding the optimal search path to intercept a boat in a harbor based on a set of fixed points that it must visit in the harbor and an explicit prior distribution.
- Tuesday: Work on Monday's project.
- Wednesday: Implement a power-series approximation for your earlier implementation of Monte-Carlo simulation for the Lotka-Volterra equation model (week 3).
- Thursday: Work on Wednesday's project.

Week of November 10

Week 10: Econometrics and Time Series

- (M) Hypothesis testing (Nicholas)
- (M) Sample analogues of probability theory methods (Nicholas)

- (T) Gauss-Markov assumptions for linear models and the 3 things your model SHOULD NOT EVER HAVE IF YOU PLAN ON USING IT (Timothy)
- (T) What to do about heteroskedasticity (Timothy)
- (W) Vector Auto-Regression (Nicholas)
- (W) ARCH and GARCH (Timothy)
- (W) ARIMA (Timothy)

Homework Topics:

- Monday: Practice rote hypothesis testing.
- Tuesday: Write a short paper (1-2 pages) explaining each of the GM assumptions. Find a Kaggle dataset, build a model, discuss it (3-4 pages).
- Wednesday: Do the same thing, but for a time series dataset.

Review Session: Thursday, November 13

Cumulative Exam: Friday, November 14

Celebration Event: Saturday, November 15

Research Brainstorming: Monday and Tuesday November 17-18

Research Pitch: Wednesday November 19

Mon-Thu Study Halls

Final Pitches: Last Week of Classes