

# Madison Institute for Mathematical Finance: Week Two Homework

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*PLEASE NOTE:*

*This homework is not complete yet because I (Tim) am writing it day by day. Check DAILY for an updated list of problems on Github.*

**Problem 1:** For each of the following lists of vectors, determine if a list is a basis using the long way (show linear independence and spanning, or lack thereof). Then, try the short way ( $A^T A$  is diagonal iff the columns of  $A$  are an orthogonal basis for  $V$ ). Can you conclude orthogonality or orthonormality?

0.0.1 Problem 1.1

$$\beta = \left\{ \begin{bmatrix} 4 \\ 5 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \\ -1 \end{bmatrix} \right\} \quad (1)$$

0.0.2 Problem 1.2

$$\beta = \left\{ \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\} \quad (2)$$

## 0.0.3 Problem 1.3

$$\beta = \left\{ \begin{bmatrix} 1 \\ 5 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -3 \\ -5 \end{bmatrix}, \begin{bmatrix} 6 \\ 8 \\ -4 \\ -2 \end{bmatrix} \right\} \quad (3)$$

## 0.0.4 Problem 1.4

$$\beta = \left\{ \begin{bmatrix} -3 \\ -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -1 \\ 2 \end{bmatrix} \right\} \quad (4)$$

**Problem 2:** Write a short explanation, using an example from question 1, about why we can't just use the  $A^T A$  trick to check if something is a basis. What *can* we use it for?

**Problem 3:** Come up with an orthogonal basis for  $\mathbb{R}^3$  like we did in class, then make it orthonormal.

**Problem 4:** Check to make sure that you can multiply the matrices below, then, if possible, perform the matrix multiplication.

## Problem 4.1

$$\begin{bmatrix} 3 & 4 \\ -1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ -1 & -2 & 5 \end{bmatrix} \quad (5)$$

## Problem 4.2

$$\begin{bmatrix} -1 & 0 & 11 \\ 2 & -2 & -4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 & -1 \\ 5 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad (6)$$

## Problem 4.3

$$\begin{bmatrix} 0 & 2 \\ -1 & 5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix} \quad (7)$$