Madison Institute for Mathematical Finance: Week One Homework

September 2, 2025

Timothy Tarter
Nicholas Harsell

James Madison University
Department of Mathematics

Below are the homework problems for week one of the MIMF lecture process. They are due by the following class, and should (hopefully) not take very long to complete. Most problems are graded, though some are only highly recommended. Please do not hesitate to reach out to a lecturer if you have questions!

Differentiation Problems

Graded Problems: 1, 2, 3, 8, 11, 14, 16, 19

1. $g(x) = 8 - 4x^3 + 2x^8$
2. $f(z) = z^{10} - 7z^5 + 2z^3 - z^2$
3. $y = 8x^4 - 10x^3 - 9x + 4$
4. $f(x) = 3x^{-4} + x^4 - 3x$
5. $R(t) = 9t^{10} + 8t^{-10} + 12$
6. $h(y) = 3y - 6y^3 - 8y^{-3} + 9y^{-1}$
7. $g(t) = t^7 + 2t^3 - 6t^{-2} + 8t^4 - 1$
8. $z = \sqrt{x} - 7\sqrt[4]{x} + 3\sqrt[3]{x}$
9. $f(x) = 7\sqrt{x^4} - 2\sqrt[6]{x^7} + \sqrt[3]{x^4}$

10. $h(y) = 6\sqrt{y} + \sqrt[4]{y^5} + \dfrac{7}{\sqrt[3]{y^2}}$

11. $g(z) = \dfrac{4}{z^2} + \dfrac{1}{7z^5} - \dfrac{1}{2z}$

12. $y = \dfrac{2}{3t^9} + \dfrac{1}{7t^3} - 9t^2 - \sqrt[3]{t^3}$

13. $W(x) = x^3 - \dfrac{1}{x^{16}} + \dfrac{1}{\sqrt[3]{x^2}}$

14. $g(w) = (w-5)(w^2+1)$

15. $h(x) = \sqrt{x}(1-9x^3)$

16. $f(t) = (3-2t^3)^2$

17. $g(x) = (1+2x)(2-x+x^2)$

18. $y = \dfrac{4-8x+2x^2}{x}$

19. $Y(t) = \dfrac{t^4-2t^2+7t}{t^3}$

20. $S(w) = \dfrac{w^2(2-w)+w^5}{3w}$

U-Substitution Practice Problems

All Graded

1. $\displaystyle\int (3x^2+2)\sqrt{x^3+2x+5}\,dx$

2. $\displaystyle\int_1^3 x\cos(x^2)\,dx$

3. $\displaystyle\int \frac{e^{5x}}{1+e^{5x}}\,dx$

4. $\displaystyle\int 4x(x^2+7)^5\,dx$

5. $\displaystyle\int_0^{\ln 2} \frac{e^x}{1+e^x}\,dx$

6. $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$

7. $\displaystyle\int \frac{5x^4}{(x^5+1)^3}\,dx$

8. $\displaystyle\int_0^{\pi} \cos(2x+1)\,dx$

9. $\displaystyle\int x\sqrt{1+x^2}\,dx$

10. $\displaystyle\int \frac{x^3}{(x^4+9)^2}\,dx$

Integration by Parts

Graded Problems: 1-9

1. $\displaystyle\int 8te^{7t}\,dt$

2. $\displaystyle\int_{\pi}^{2\pi} (1-3x)\sin\left(\frac{1}{2}x\right)\,dx$

3. $\displaystyle\int_{-1}^{2} w^2e^{4w}\,dw$

4. $\displaystyle\int_{-2}^{3} (2-x^2)\ln(4x)\,dx$

5. $\displaystyle\int (6+3z)\cos(1+4z)\,dz$

6. $\displaystyle\int \sqrt{x^3}\ln(\sqrt{x^3})\,dx$

7. $\displaystyle\int (2w^2-w)e^{7w-1}\,dw$

8. $\displaystyle\int_0^{\frac{\pi}{2}} e^{-x}\sin(4x)\,dx$

9. $\displaystyle\int e^{4t}\cos(2t)\,dt$

10. $\int e^{3-x^2} \sin(2+z)\, dz$

11. $\int_{-1}^{0} 2x^{17} e^{1+x^2}\, dx$

12. $\int 9t^{11} \cos(1-t^6)\, dt$

Partial Derivative Practice Problems

<u>All Graded</u>

1. Given $f(x,y) = 3x^2 + 2y$, find $\frac{\partial f}{\partial x}$.
2. Given $f(x,y) = x^2y + y^3$, find $\frac{\partial f}{\partial y}$.
3. Given $f(x,y) = \sin(xy)$, find $\frac{\partial f}{\partial x}$.
4. Given $f(x,y) = e^{x+y}$, find $\frac{\partial^2 f}{\partial x \partial y}$.
5. Given $f(x,y,z) = xyz + x^2z^2$, find $\frac{\partial f}{\partial z}$.
6. Given $f(x,y) = \ln(x^2+y^2)$, find $\frac{\partial f}{\partial x}$.
7. Given $f(x,y) = \frac{x^2y}{x^2+y^2}$, find $\frac{\partial f}{\partial y}$.
8. Given $f(x,y,z) = x^2\cos(yz)$, find $\frac{\partial^2 f}{\partial y \partial z}$.
9. Given $f(r,\theta) = r^2\sin(\theta) + \theta^3\ln(r)$, find $\frac{\partial f}{\partial r}$.
10. Given $f(x,y) = e^{xy}\sin(x+y)$, find $\frac{\partial^2 f}{\partial x^2}$.

Gradient Vector Practice Problems

<u>All Graded</u>

Find the gradient vector, ∇f, of each function.

1. $f(x,y) = 3x^2 + 2y$
2. $f(x,y) = x^2y + y^3$
3. $f(x,y) = \sin(xy)$

4. $f(x, y) = e^{x+y}$
5. $f(x, y, z) = xyz + x^2z^2$
6. $f(x, y) = \ln(x^2 + y^2)$
7. $f(x, y) = \frac{x^2y}{x^2+y^2}$
8. $f(x, y, z) = x^2 \cos(yz)$
9. $f(r, \theta) = r^2 \sin(\theta) + \theta^3 \ln(r)$
10. $f(x, y) = e^{xy} \sin(x + y)$

Single-Variable Optimization Problems

Solve the following optimization problems. For each, identify the variable, set up the objective function, find critical points, and determine maximum or minimum values as appropriate.

1. **Geometry:** A farmer wants to build a rectangular pen next to a straight river. No fencing is needed along the river. If the farmer has 120 meters of fencing, what dimensions will maximize the area of the pen?
2. **Cost Minimization:** A cylindrical can is to hold 500 cm^3 of liquid. The material for the sides of the can costs 0.02 per cm^2, and the material for the top and bottom costs 0.05 per cm^2. What dimensions minimize the cost of the can?
3. **Revenue Maximization:** A theater charges \$10 per ticket and sells 300 tickets per night. Market research suggests that for every \$1 increase in ticket price, 20 fewer people will attend. What ticket price maximizes revenue?
4. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.