Math 248: Lab 6

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Problem 1

Fixed-Point Iteration Theorem: Suppose $g(x)$ and $g'(x)$ are continuous functions on $[a, b]$ where $p \in (a, b)$ is the unique fixed point of $g(x)$ in that interval and let $x_0 \neq p$ be any starting approximation to p within this interval. Then,

1. if $|g'(x)| \leq K < 1$ for all $a \leq x \leq b$ then the FPI $x_{i+1} \leftarrow g(x_i)$ converges to p, and p is called an **attracting fixed point** or simply an **attractor**.
2. if $|g'(x)| > 1$ for all $a \leq x \leq b$ then the FPI diverges and p is called an **repelling fixed point** or simply an **repeller/repulsor**.

1. **[12 pts]** Use the Fixed-Point Iteration Theorem to show that $g(x) = 2^{-x}$ has a unique attracting fixed point on [1/3,1].

Let

$$g(x) = 2^{-x}. \tag{1}$$

Then

$$g'(x) = -ln(2)2^{-x} \tag{2}$$

and

$$|g'(x)| = ln(2)2^{-x}. \tag{3}$$

Therefore,

$$|g'(\frac{1}{3})| = ln(2)2^{-\frac{1}{3}} \simeq 0.55 \tag{4}$$

$$|g'(1)| = ln(2)2^{-1} \simeq 0.34. \tag{5}$$

Since $ln(2)2^{-x}$ is a monotonically decreasing function, we know that $|g'((\frac{1}{3}, 1))| \subseteq (0.34, 0.55) \subseteq (0, 1)$. Then by FPIT, $g(x_i)$ converges to p which is an attracting fixed point in $[\frac{1}{3}, 1]$.

$\square$

Problem 2

2. **[3 pts]** Use the `fpi.m` function you coded to find an approximation to the fixed point of g using an initial condition of 1/3 and a relative error tolerance of 10^{-4}. How many iterations did it take and what was your output correct to 4 decimal places?

The estimated fixed point is $x = 0.6412$ and it took 11 iterations.