Math 248: Lab 8

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Consider approximating the integral $\int_0^2 e^x dx$ using the composite Midpoint and Trapezoid Rules.

1. **[20 pts]** Recall the error formulas:

$$\int_a^b f(x)dx = M(n) + \frac{(b-a)h^2}{6} f''(\xi), \quad \int_a^b f(x)dx = T(n) - \frac{(b-a)h^2}{12} f''(\xi)$$

   where $M(n)$ and $T(n)$ are the Composite Midpoint and Composite Trapezoid approximations using n subintervals of width $h = \frac{b-a}{n}$.

   Take a linear combination of $M(n)$ and $T(n)$ that eliminates the $O(h^2)$ term and write the resulting formula for the integral in terms of $M(n)$ and $T(n)$. We'll name this new, better integral, $I(n)$. This is the formula that your MATLAB function will implement in Coding Part 1.

   (after this part, do the entire Coding Part, then return here)

2. **[10 pts]** For the ratio of successive errors for $M(n)$ and $T(n)$ (from coding Part 3), explain why the errors decrease roughly by a factor of 4.

3. **[10 pts]** Based on these ratio of successive errors for $I(t)$ (from coding Part 3), determine the order of convergence of this improved method.

Part 1

$$I(n) = \frac{1}{3}M(n) + \frac{2}{3}T(n)$$

since

$$\frac{1}{3}\left[M(n) + \frac{(b-a)h^2}{6} f''(\xi)\right] + \frac{2}{3}\left[T(n) - \frac{(b-a)h^2}{12} f''(\xi)\right] =$$

$$= \frac{1}{3}M(n) + \frac{2}{3}T(n) + \frac{(b-a)h^2}{18} f''(\xi) - \frac{(b-a)h^2}{18} f''(\xi) = \int_a^b f(x)dx.$$

Part 2

The successive errors of $M(n)$ and $T(n)$ decrease by a factor of 4 because both the midpoint and trapezoid method have error $E(h)$ proportional to h^2. Accordingly, if we halve the step size h by doubling n, then the successive error is

$$\frac{E(h)}{E(\frac{1}{2}h)} = 4.$$

Part 3

The successive error of I decreases by a factor of 16. Thus, our error term is roughly proportional to h^4.