Math 248: Lab 3

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1. **[10 pts]** I claim that for this sequence

$$\lim_{k\to\infty} a_k = L$$

where $L = \dfrac{a_1 + 2a_2}{3}$ for any $a_1, a_2 \in \mathbb{R}$. Try to determine the validity of this claim. I am NOT asking for a mathematical proof, but merely evidence. Use **at least 3 distinct test cases** to support your reasoning. In each case,

- show the work for what the limit should be according to my claim.
- describe the pattern that you see in your code.
- give the final sequence value from your code as well as what value of n was used. (Don't round the output. Write down exactly what it says.)

Make n big enough to show a clear pattern. Be sure to give a well-written response.

Example 1: $n = 1000, a_1 = 100, a_2 = -100$

What should the limit be?

$$L = \frac{a_1 + 2a_2}{3} = \frac{100 - 200}{3} = \frac{-100}{3} \simeq -33.\overline{33} \tag{1}$$

What's the pattern in the code?

The numbers start to converge pretty quickly, around n = 20.

What's the final value?

The final value at n = 1000 was $-33.\overline{33}$.

Example 2: $n = 1000, a_1 = 10, a_2 = -100$

What should the limit be?

$$L = \frac{a_1 + 2a_2}{3} = \frac{10 - 200}{3} = \frac{-190}{3} \simeq -63.\overline{33} \tag{2}$$

What's the pattern in the code?

The numbers start to converge pretty quickly, around n = 18.

What's the final value?

The final value at n = 1000 was $-63.\overline{33}$.

Example 3: $n = 1000, a_1 = 3, a_2 = 6$

What should the limit be?

$$L = \frac{a_1 + 2a_2}{3} = \frac{3 + 12}{3} = \frac{15}{3} \simeq 5 \tag{3}$$

What's the pattern in the code?

The numbers start to converge pretty quickly, around n = 16.

What's the final value?

The final value at n = 1000 was 5.