Math 248: Lab 9

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Part 1: Solving the system by hand.

$$\begin{aligned}
x_1 &= 2 - x_2 \\
a(2 - x_2) + x_2 &= 1 \\
(1 - a)x_2 + 2a &= 1 \\
(1 - a)x_2 &= 1 - 2a \\
x_2 &= \frac{1 - 2a}{1 - a}
\end{aligned}$$

$$\begin{aligned}
x_1 &= 2 - \frac{1 - 2a}{1 - a} \\
x_1 &= \frac{2(1 - a) - 1 + 2a}{1 - a} \\
x_1 &= \frac{1}{1 - a}
\end{aligned}$$

So,

$$x_1 = \frac{1}{1 - a} \tag{1}$$

$$x_2 = \frac{1 - 2a}{1 - a}. \tag{2}$$

Part 2: Gaussian elimination.

ans = 1×2 table

	x1_error	x2_error
1	2.8133e-13	0

ans = 1×2 table

	x1_error	x2_error
1	4.9752e-09	0

ans = 1×2 table

	x1_error	x2_error
1	2.2122e-05	0

ans = 1×2 table

	x1_error	x2_error
1	1.2204	1.1102e-16

Figure 1: *Gauss* absolute error table, $a = \{10^{-4}, 10^{-8}, 10^{-12}, 10^{-16}\}$ respectively

It seems like as a gets smaller, the error gets larger in both terms, but especially for the term with a in it.

Part 3: MATLAB \ feature.

ans = *1×2 table*

	x1_error	x2_error
1	2.2204e-16	0

ans = *1×2 table*

	x1_error	x2_error
1	2.2204e-16	0

ans = *1×2 table*

	x1_error	x2_error
1	0	0

ans = *1×2 table*

	x1_error	x2_error
1	2.2204e-16	0

Figure 2: *MATLAB solver* absolute error table, $a = \{10^{-4}, 10^{-8}, 10^{-12}, 10^{-16}\}$ respectively

It seems like the error stays pretty stagnant for a of any size.

Part 4: Reversed equations with Gaussian elimination.

ans = *1×2 table*

	x1_error	x2_error
1	2.2204e-16	0

ans = *1×2 table*

	x1_error	x2_error
1	2.2204e-16	0

ans = *1×2 table*

	x1_error	x2_error
1	0	0

ans = *1×2 table*

	x1_error	x2_error
1	2.2204e-16	0

Figure 3: *Reversed EQ Gauss* absolute error table, $a = \{10^{-4}, 10^{-8}, 10^{-12}, 10^{-16}\}$ respectively

We get the same result as the MATLAB solver! Being able to start with the unit pivot in the top left corner makes it much easier to turn a into zero, and accordingly, our result is significantly improved.