Math 248: Lab 5

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Problem 1

1. You know from algebra that the roots of the quadratic polynomial $y = ax^2 + bx + c$ can be found using the quadratic formula There is an alternative formula which can be derived from the first. The standard formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The alternative formula is:

$$x = \frac{-2c}{b \mp \sqrt{b^2 - 4ac}}$$

   (a) **[6 pts]** Derive the alternative formula from the standard formula.

   (b) **[12 pts]** Find the roots of the equation

$$x^2 + 111.11x + 1.2121 = 0$$

   by hand using the standard formula using 5-digit **decimal** rounding arithmetic.

   (c) **[12 pts]** Repeat the previous part but using the alternative formula.

   (d) **[8 pts]** The true roots of the equation, with 5 significant digits, are -1.1110×10^2 and -1.0910×10^{-2}. Explain any differences in these roots calculated in parts (b) and (c), and the true roots. What causes these differences? Is either the standard formula or the alternative formula of the quadratic formula any better than the other? Why or why not?

Part A:

Let $D = \sqrt{b^2 - 4ac}$ be the square root of the discriminant. Then

$$x = \frac{-b \pm D}{2a}. \tag{1}$$

Multiply in $1 = \frac{-b \mp D}{-b \mp D}$, as follows:

$$\begin{aligned} x &= \frac{-b \pm D}{2a} * \frac{-b \mp D}{-b \mp D} && (2) \\ &= \frac{(-b \pm D)(-b \mp D)}{2a(-b \mp D)} && (3) \\ &= \frac{(-b)^2 - D^2}{2a(-b \mp D)} && (4) \end{aligned}$$

$$= \frac{b^2 - (b^2 - 4ac)}{2a(-b \mp D)} \tag{5}$$

$$= \frac{2c}{(-b \mp D)} \tag{6}$$

$$= \frac{2c}{-b \mp \sqrt{b^2 - 4ac}} \tag{7}$$

as desired.

Part B:

Note:

- $a = 1$
- $b = 111.11$
- $c = 1.2121$

Thus,

$$x = \frac{-111.11 \pm \sqrt{(111.11)^2 - 4 * (1.2121)}}{2} \tag{8}$$

Breaking this expression apart,

$$\frac{-111.11}{2} = -55.555 \tag{9}$$

$$(111.11)^2 = 12345.4321 \tag{10}$$

$$4 * 1.2121 = 4.8484 \tag{11}$$

So (8) becomes

$$-55.555 \pm \frac{\sqrt{12345.4321 - 4.8484}}{2}. \tag{12}$$

Then the discriminant becomes 12340.5837, whose square root is 111.08818 when rounded to the fifth digit. When we divide 111.08818 by 2, we get 55.544085, which has six decimal digits. Thus, we round it to 55.54409. So, (12) becomes

$$x = -55.555 \pm 55.54409 = -0.01091, -111.09909. \tag{13}$$

Part C:

Using

$$x = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}} \tag{14}$$

with

- $a = 1$
- $b = 111.11$
- $c = 1.2121$

and five digit rounding, we get:

$$2c = 2.4242 \tag{15}$$
$$\frac{2.4242}{-111.11 \mp \sqrt{(111.11)^2 - 4.8282}} \tag{16}$$
$$\frac{2.4242}{-111.11 \mp \sqrt{12345.4321 - 4.8282}} \tag{17}$$
$$\frac{2.4242}{-111.11 \mp \sqrt{12340.6039}} \tag{18}$$
$$\frac{2.4242}{-111.11 \mp 111.08827} \tag{19}$$

which splits into

$$x_1 = \frac{2.4242}{-111.11 + 111.08827} = \frac{2.4242}{-0.02173} = -111.56006 \tag{20}$$
$$x_2 = \frac{2.4242}{-111.11 - 111.08827} = \frac{2.4242}{-222.19827} = -0.01091 \tag{21}$$

Part D:

The true roots are -111.1 and $-.01091$ when limited to five significant digits. In part B, we got that the roots were -111.09909 and -0.01091. In part C, we got that the roots were -111.56006 and -0.01091. The only difference is in the -111 root. What causes the difference is that, for the alternative method, having the discriminant on the bottom causes excess rounding error from the square root on the bottom. Neither formula is strictly better or worse than the other, but for this use case, the standard formula is better.

Problem 2

2. A rectangular box has sides of length 3 cm, 4 cm, and 5 cm, measured to the nearest centimeter.

(a) [6 pts] What are the best upper and lower bounds for the volume of this box?

(b) [6 pts] What are the best upper and lower bounds for the surface area?

Part A:

Our two most extreme options for parameters are $(3.5, 4.5, 5.5)$ and $(2.5, 3.5, 4.5)$ for the upper and lower bound respectively. The volume formula is $w * l * h = V(w, l, h)$. Accordingly, our upper and lower bounds for volume are:

$$\text{upper bound: } V(3.5, 4.5, 5.5) = 3.5 * 4.5 * 5.5 = 86.625cm^3 \tag{22}$$
$$\text{lower bound: } V(2.5, 3.5, 4.5) = 2.5 * 3.5 * 4.5 = 39.375cm^3. \tag{23}$$

Part B:

Using the same values as before, we now apply the surface area formula for a box, $S(l, w, h) = 2(lw + wh + lh)$.

$$\text{upper bound: } S(3.5, 4.5, 5.5) = 119.5cm^2 \tag{24}$$

$$\text{lower bound: } S(2.5, 3.5, 4.5) = 71.5cm^2. \tag{25}$$