Math 360 Homework 3

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Problem 1: Find a value of z for which $1^z \neq 1$.

Recall that

$$log(z) = log_e|z| + i(Arg(z) + 2k\pi) \tag{1}$$

and

$$z^c = exp(clog(z)). \tag{2}$$

Then

$$\begin{aligned} (1+0i)^c &= exp(c[log_e|1| + i(Arg(1+0i) + 2k\pi)]) && (3)\\ &= exp(ic(Arg(1+0i) + 2k\pi)) && (4)\\ &= e^{ic(0+2k\pi)}. && (5) \end{aligned}$$

Then

$$2cik\pi \neq 0. \tag{6}$$

So letting $c = a + bi$,

$$(a+bi)(2i\pi k) = -2b\pi k + i(2a\pi k). \tag{7}$$

(7) is nonzero for any nonzero b and $a \in \mathbb{Z}$.

Problem 2: Explain why $sin(z)$ and $cos(z)$ are unbounded as complex functions.

Recall that

$$cos(z) = \frac{e^{iz} + e^{-iz}}{2} \tag{8}$$

and

$$sin(z) = \frac{e^{iz} - e^{-iz}}{2}. \tag{9}$$

$$\tag{10}$$

Problem 3: Prove that $cos(2z) = cos^2(z) - sin^2(z)$ **holds for any** $z \in \mathbb{C}$**.**

Problem 4: Find all values of z **which satisfy** $log(z) = \frac{i\pi}{4}$**.**