Math 435 11/17/2025 Notes

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Review

Today

- Category theory 101!

Remark 1. *π_1 takes map between spaces to maps between fundamental groups. I.e., it is a functor.*

Example 1. ***Set*** *is the category of sets and functions. It has*

- *Objects: Sets*
- *Morphisms: Functions*

$$A \xrightarrow{\quad f \quad} B$$

denotes a relation between sets $A, B \in \mathbf{Set}$ with $f \in \mathrm{Mor}(A, B)$.

Definition 1. *A category $\mathcal{C}$ consists of two things,*

1. *A collection of objects, $Ob(\mathcal{C})$, and,*
2. *Morphisms between those objects, $Mor(\mathcal{C})$.*

These have the following properties.

- *Each morphism $f \in hom(\mathcal{C})$ has a **source** $src \in Ob(\mathcal{C})$ and a **target** $tgt \in Ob(\mathcal{C})$.*
- *There is a composition operation such that given objects $A, B, C \in Ob(\mathcal{C})$, $\circ : Mor(B, C) \times Mor(A, B) \to Mor(A, C)$ such that $\circ$ is associative.*
- *There exists an identity morphism, for all $A \in \mathcal{C}$, $\exists id_A \in Mor(A, A)$ with $f \circ id_A = f$ and $id_A \circ f = f$ for any $f \in Mor(A, A)$.*

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[rrr, bend left, "(h\circ g)\circ f"] \arrow[rr, bend right, "g\circ f"'] \arrow[rrr, bend right, "h\circ(g\circ f)"'] & B \arrow[r, "g"] \arrow[rr, bend left, "h\circ g"] & C \arrow[r, "h"] & D \end{tikzcd}$$