

# Math 435 11/19/2025 Notes

November 20, 2025

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## Review

**Definition 1.** A category  $\mathcal{C}$  consists of two things,

1. A collection of objects,  $Ob(\mathcal{C})$ , and,
2. Morphisms between those objects,  $Mor(\mathcal{C})$ .

These have the following properties.

- Each morphism  $f \in hom(\mathcal{C})$  has a **source**  $src \in Ob(\mathcal{C})$  and a **target**  $tgt \in Ob(\mathcal{C})$ .
- There is a composition operation such that given objects  $A, B, C \in Ob(\mathcal{C})$ ,  $\circ : Mor(B, C) \times Mor(A, B) \rightarrow Mor(A, C)$  such that  $\circ$  is associative.
- There exists an identity morphism, for all  $A \in \mathcal{C}$ ,  $\exists id_A \in Mor(A, A)$  with  $f \circ id_A = f$  and  $id_A \circ f = f$  for any  $f \in Mor(A, B)$  and  $id_A \circ g = g$  for any  $g \in Mor(B, A)$ .

## Today

- Category theory examples
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**Example 1.** *Set* has

- $Ob(Set)$  contains sets
- $Mor(Set)$  contains functions

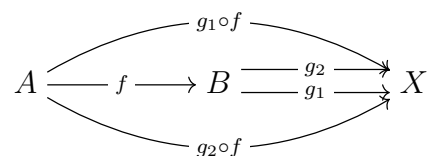
**Example 2.** *Top*<sub>\*</sub> has

- $Ob(Top_*)$  contains pointed / based topological spaces
- $Mor(Top_*)$  contains pointed / based continuous functions

**Example 3.** *Grp* has

- $Ob(Grp)$  contains groups
- $Mor(Grp)$  contains group homomorphisms

**Example 4.** Consider:  $f : A \rightarrow B$  is an epimorphism (surjective) iff  $g_1 \circ f = g_2 \circ f$  implies  $g_1 = g_2$  for any  $g_1, g_2 : B \rightarrow X$ .



This holds universally for any category! So if we have any such diagram, in any category,  $f$  is an epimorphism.

**Remark 1.** Principle of duality: if we reverse the arrows of the diagram in example 4,  $f : B \rightarrow A$  is an monomorphism (injection). I.e.,  $f \circ g_1 = f \circ g_2$  implies  $g_1 = g_2$ .

**Example 5.** Unique factorization of the product:

