# Math 435 09/26/2025 Notes

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## Review

**Definition 1.** *Let $X$ be a space and let $X^*$ be a partition of $X$. Then, the natural surjection $p : X \to X^*$ sends each element to the set which contains it, which induces the quotient topology $U_p$ on $X^*$.*

## Today

- Constructing $S^1$

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**Example 1.** *Constructing the Circle, $S^1$: Let $X = [0,1]$ with the subspace topology where $\mathbb{R}$ has the standard topology. Define $X^* = \{\{x\}|0 < x < 1\} \cup \{0,1\}$. Which sets are in $\mathscr{U}_p$? I.e., which sets are open in $X^*$? Well, $U \in \mathscr{U}_p \iff p^{-1}(U) \overset{\circ}{\subset} [0,1]$. So which sets in $\mathscr{U}_p$ have an open inverse image? If $(a,b) \subseteq [0,1]$, then $p(a,b) \subseteq X^*$ is open. Since we endowed $X$ with the subspace topology, we have open intervals $[0,a)$ and $(b,1]$ in $X$. But in $X^*$, it isn't open, unless we say $0 = 1 = \{0,1\}$ with $(a,\{0,1\}] \cup [\{0,1\},b)$. Notably it cannot be open without both $a$ and $b$. So the open sets of $\mathscr{U}$ are $\{\{(a,b)|0 < a < b < 1\}, \{[\{0,1\},a) \cup (b,\{0,1\}]\}\}$.*