Math 435 10/06/2025 Notes

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Review

Definition 1. *A map $f : X \to Y$ is a continuous function if inverse images of open sets of Y are open in X.*

Lemma 1. *(Pasting Lemma) If you have an open set $A, B \in X$ with $A \cup B = X$ and a continuous function from $f : A \to Y$ and $g : B \to Y$, and $f|_{A\cap B} = g|_{a\cap b}$, then $h : X \to Y$ is continuous by*

$$h \to \begin{cases} f(x) \text{ if } x \in B \\ g(x) \text{ if } x \in A \end{cases} \tag{1}$$

Today

- A caution
- An example
- Homeomorphisms

Remark 1. *If $f : X \to Y$ is continuous, it is not necessarily the case that the image of an open set is open.*

Example 1. *Let $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2$. $(-1, 1) \subseteq dom(f)$ is open in the domain, and has $f((-1, 1)) = [0, 1) \subseteq im(f)$ which is not open in the image.*

Remark 2. *Cech-closure spaces are spaces constructed on graphs where the closure of a node is the set of nodes and edges connected to it.*

Definition 2. *Let X and Y be spaces. X and Y are homeomorphic if there exists a continuous bijection, $f : X \to Y$ such that $f^{-1} : Y \to X$ is continuous. Equivalently, images of open sets are open in the image of f. If X is homeomorphic to Y, we write $X \cong Y$.*

Theorem 1. *A continuous map is a homeomorphism iff $f : X \to Y$ is a bijection and f is a bijection on $f : \mathscr{U}_x \to \mathscr{U}_y$.*