

Math 440 Exam 2 Practice Test

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1 Describe / Explain:

- 1.1 Laplace's Equation (include well-posedness, uniqueness, and the mean value proposition).

answer goes here

- 1.2 Linear Operators, provide an example of something which is a linear operator, and provide an example of something which is not a linear operator.

answer goes here

2 Prove Orthogonality of Sines.

answer goes here

- 3 Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ with certain BC's and IC's, addressing $\lambda > 0, = 0, < 0$.

answer goes here

- 4 Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ at equilibrium in polar coordinates over the interval $(-L, L)$.

Note that the 1D heat equation can be expressed in polar coordinate with axial symmetry (meaning that $\frac{\partial^2 u}{\partial \theta^2} = 0$) as follows:

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right). \quad (1)$$

We also get the following BCs for free:

$$u(-L) = u(L) \quad (2)$$

$$\frac{\partial u}{\partial r}(-L) = \frac{\partial u}{\partial r}(L) \quad (3)$$

$$|u(0)| < \infty. \quad (4)$$

5 Solve Laplace's Equation on a rectangle, with LHS, Top, and Bottom = 0, RHS = $f_2(y)$.

6 Extra Resources Which You Gotta Know or You Might Be Cooked:

Trig Identities:

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \quad (5)$$

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b) \quad (6)$$

$$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b) \quad (7)$$

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b) \quad (8)$$

If BC's are

- $u(0, t) = 0 = u(L, t)$ then

$$u(x, t) = \sum_{n=1}^{\infty} A_n e^{-kt(\frac{n\pi}{L})^2} \sin\left(\frac{n\pi x}{L}\right) \quad (9)$$

- $\frac{\partial u}{\partial x}(0, t) = 0 = \frac{\partial u}{\partial x}(L, t)$ then

$$u(x, t) = \sum_{n=1}^{\infty} B_n e^{-kt(\frac{n\pi}{L})^2} \cos\left(\frac{n\pi x}{L}\right) \quad (10)$$

where

$$A_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx \quad (11)$$

$$\text{and} \quad (12)$$

$$B_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx. \quad (13)$$

Orthogonality of sines and cosines (formulas):

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \begin{cases} 0; & \text{if } n \neq m \\ \frac{L}{2}; & \text{if } n = m. \end{cases} \quad (14)$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right) \cos\left(\frac{m\pi x}{L}\right) dx = \begin{cases} 0; & \text{if } n \neq m \\ \frac{L}{2}; & \text{if } n = m \neq 0 \\ L; & \text{if } n = m = 0 \end{cases} \quad (15)$$