Stacks and Queues

The Stack Abstract Data Type

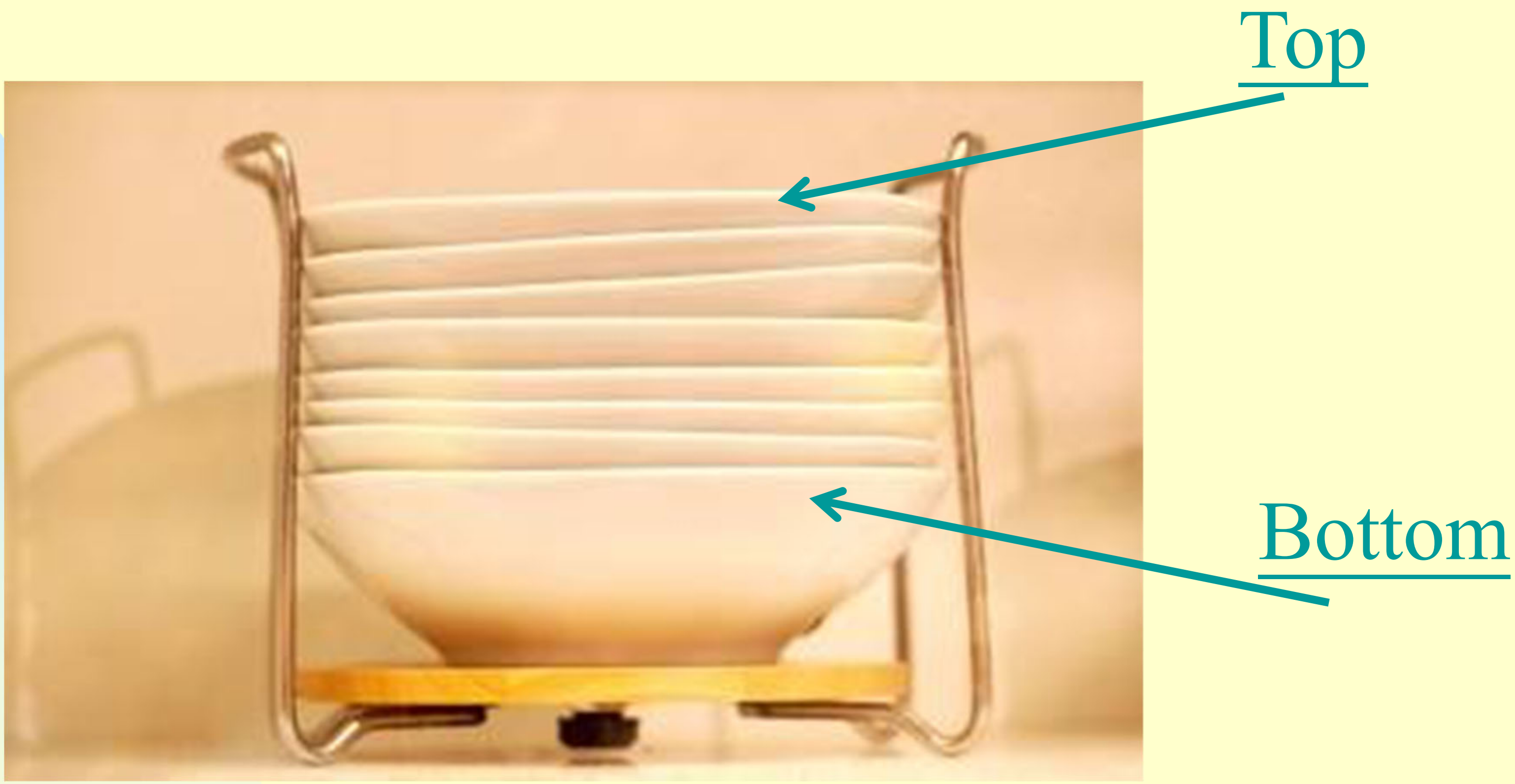
Linear list.

One end is called top.

Other end is called bottom.

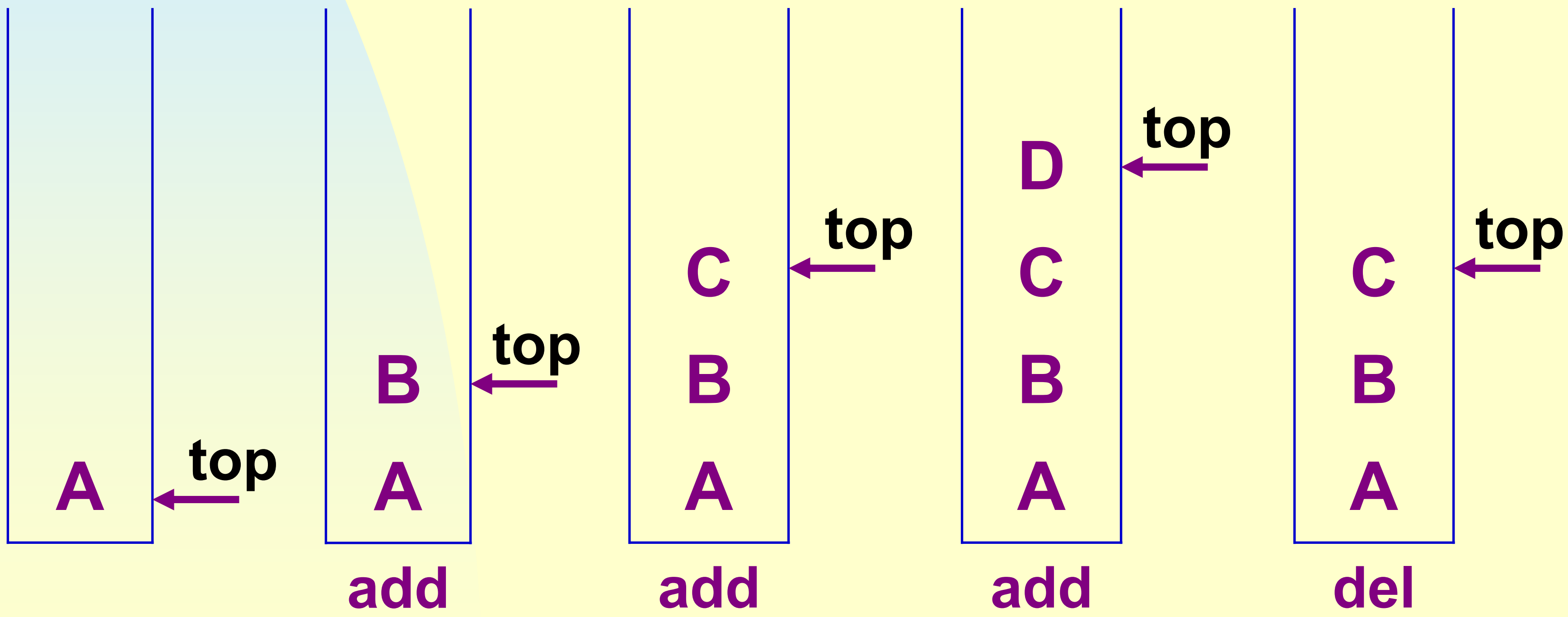
Additions to and removals from the top end only.

Stack Of Bowls



- Add a bowl to the stack.
- Remove a bowl from new stack.
- A stack is a LIFO list.

Inserting and deleting elements in a stack:



ADT 3.1 Stack

template <**class** *T*>
class *Stack*
{ // A finite ordered list with zero or more elements.
public:
Stack (**int** *stackCapacity* = 10)**;**
//Creates an empty stack with initial capacity of *stackCapacity*

bool *IsEmpty*() **const;**
//If number of elements in the stack is 0, **true** else **false**

T& *Top*() **const;**
// Return the top element of stack

void *Push*(**const** *T*& *item*)**;**
// Insert *item* into the top of the stack

void *Pop*()**;**
// Delete the top element of the stack.
};

To implement STACK ADT, we can use

- an array, *stack*[]
- a variable, *top*

Initially *top* is set to

–1.

So we have the following data members of Stack:

private:

*T** *stack*;

int *top*;

int *capacity*;

```
template <class T>
Stack<T>::Stack(int stackCapacity): capacity(stackCapacity)
{
    if (capacity < 1) throw "Stack capacity must be > 0";
    stack = new T[capacity];
    top = -1;
}

template <class T>
inline bool Stack<T>::IsEmpty() const
{
    return(top == -1);
}
```

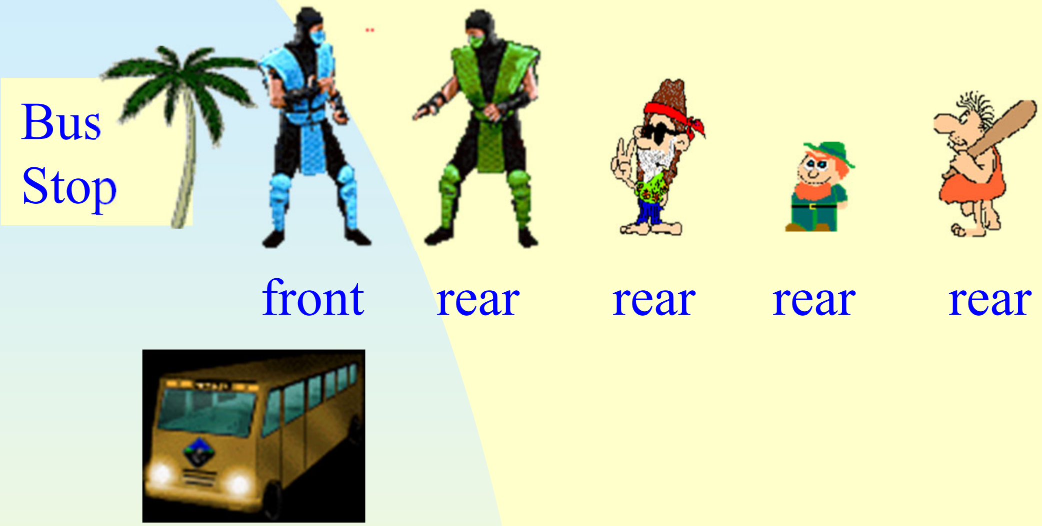
```
template <class T>
inline T& Stack<T>::Top() const
{
   if (IsEmpty()) throw "Stack is Empty";
   return stack[top];
}

template <class T>
void Stack<T>::Push(const T& x)
{
   if (top == capacity - 1)
   {
      ChangeSize1D(stack, capacity, 2*capacity);
      capacity *= 2;
   }
   stack[++top] = x;
}
```

The template function *ChangeSize1D* changes the size of a 1-Dimensional array of type *T* from *oldSize* to *newSize*.

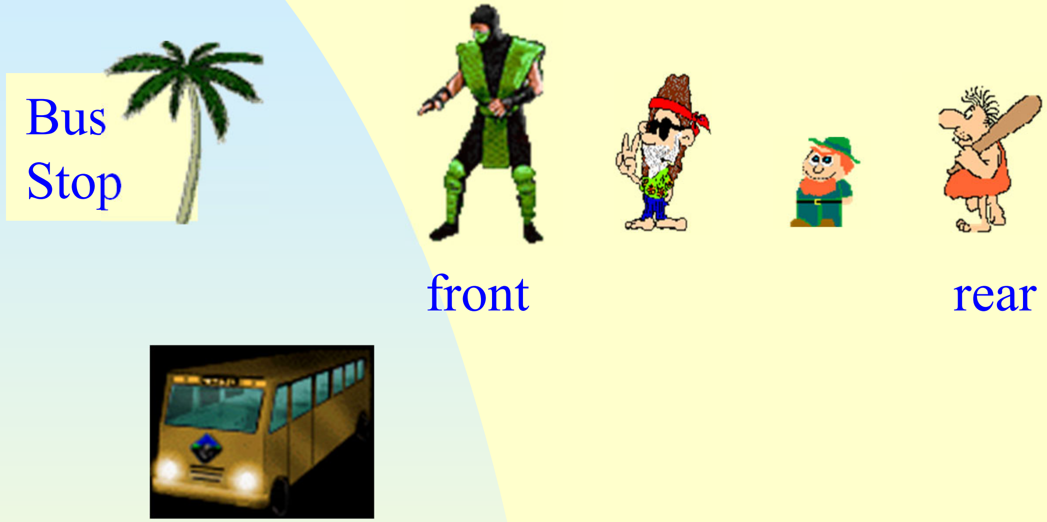
```
template <class T>
void ChangeSize1D(T* a, const int oldSize, const int newSize)
{
   if (newSize < 0) throw "New length must be >= 0";
   T* temp = new T[newSize];
   int number = min(oldSize, newSize);
   copy(a, a + number, temp);
   delete [] a;
   a = temp;
}
```

```
template <class T>
void Stack<T>::Pop()
{ // Delete top element of stack.
    if (IsEmpty()) throw "Stack is empty. Cannot delete.";
    stack[top--].~T();  // destructor for T
}
```

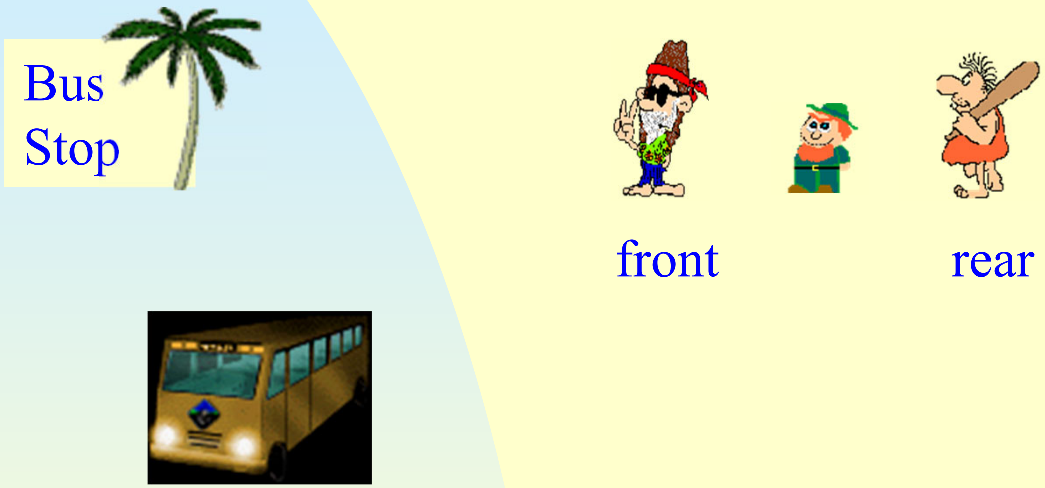
Bus Stop Queue



Bus Stop Queue



Bus Stop Queue



Bus Stop Queue

Bus
Stop

front rear rear

3.3 The Queue Abstract Data Type

Linear list.

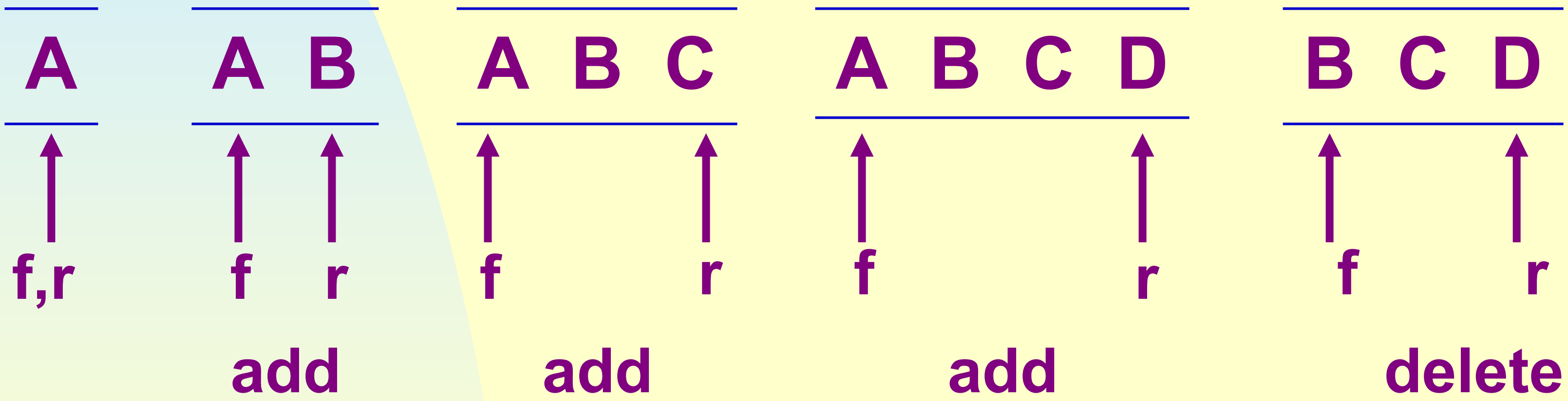
One end is called front.

Other end is called rear.

Additions are done at the rear only.

Removals are made from the front only.

3.3 The Queue Abstract Data Type



f = queue front   r = queue rear

A queue is a FIFO List.

ADT 3.2 Queue

```
template <class T>
class Queue
 { // A finite ordered list with zero or more elements.
 public:
    Queue (int queueCapacity = 10);
    // Create an empty queue with initial capacity of queueCapacity

    bool IsEmpty() const;

    T& Front() const; //Return the front element of the queue.

    T& Rear() const; //Return the rear element of the queue.

    void Push(const T& item);
     //Insert item at the rear of the queue.

    void Pop();
    // Delete the front element of the queue.
};
```

To implement this QUEUE ADT, we can use

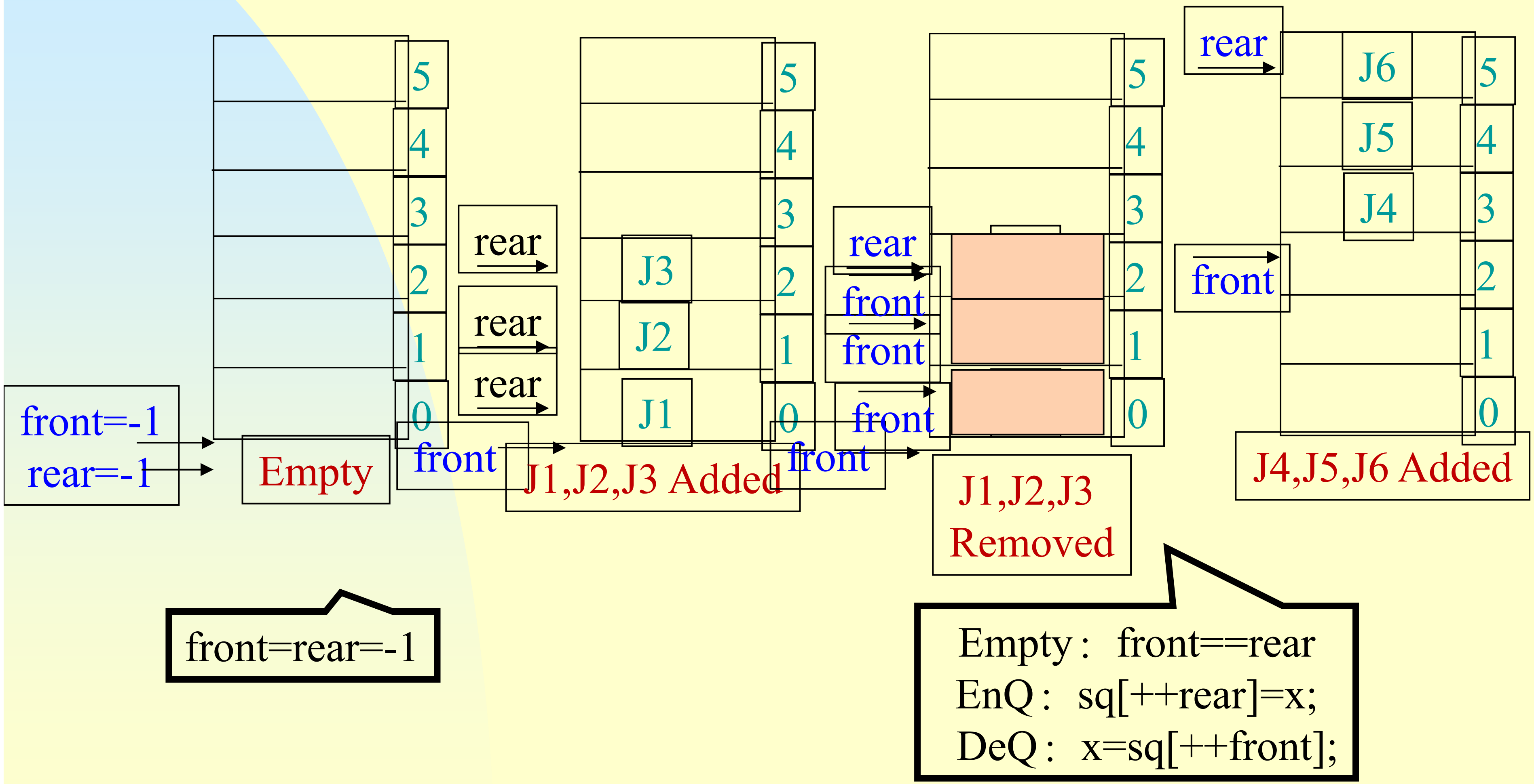
an array, *queue*[]

two variables, *front* and *rear*

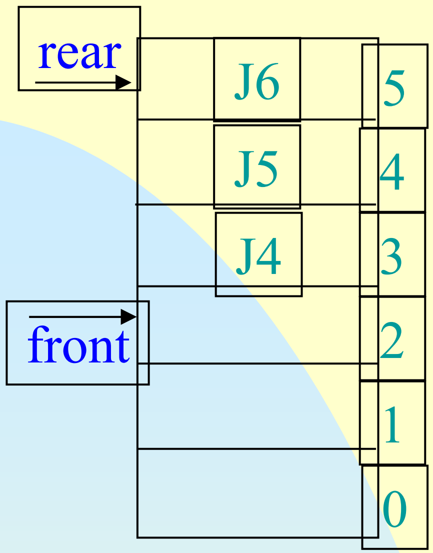
front being one less than the position of the first element

So we have the following data members of Queue:

private:
*T** *queue***;**
int *front*,
rear,
*capacity***;**



front=-1
rear=-1
Empty
5
4
3
2
1
0
rear
rear
rear
front
J3
J2
J1
J1,J2,J3 Added
rear
front
front
front
front
J1,J2,J3
Removed
rear
front
J6
J5
J4
J4,J5,J6 Added
front=rear=-1
Empty : front==rear
EnQ : sq[++rear]=x;
DeQ : x=sq[++front];



Problem

EnQueue: Add an element

Overflow!

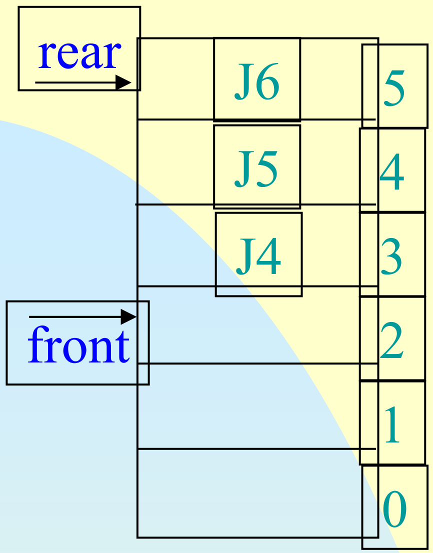
Space Available! →

False Overflow

Solution?

Elements movement

?



Problem

False Overflow

Solution?

$6 \rightarrow 2$

$6 \rightarrow 1$

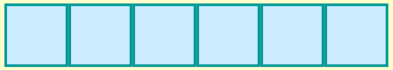
$6 \rightarrow 0$

Capacity = 6

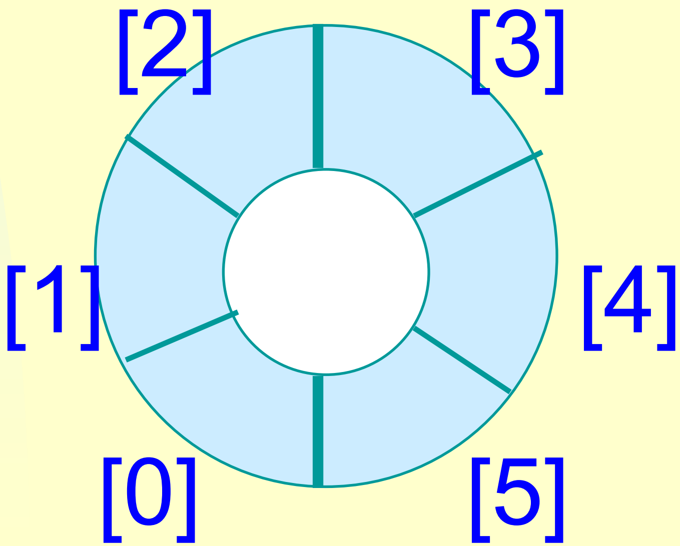
$6 \% 6 = 0$

Array Queue

Use a 1D array *queue*.

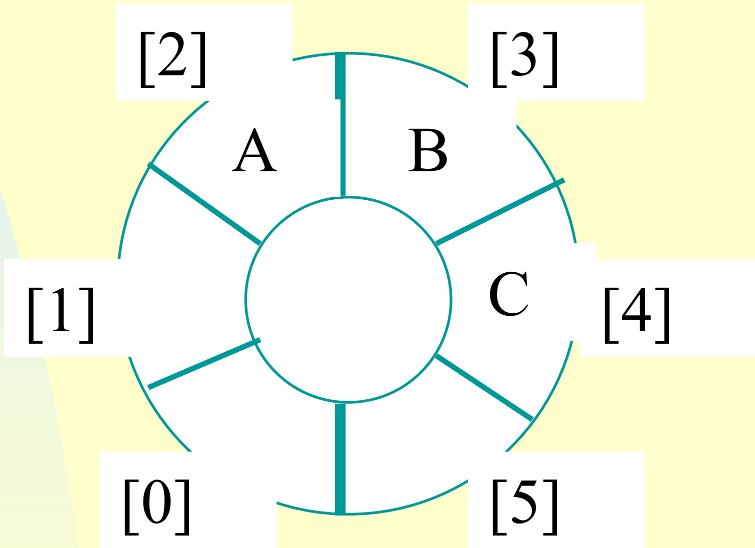
queue[] 

- Circular view of array.



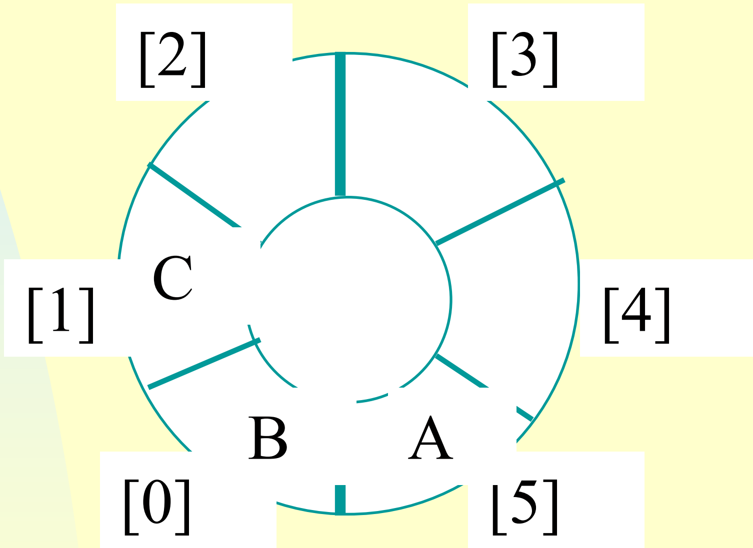
Array Queue

- Possible configuration with 3 elements.



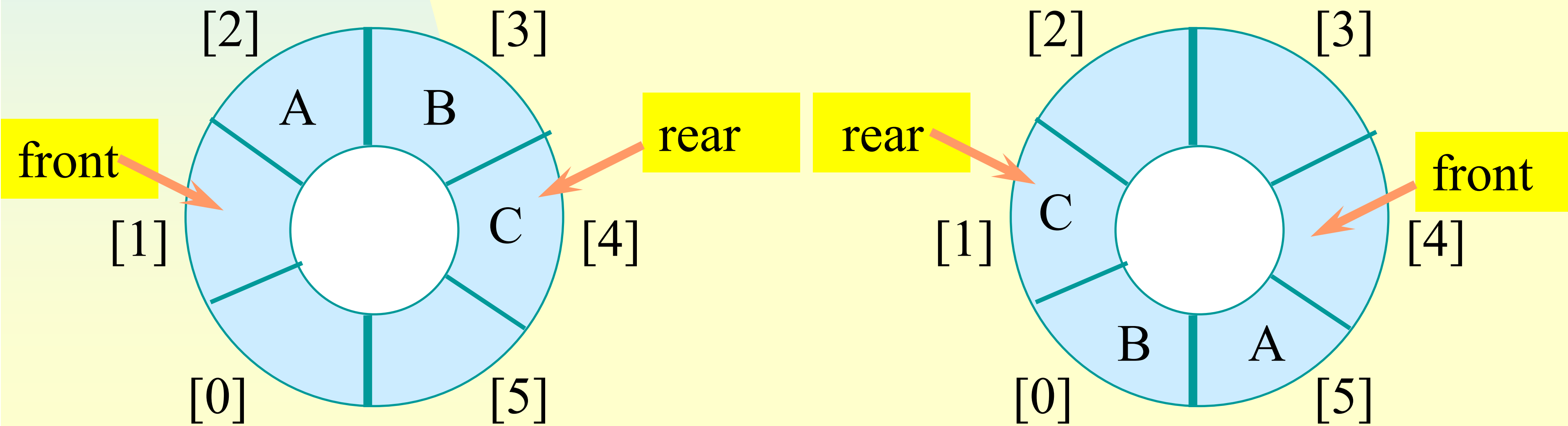
Array Queue

- Another possible configuration with 3 elements.



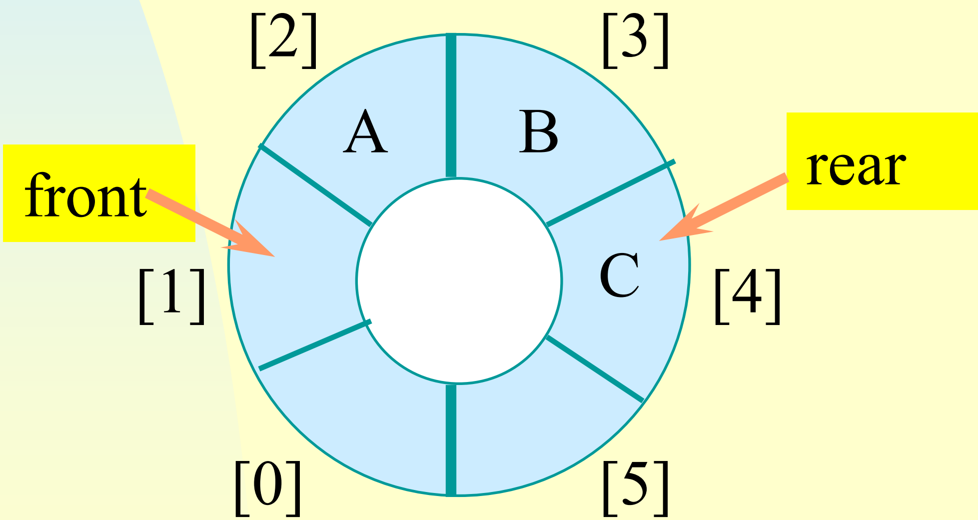
Array Queue

- Use integer variables front and rear.
  - front is one position counterclockwise from first element
  - rear gives position of last element



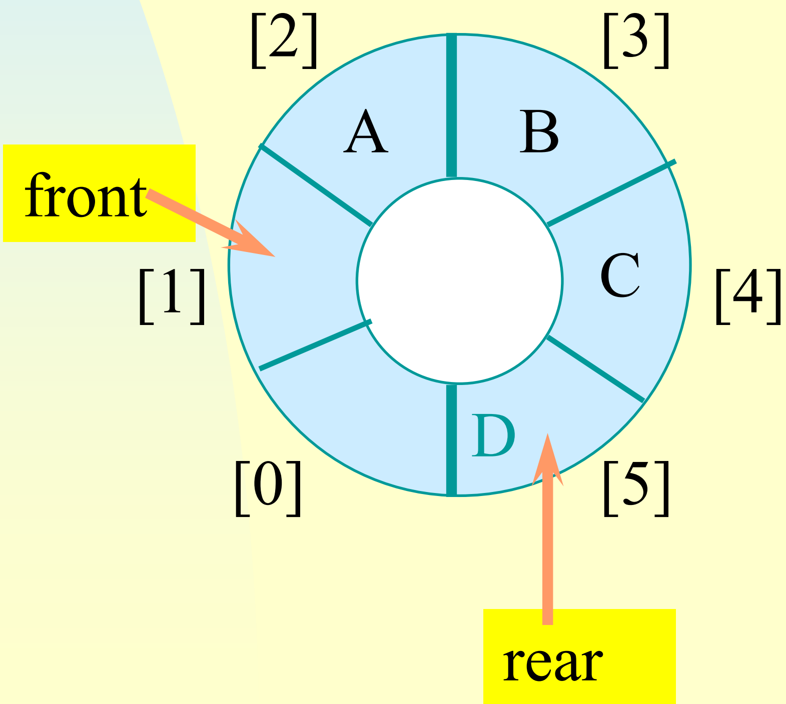
Add An Element

- Move rear one clockwise.



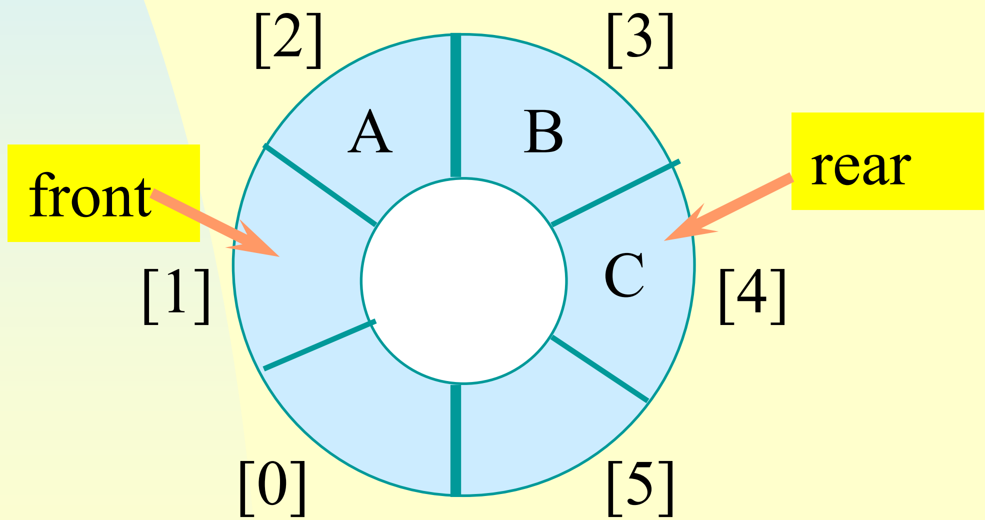
Add An Element

- Move rear one clockwise.
- Then put into queue[rear].



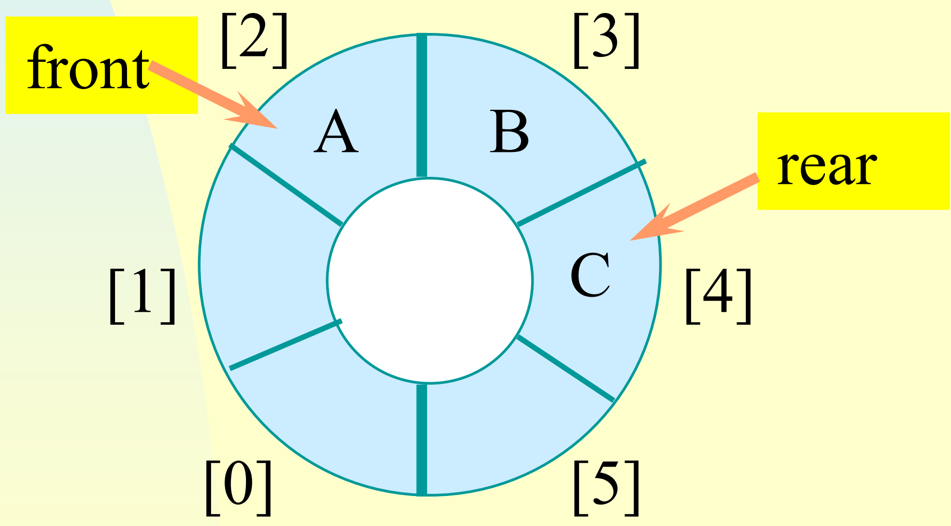
Remove An Element

- Move front one clockwise.



Remove An Element

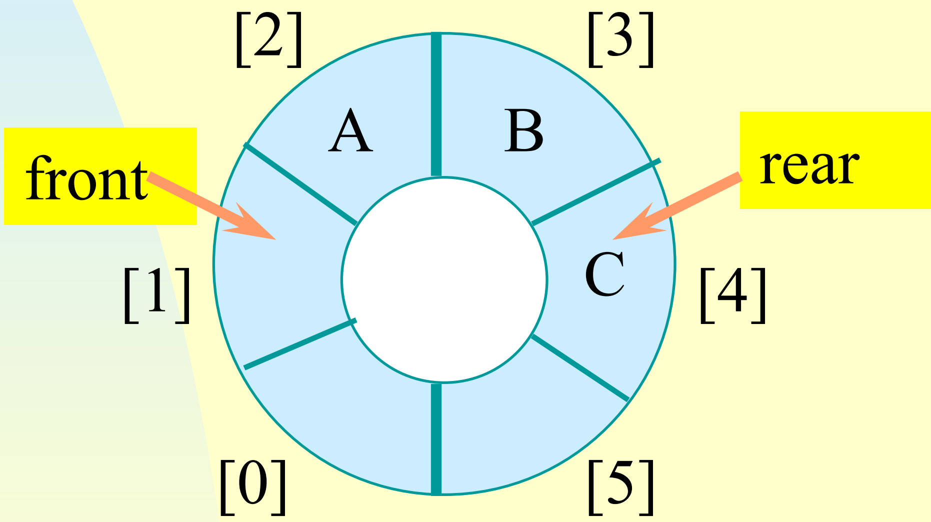
- Move front one clockwise.
- Then extract from queue[front].



Moving rear Clockwise

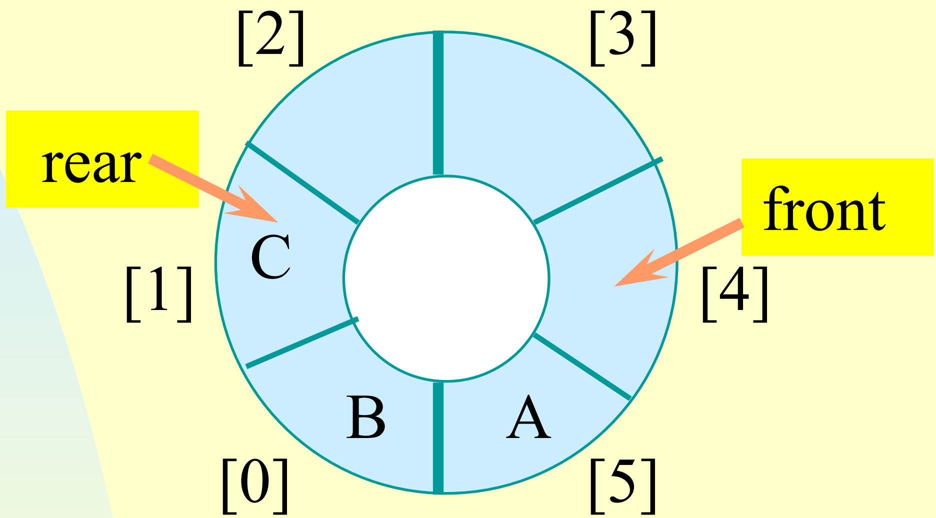
- rear++;

  if (rear = = queue.length) rear = 0;

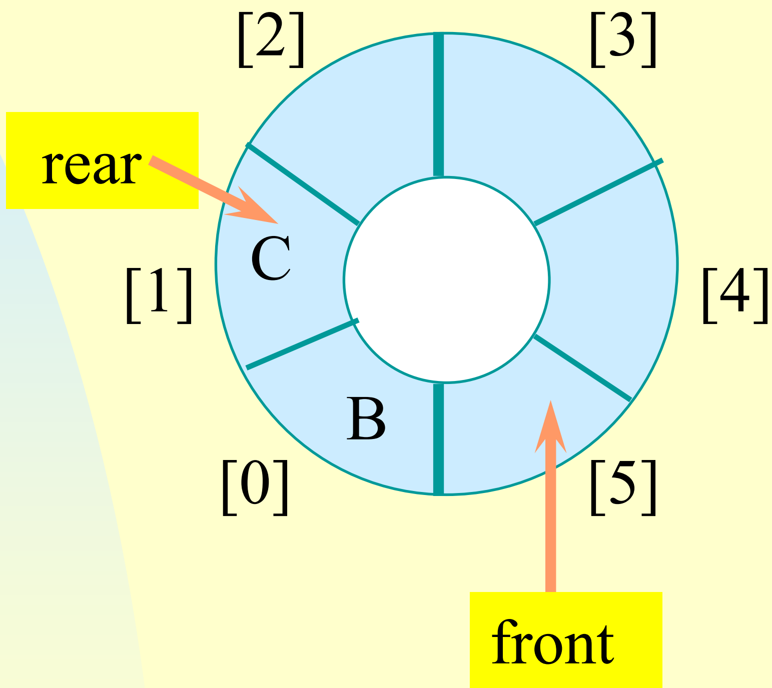


- rear = (rear + 1) % queue.length;

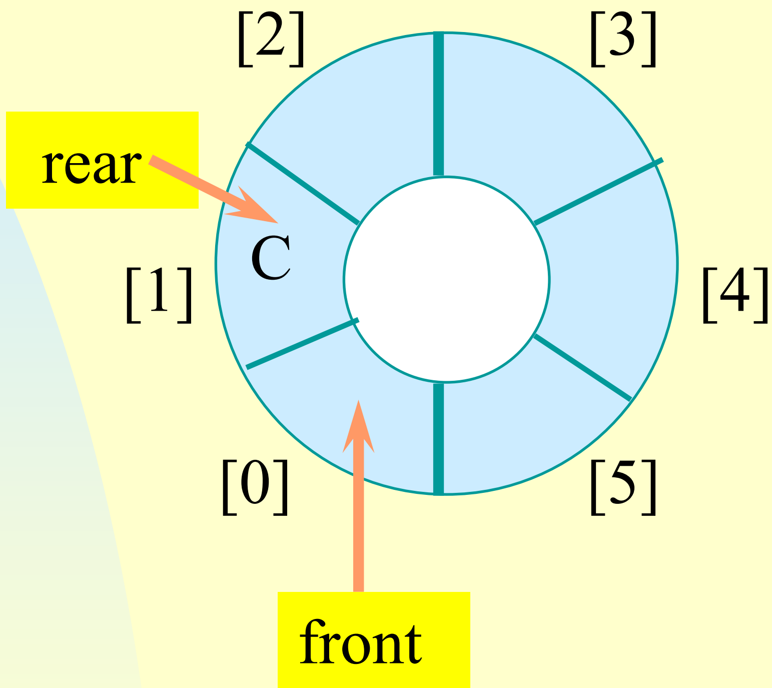
Empty That Queue



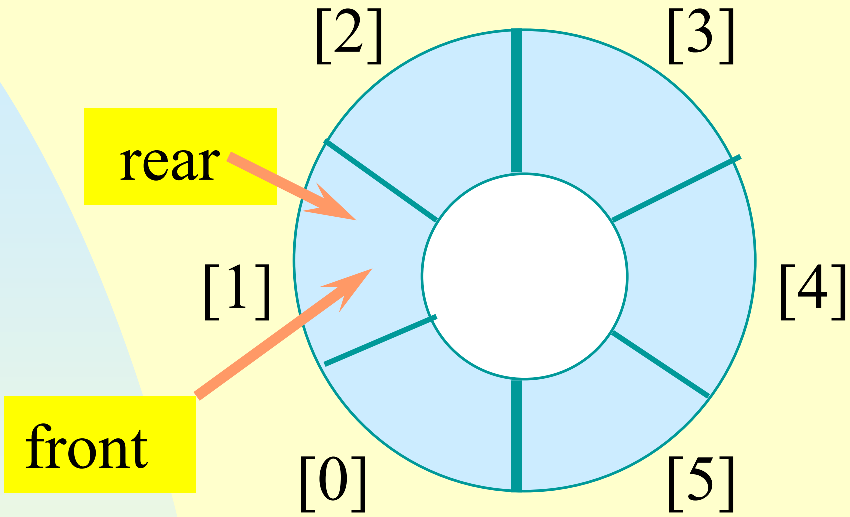
Empty That Queue



Empty That Queue



Empty That Queue

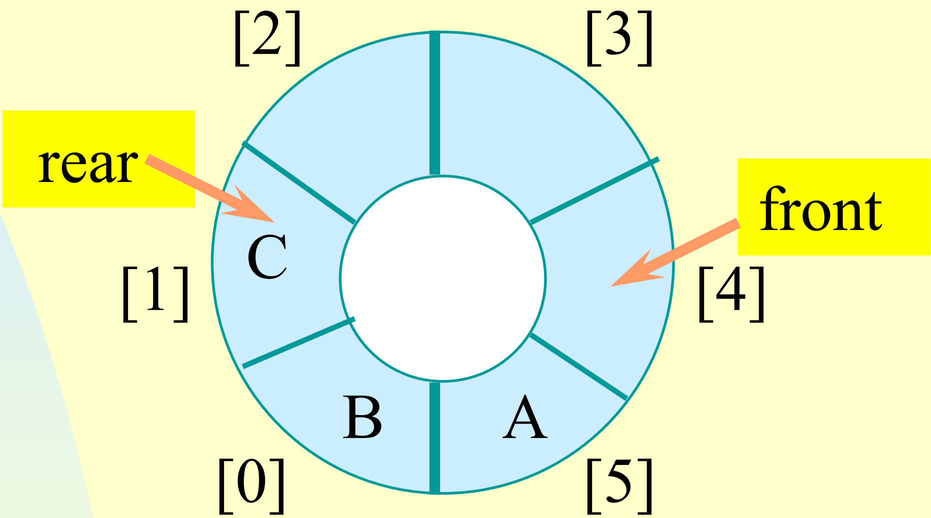


When a series of removes causes the queue to become empty, front = rear.

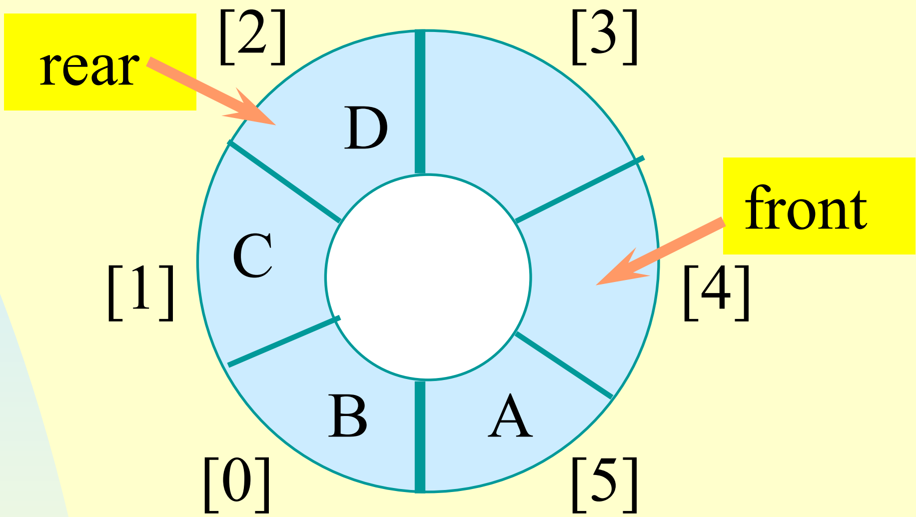
When a queue is constructed, it is empty.

So initialize front = rear = 0.

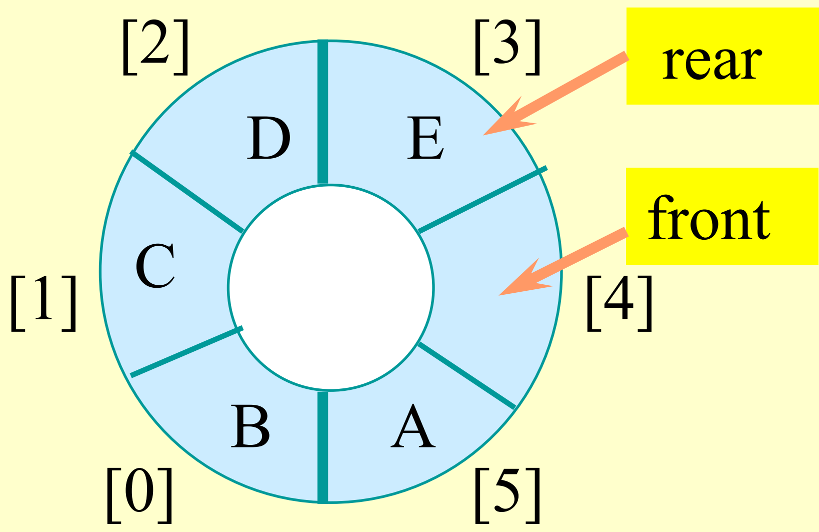
A Full Tank Please



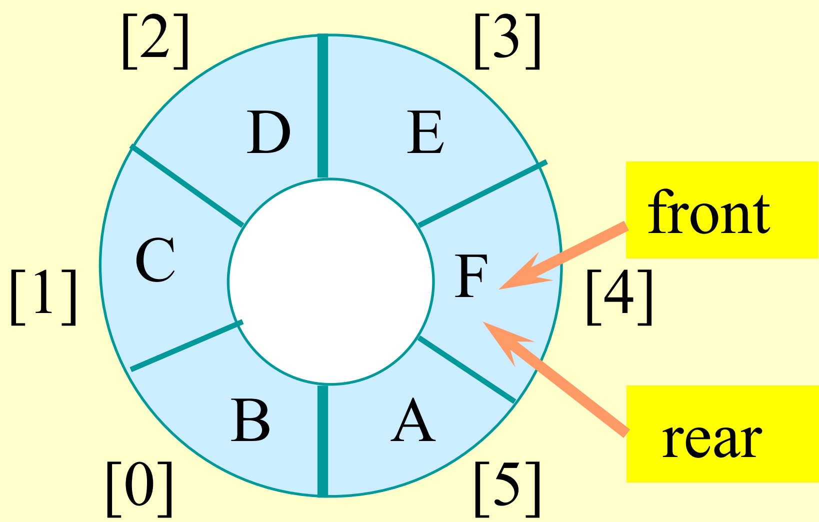
A Full Tank Please



A Full Tank Please



A Full Tank Please



- When a series of adds causes the queue to become full, front = rear.
- So we cannot distinguish between a full queue and an empty queue!

Ouch!!!!!

Remedies.

- Don’t let the queue get full.
  - When the addition of an element will cause the queue to be full, increase array size.
  - This is what the text does.
- Define a boolean variable lastOperationIsPut.
  - Following each put set this variable to true.
  - Following each remove set to false.
  - Queue is empty iff (front == rear) && !lastOperationIsPut
  - Queue is full iff (front == rear) && lastOperationIsPut

Ouch!!!!!

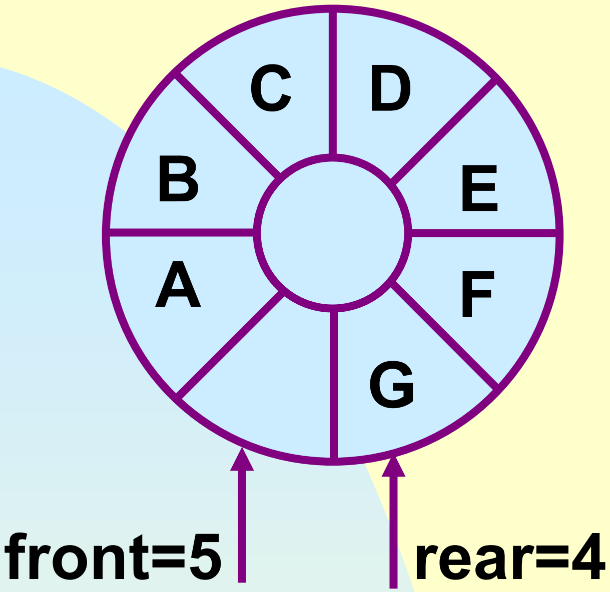
- Remedies (continued).
  - Define an integer variable size.
    - Following each put do size++.
    - Following each remove do size--.
    - Queue is empty iff (size == 0)
    - Queue is full iff (size == queue.length)

```
template <class T>
Queue<Type>::Queue(int queueCapacity):
                                   capacity(queueCapacity)
{
   if (capacity < 1) throw "Queue capacity must > 0";
   queue = new T[capacity];
   front = rear = 0;
}
```

```
template <class T>
Inline bool Queue<T>::IsEmpty()
{ return front == rear };

template <class T>
inline T& Queue<T>::Front()
{
   if (IsEmpty()) throw "Queue is empty. No front element";
   return queue[(front+1)%capacity];
}

template <class T>
inline T& Queue<T>::Rear()
{
   if (IsEmpty()) throw "Queue is empty. No rear element";
   return queue[rear];
}
```

```
template <class T>
void Queue<T>::Push(const T& x)
{ // add x at rear of queue
   if ((rear+1)%capacity == front)
   { // queue full, double capacity
     // code to double queue capacity comes here
   }
   rear = (rear+1)%capacity;
   queue[rear] = x;
}
```

We can double the capacity of queue in the way as shown in the next slide:



queue

0	1	2	3	4	5	6	7
C	D	E	F	G		A	B

front=5, rear=4

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	B	C	D	E	F	G									

front=15, rear=6

This configuration may be obtained as follows:

(1) Create a new array newQueue of twice the capacity.

(2) Copy the second segment to positions in newQueue beginning at 0.

(3) Copy the first segment to positions in newQueue beginning at capacity-front-1.

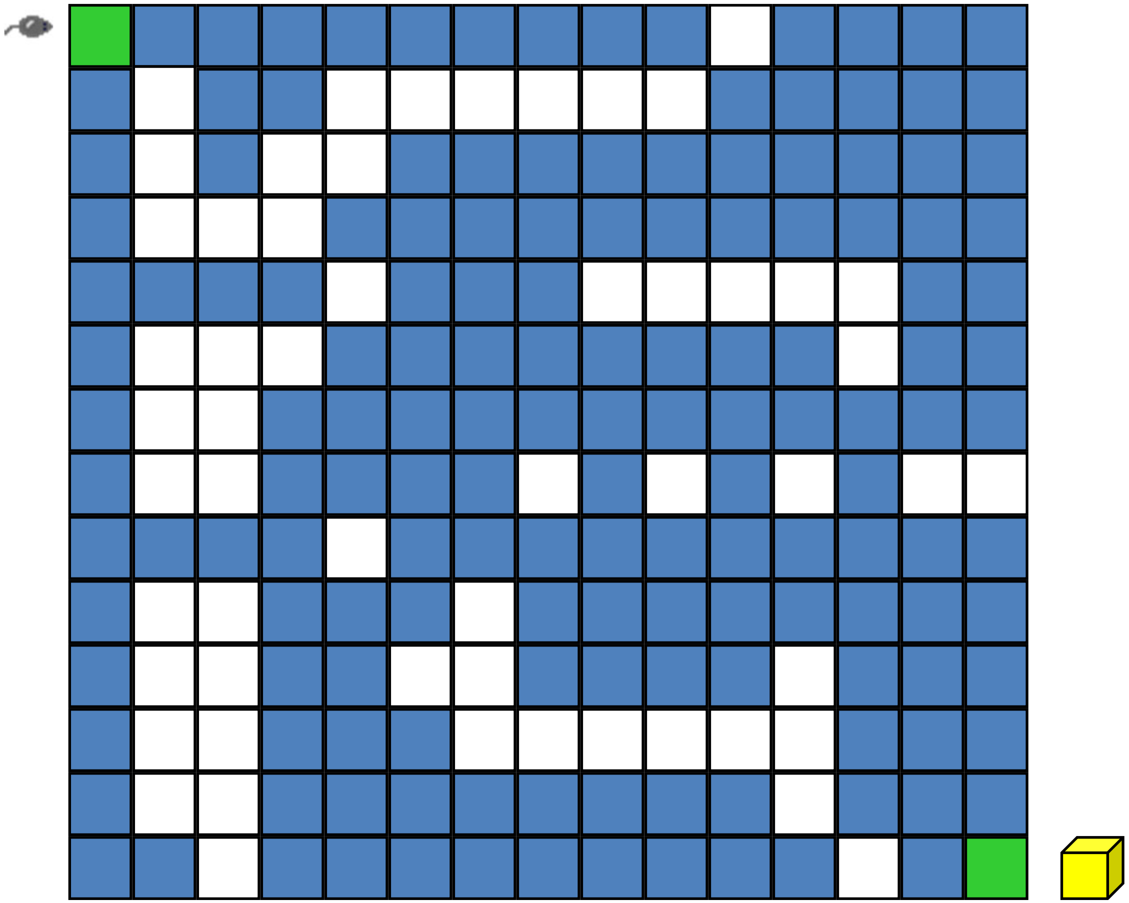
The code is in the next slide:

```
// allocate an array with twice the capacity
T*  newQueue = new T[2*capacity];

// copy from queue to newQueue
int start = (front+1)%capacity;
if (start < 2)
    // no wrap around
    copy(queue+start, queue+start+capacity-1, newQueue);
else
{ // queue wraps around
    copy(queue+start, queue+capacity, newQueue);
    copy(queue, queue+rear+1, newQueue+capacity-start);
}

// switch to newQueue
front = 2*capacity-1;  rear =  capacity-2; capacity *= 2;
delete [] queue;
queue = newQueue;
```

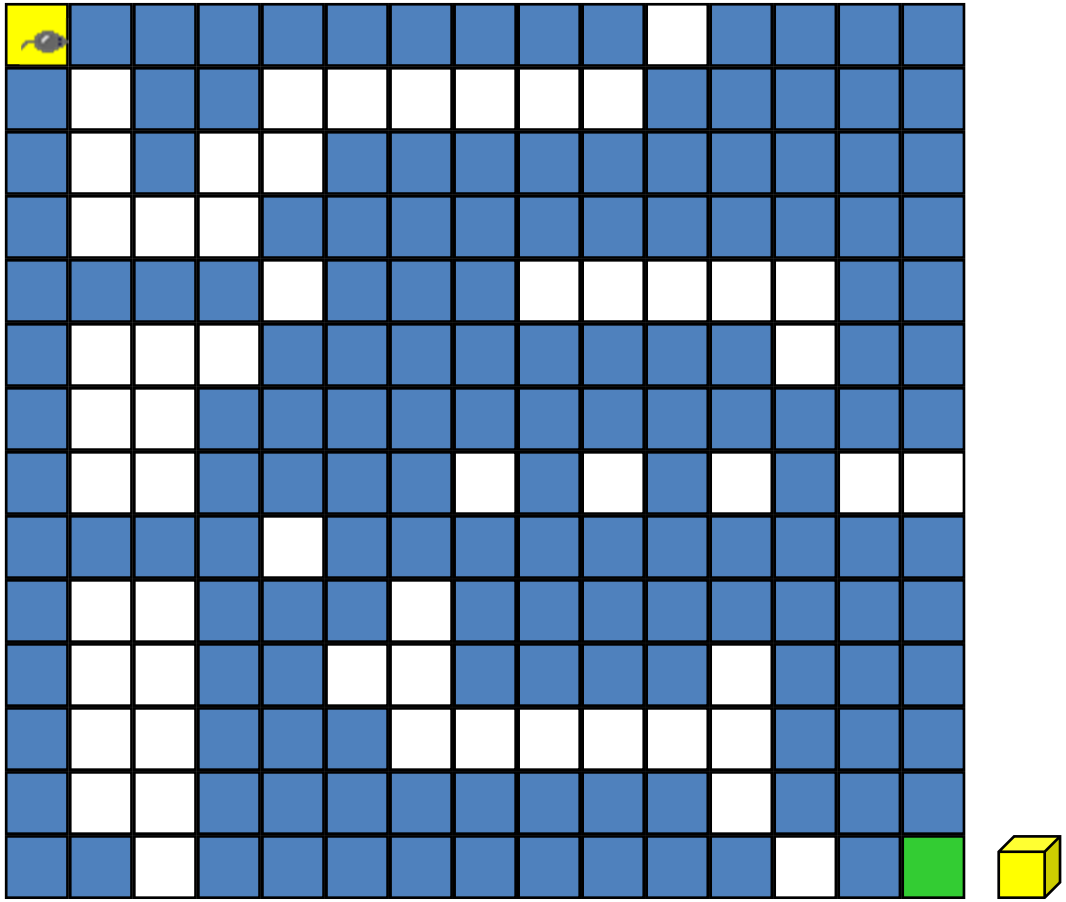
```
template <class T>
void Queue<T>::Pop()
{ // Delete front element from queue
   if (IsEmpty()) throw "Queue is empty. Cannot delete.";
   front = (front+1)%capacity;
   queue[front].~T();      //destructor for T
}
```

For the circular representation, the worst-case add and delete times (assuming no array resizing is needed) are O(1).

Rat In A Maze

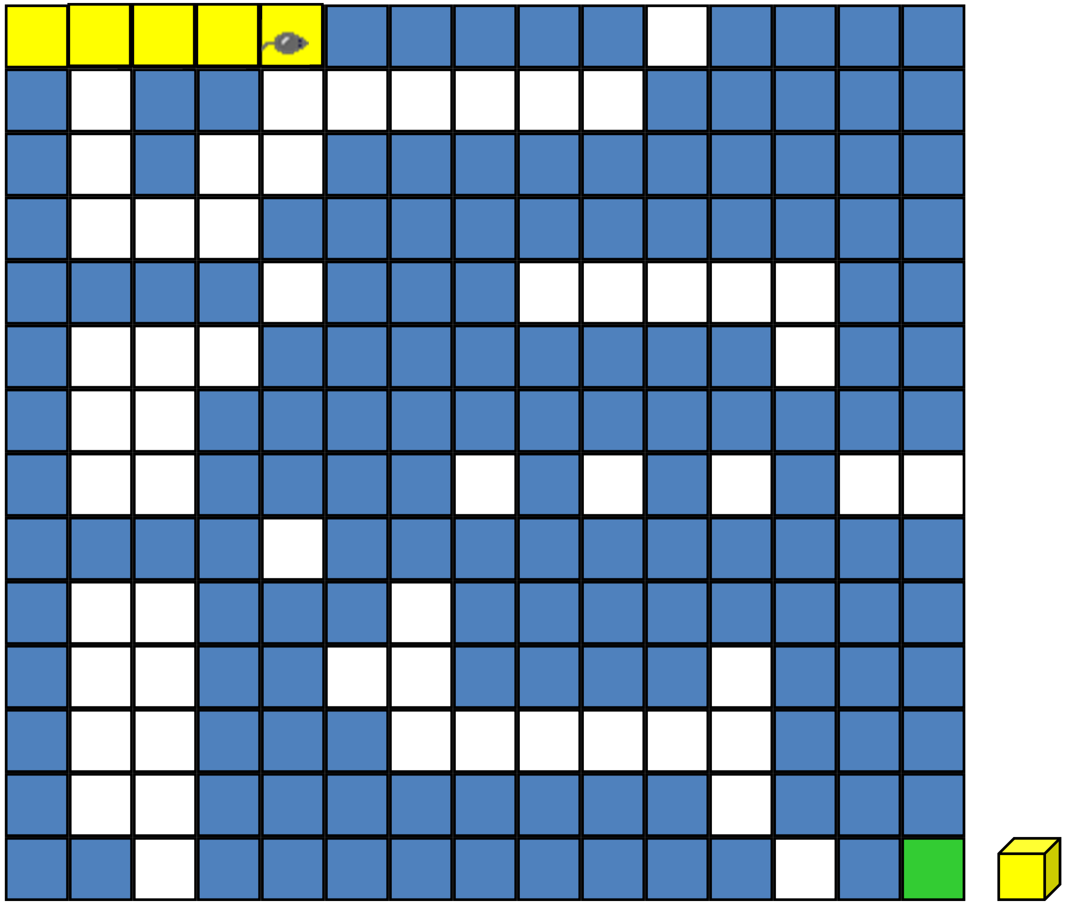


Rat In A Maze



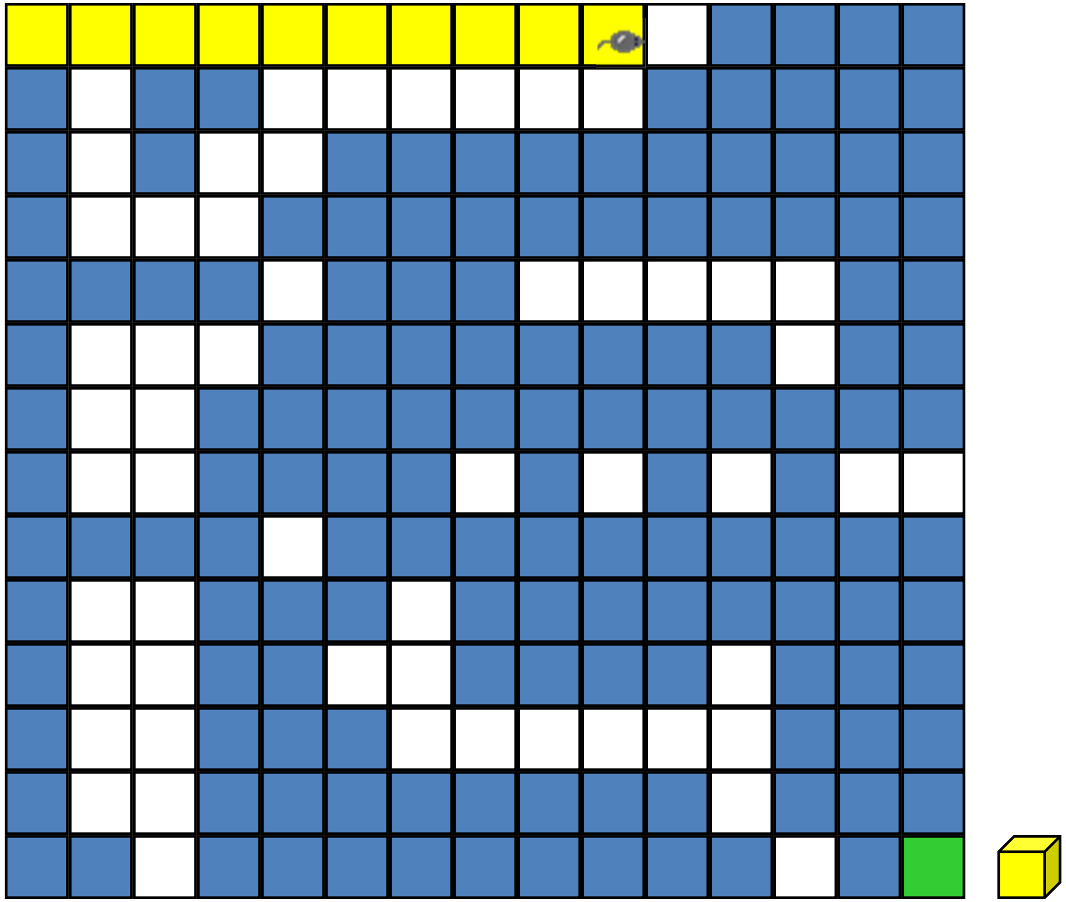
- Move order is: right, down, left, up
- Block positions to avoid revisit.

Rat In A Maze



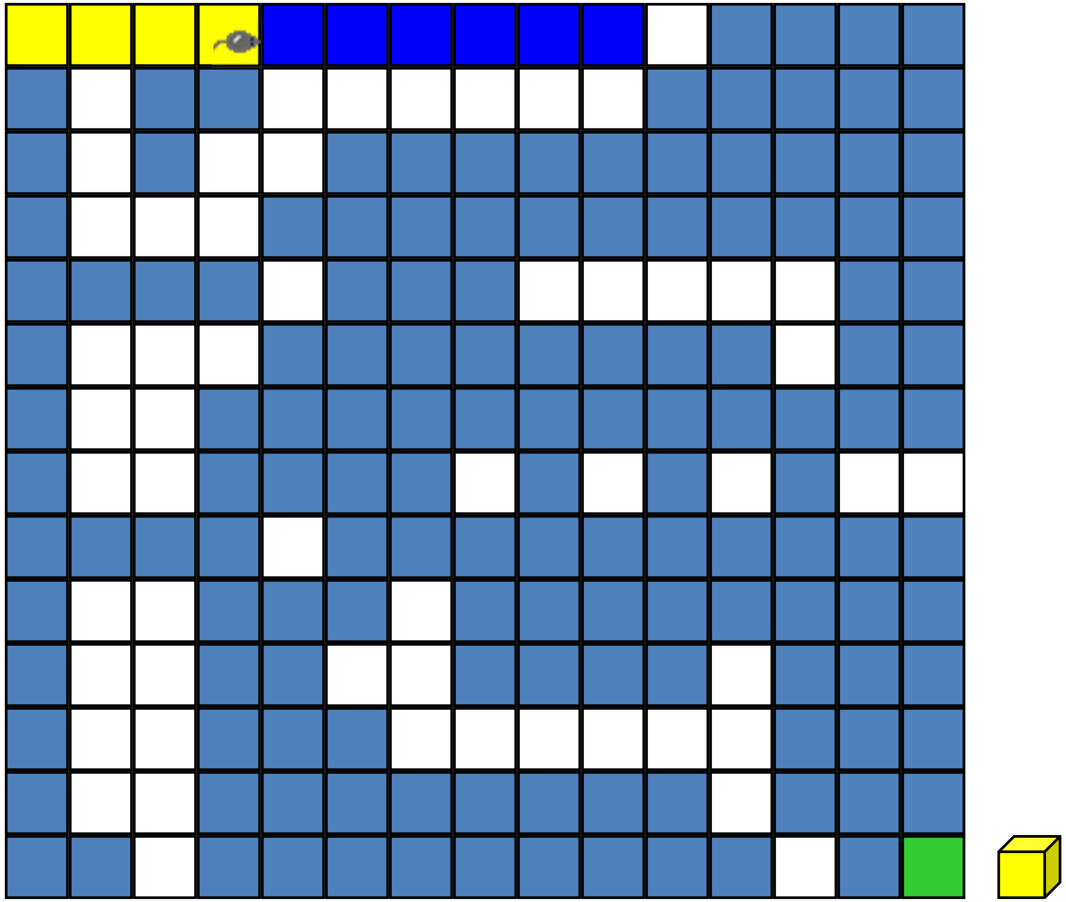
- Move order is: right, down, left, up
- Block positions to avoid revisit.

Rat In A Maze



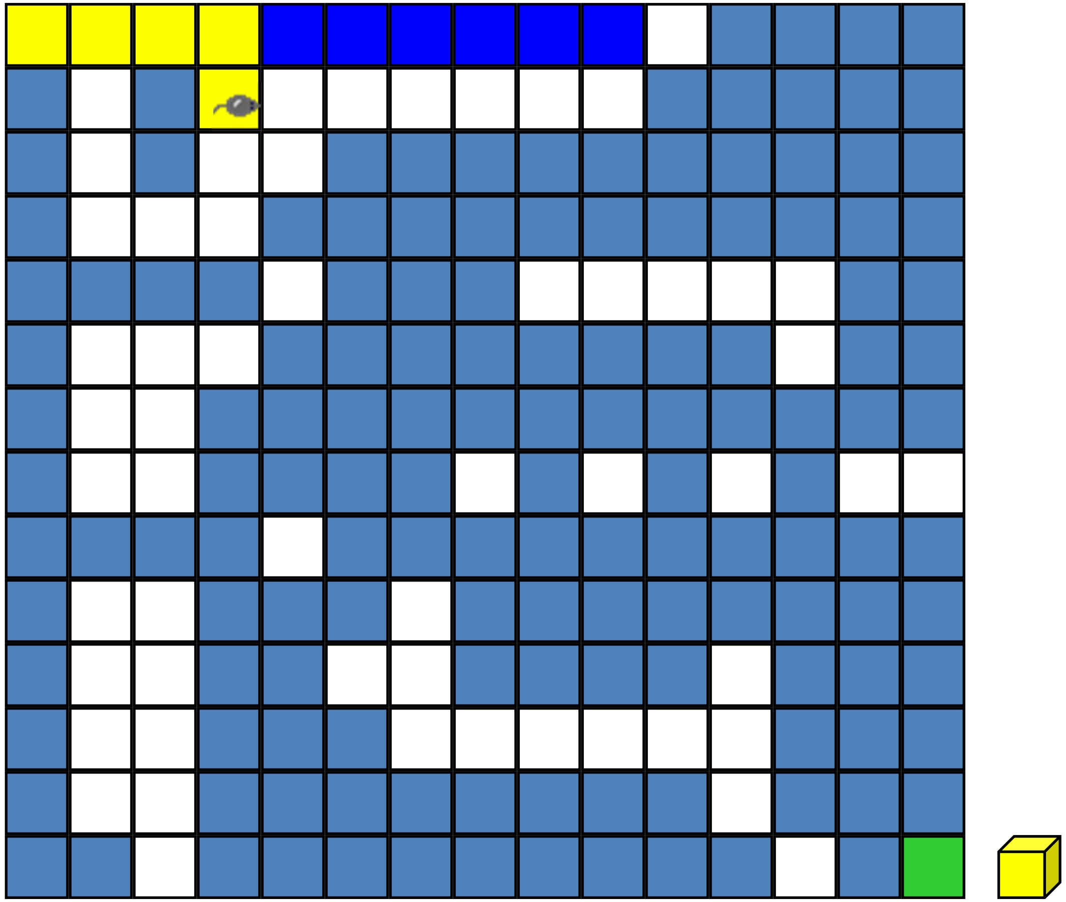
- Move backward until we reach a square from which a forward move is possible.

Rat In A Maze



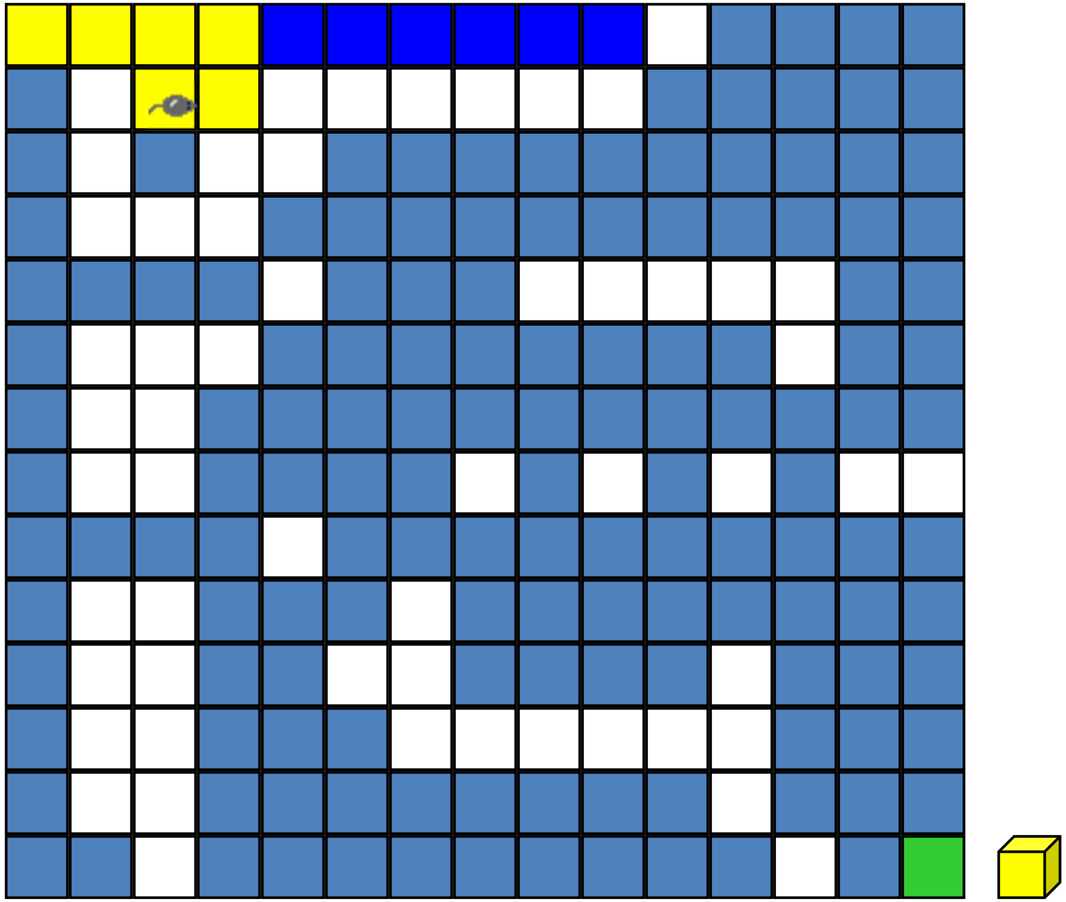
- Move down.

Rat In A Maze



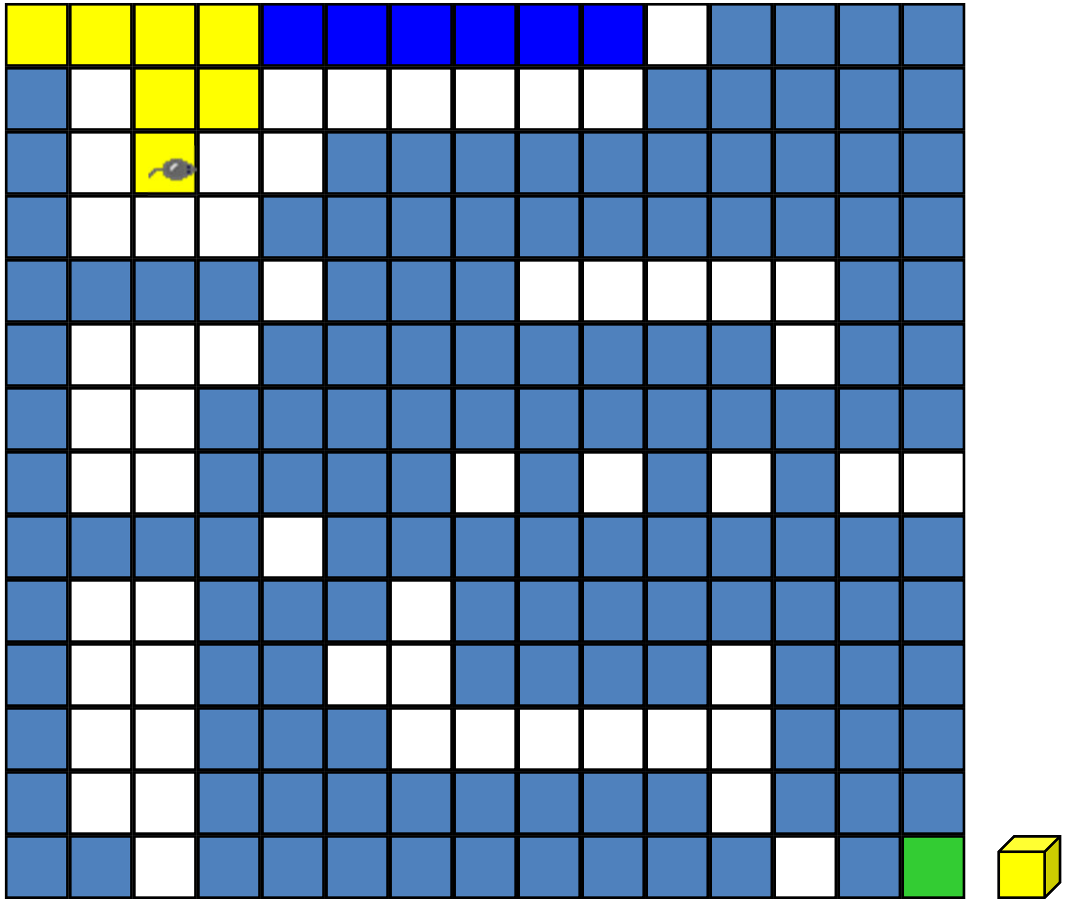
- Move left.

Rat In A Maze



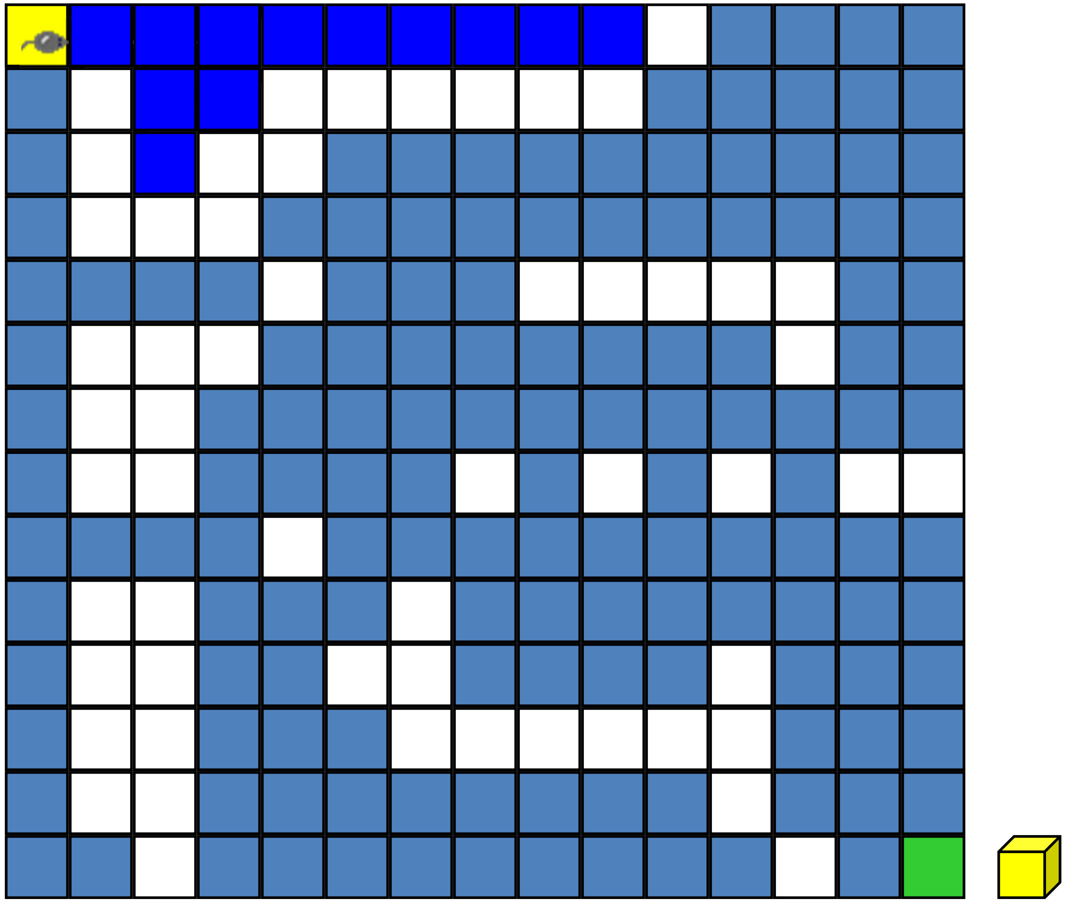
- Move down.

Rat In A Maze



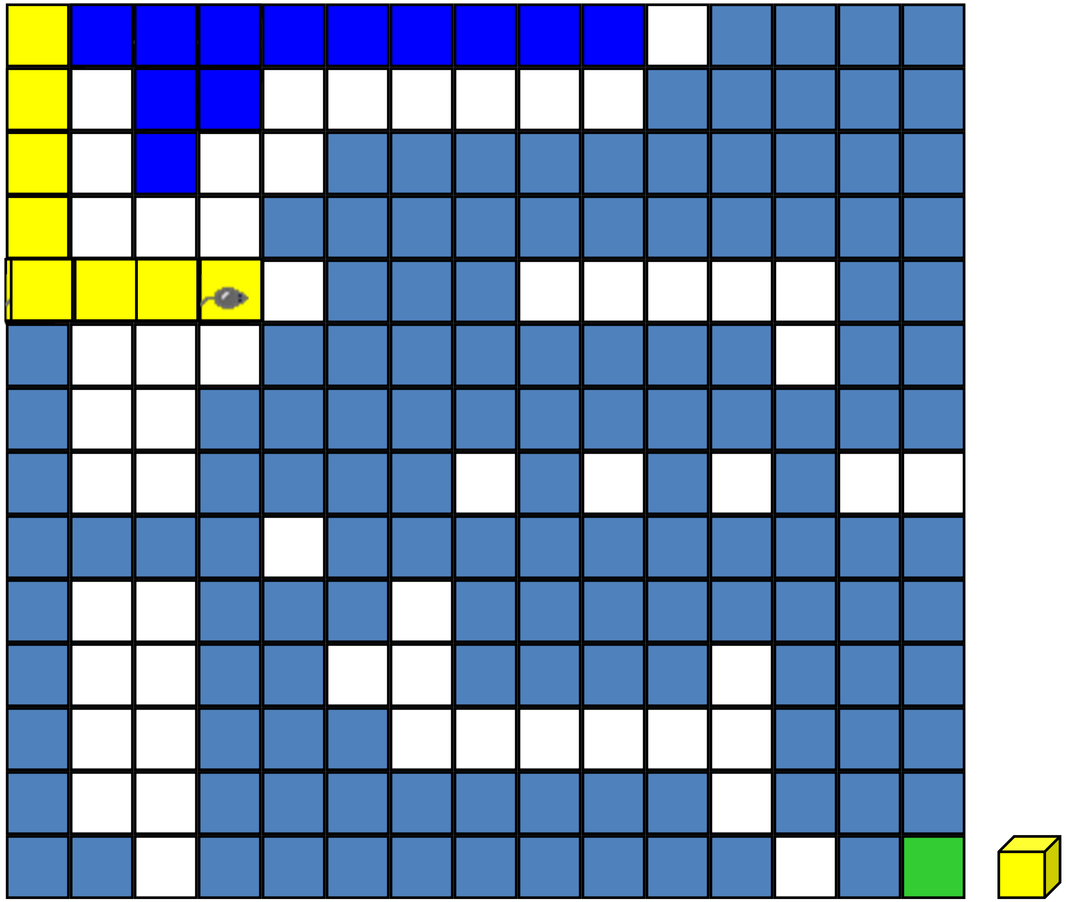
- Move backward until we reach a square from which a forward move is possible.

Rat In A Maze



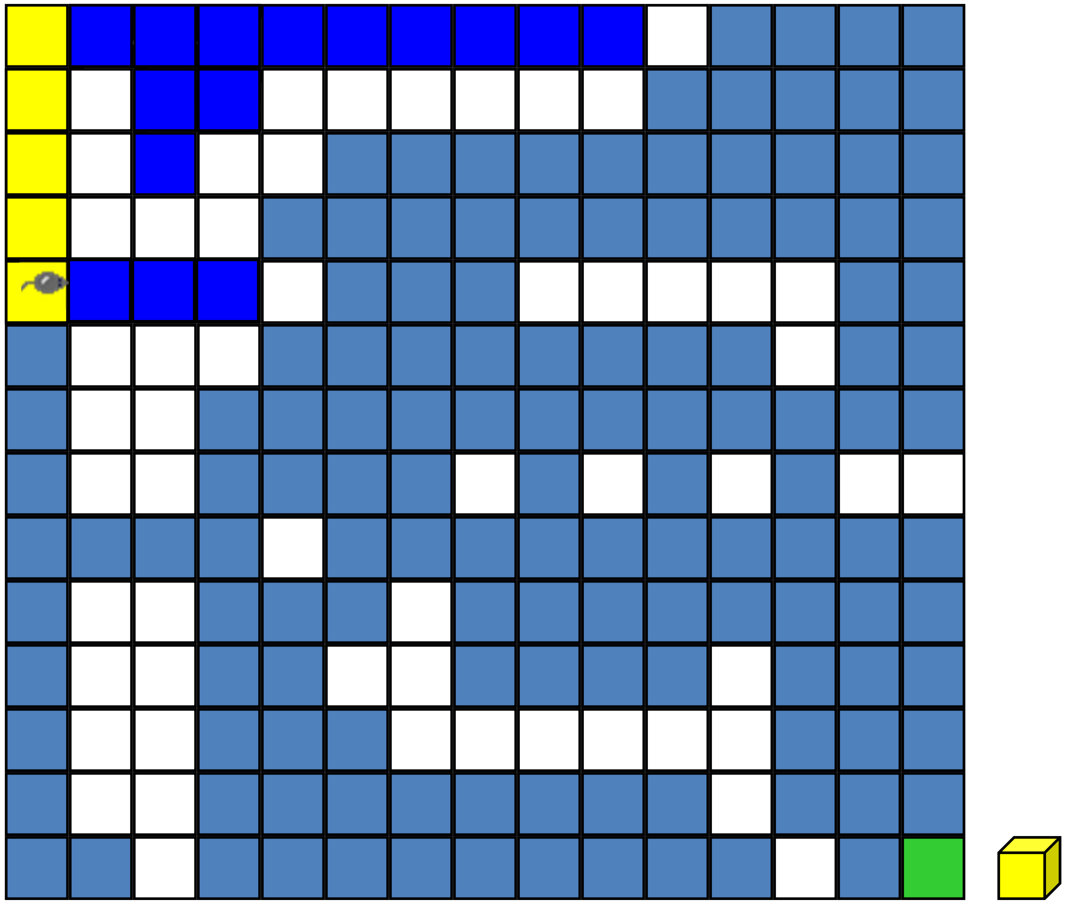
- Move backward until we reach a square from which a forward move is possible.
- Move downward.

Rat In A Maze



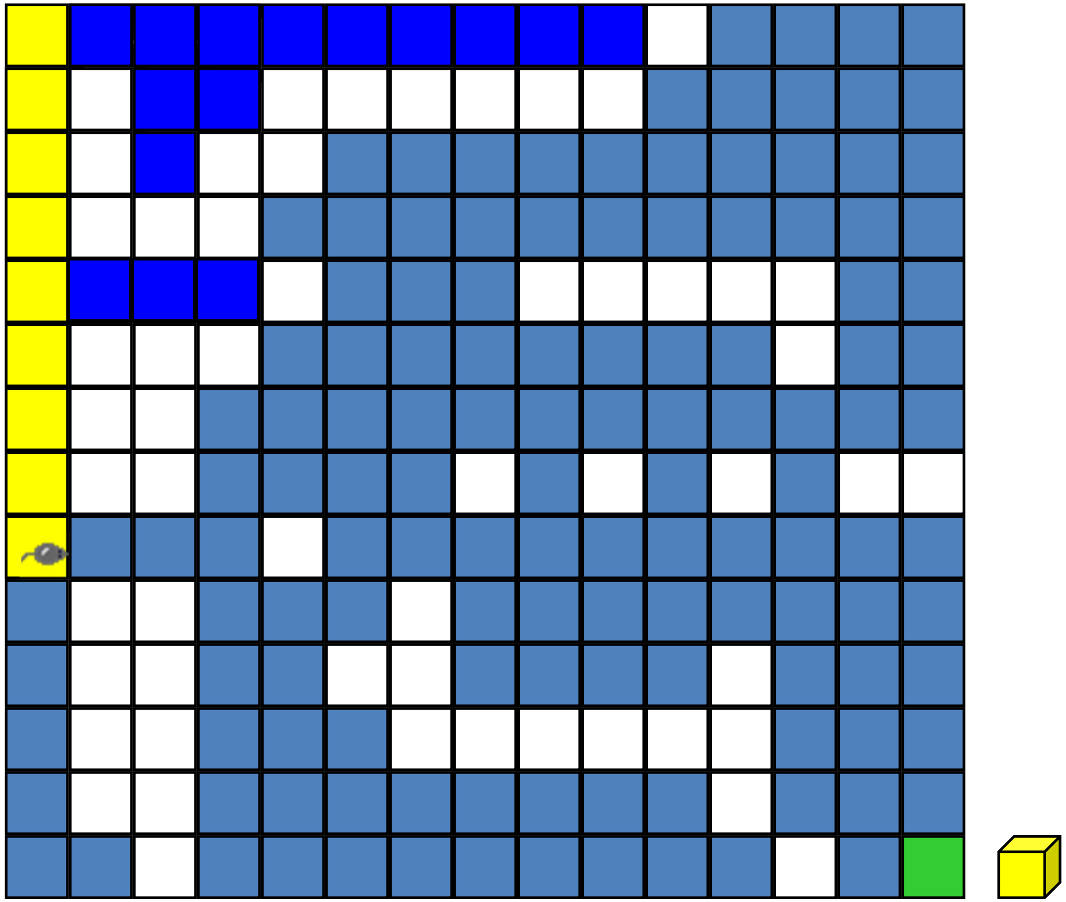
- Move right.
- Backtrack.

Rat In A Maze



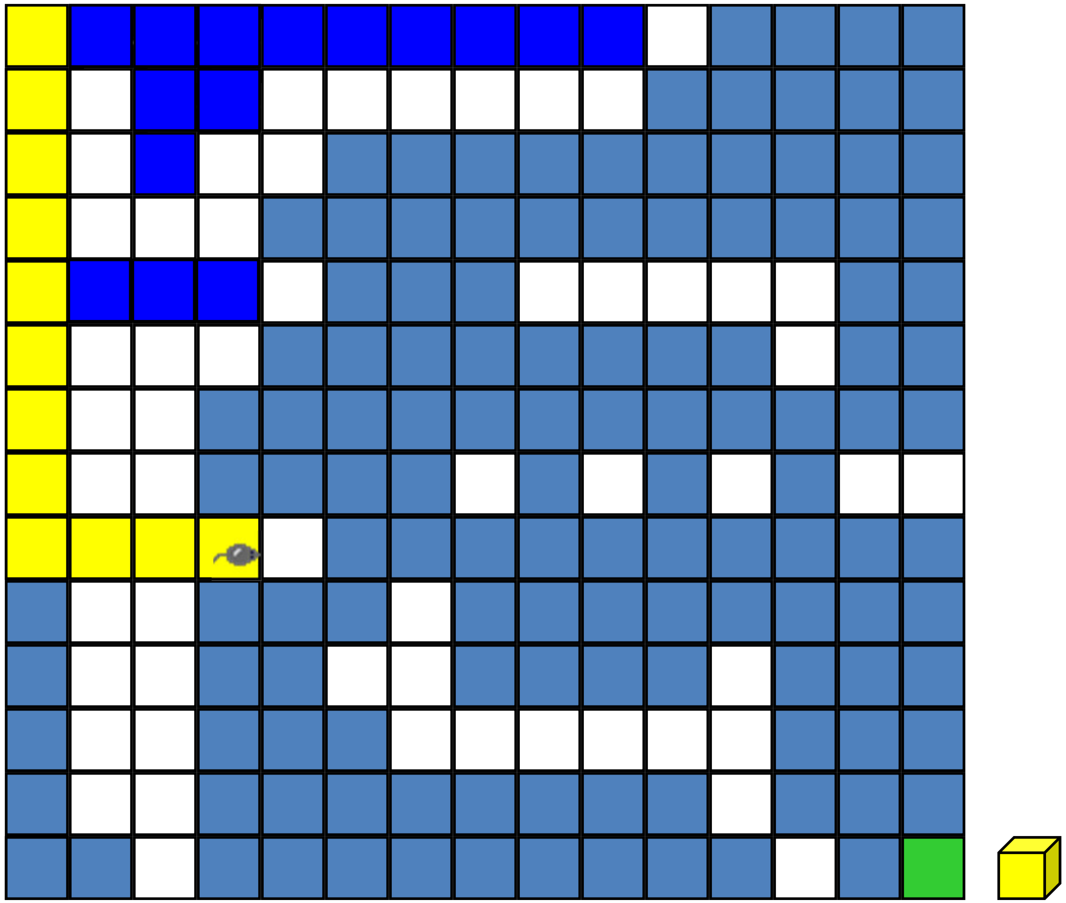
- Move downward.

Rat In A Maze



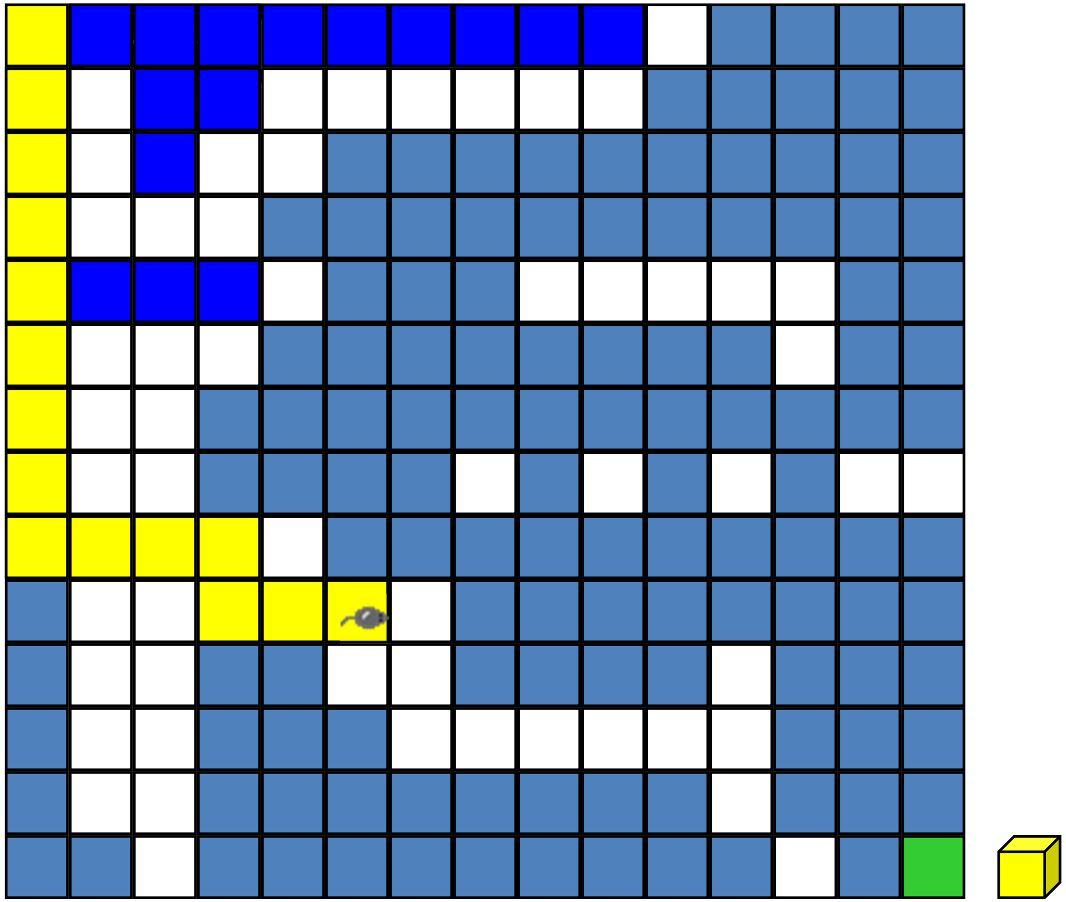
- Move right.

Rat In A Maze



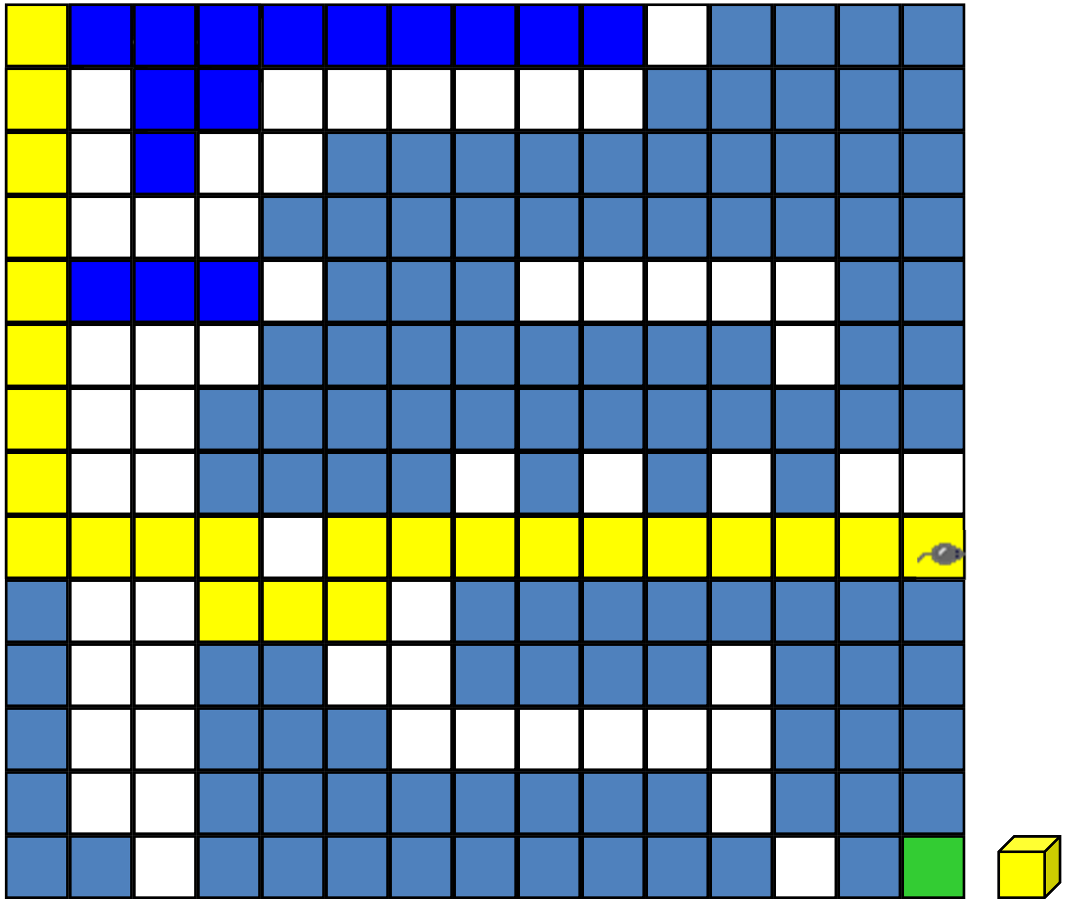
- Move one down and then right.

Rat In A Maze



- Move one up and then right.

Rat In A Maze



- Move down to exit and eat cheese.

Standing... Wondering...

- Move forward whenever **possible**
  - No wall & not visited
- Move back ---- HOW?
  - Remember the footprints
  - OR …… Better?
  - NEXT possible move from previous position
- Storage?
  - STACK

Path from maze entry to current position operates as a stack!

It’s a LONG life …

- How to put an end to this misery?
  - God bless it!
  - Dame it!

    - Whenever exist a possible move from previous positions
    - Whenever the stack is not empty

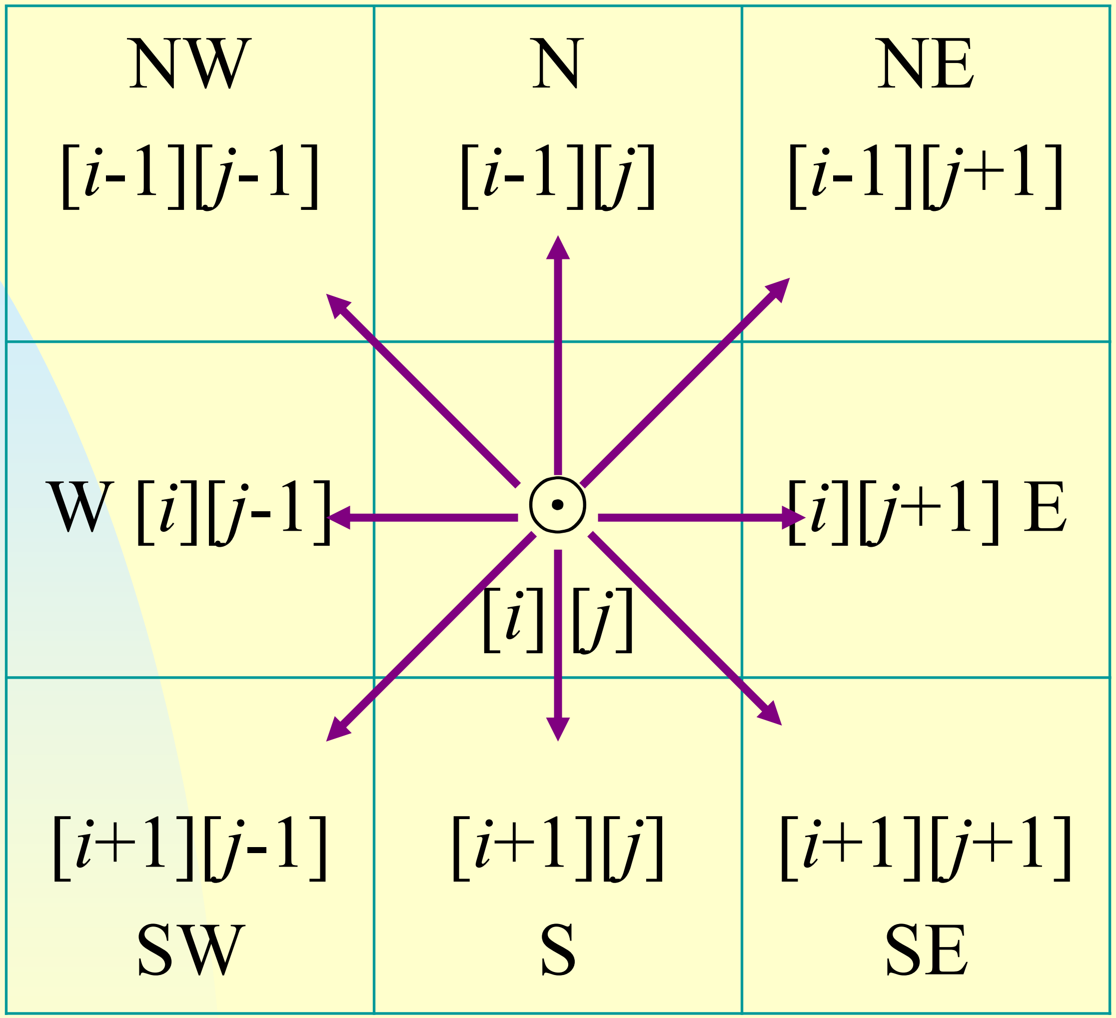
To Do: A Mazing Problem

Problem: find a path from the entrance to the exit of a maze.

entrance	**0**	1	0	0	1	1	0	1	1	
	1	**0**	**0**	1	0	0	1	1	1	
	0	1	1	**0**	1	1	1	0	1	
	1	1	0	**0**	1	**0**	**0**	1	0	
	1	0	0	1	**0**	1	1	**0**	1	
	0	0	1	1	0	1	**0**	1	1	
	0	1	0	0	1	1	**0**	**0**	**0**	**exit**

Representation:

- **maze[i][j], $1 \le i \le m$, $1 \le j \le p$.**
- **1--- blocked, 0 --- open.**
- **the entrance: maze[1][1], the exit: maze[m][p].**
- **current point: [i][j].**
- **boarder of 1’s, so a maze[m+2][p+2].**
- **8 possible moves: N, NE, E, SE, S, SW, W and NW.**


NW
[i-1][j-1]
N
[i-1][j]
NE
[i-1][j+1]
W [i][j-1]
[i] [j]
[i][j+1] E
[i+1][j-1]
SW
[i+1][j]
S
[i+1][j+1]
SE

To predefine the 8 moves:

```
struct offsets
{
   int a, b;
};
enum directions {N, NE, E, SE, S, SW, W, NW};
offsets move[8];
```

q	move[q].a	move[q].b
N	-1	0
NE	-1	1
E	0	1
SE	1	1
S	1	0
SW	1	-1
W	0	-1
NW	-1	-1

Table of moves

Thus, from [i][j] to [g][h] in SW direction:

$g = i$ + move[SW].a;

$h = j$ + move[SW].b;

The basic idea:

Given current position [*i*][*j*] and 8 directions to go, we pick one direction *d*, get the new position [*g*][*h*].

If [*g*][*h*] is the goal, success.

If [*g*][*h*] is a legal position, save [*i*][*j*] and *d*+1 in a stack in case we take a false path and need to try another direction, and [*g*][*h*] becomes the new current position.

Repeat until either success or every possibility is tried.

In order to prevent us from going down the same path twice:

use another array, *mark*[*m*+2][*p*+2], which is initially 0.

mark[*i*][*j*] is set to 1 once the position is visited.

First pass:

Initialize *stack* to the maze entrance coordinates and direction east**;**

while (*stack* is not empty)

{

    (i, j, *dir*) = coordinates and direction from top of *stack***;**

    pop *stack***;**

    while (there are more moves from (i, j))

    {

        (g, h) = coordinates of next move **;**

        if (($g == m$) **&&** ($h == p$)) success**;**

```
if ((!maze[g][h]) && (!mark[g][h])) // legal and not visited
{
    mark[g][h] = 1;
    dir = next direction to try;
    push (i, j, dir) to stack;
    (i, j, dir) = (g, h, N);
}
}
}
cout << "No path in maze."<< endl;
```

We need a stack of items:

```
struct Items {
        int x, y, dir;
};
```

Also, to avoid doubling array capacity during stack pushing, we can set the size of stack to *m*p*.

Now a precise maze algorithm.

```
void Path(const int m, const int p)
{ //Output a path (if any) in the maze; maze[0][i] = maze[m+1][i]
  // = maze[j][0] = maze[j][p+1] = 1, 0 ≤ i ≤ p+1, 0 ≤ j ≤ m+1.
    // start at (1,1)
    mark[1][1]=1;
    Stack<Items> stack(m*p);
    Items temp(1, 1, E);
    stack.Push(temp);
    while (!stack.IsEmpty())
    {
        temp = stack.Top();
        stack.Pop();
        int i = temp.x; int j = temp.y; int d = temp.dir;
```

```
while ( d < 8)
{
    int g = i + move[d].a; int h = j + move[d].b;
    if ((g == m) && (h == p)) { // reached exit
   // output path
        cout << stack;
        cout << i << " " << j << " " << d << endl;
        cout << m<< " " << p << endl;   // last two points
        return;
    }
```

```
            if ((!maze[g][h]) && (!mark[g][h])) { //new position
                mark[g][h]=1;
                temp.x = i; temp.y = j; temp.dir = d+1;
                stack.Push(temp);
                i = g ; j = h ; d = N;   // move to (g, h)
            }
            else d++;  // try next direction
        }
    }
    cout << "No path in maze."<< endl;
}
```

The operator << is overloaded for both *Stack* and *Items* as:

```
template <class T>
ostream& operator << (ostream& os, Stack<T>& s)
{
   os << “top = “<<s.top<< endl;
   for (int i = 0; i <= s.top; i++);
      os << i << “:” << s.stack[i] << endl;
   return os;
}
```

We assume << can access the private data member of *Stack* through the friend declaration.

```
ostream& operator << (ostream& os, Items& item)
{
   return os<<item.x<<“,”<<item.y<<“,”<<item.dir-1;
   // note item.dir is the next direction to go so the current
   // direction is item.dir-1.
}
```

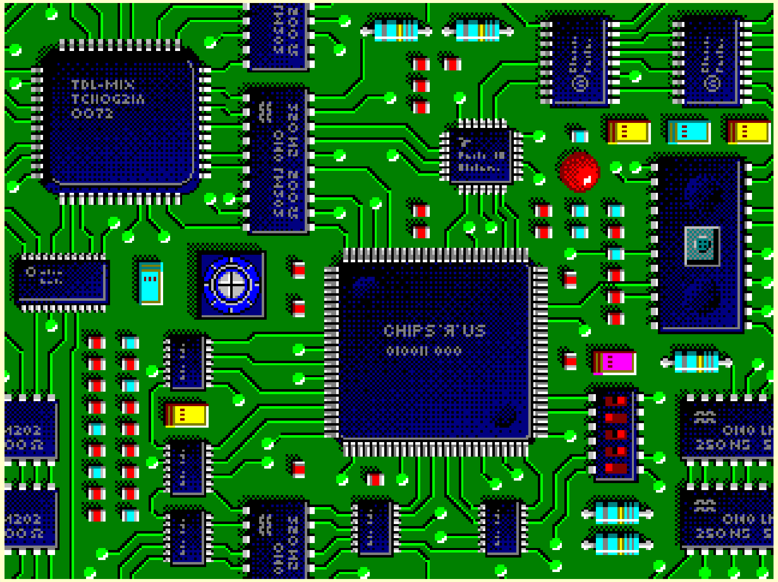
Since no position is visited twice, the worst case computing time is O(*m*p*).**

Special Case

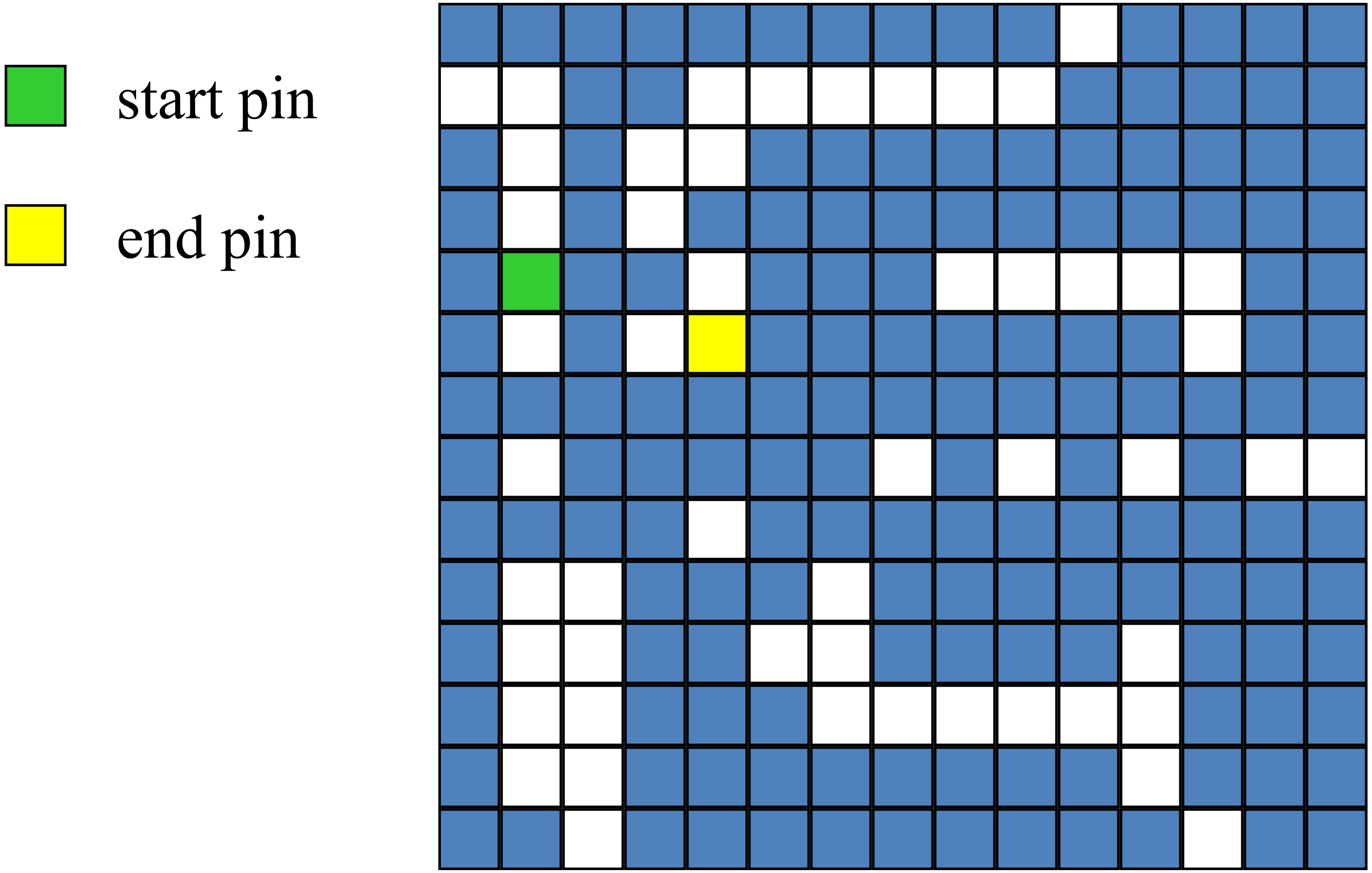
entrance	0	0	0	0	0	0	0	0	1	
	1	1	1	1	1	1	1	1	0	
	1	0	0	0	0	0	0	0	1	
	0	1	1	1	1	1	1	1	1	
	1	0	0	0	0	0	0	0	1	
	1	1	1	1	1	1	1	1	0	
	1	0	0	0	0	0	0	0	1	
	0	1	1	1	1	1	1	1	1	
	1	0	0	0	0	0	0	0	0	exit

Queue instead of Stack?

Wire Routing

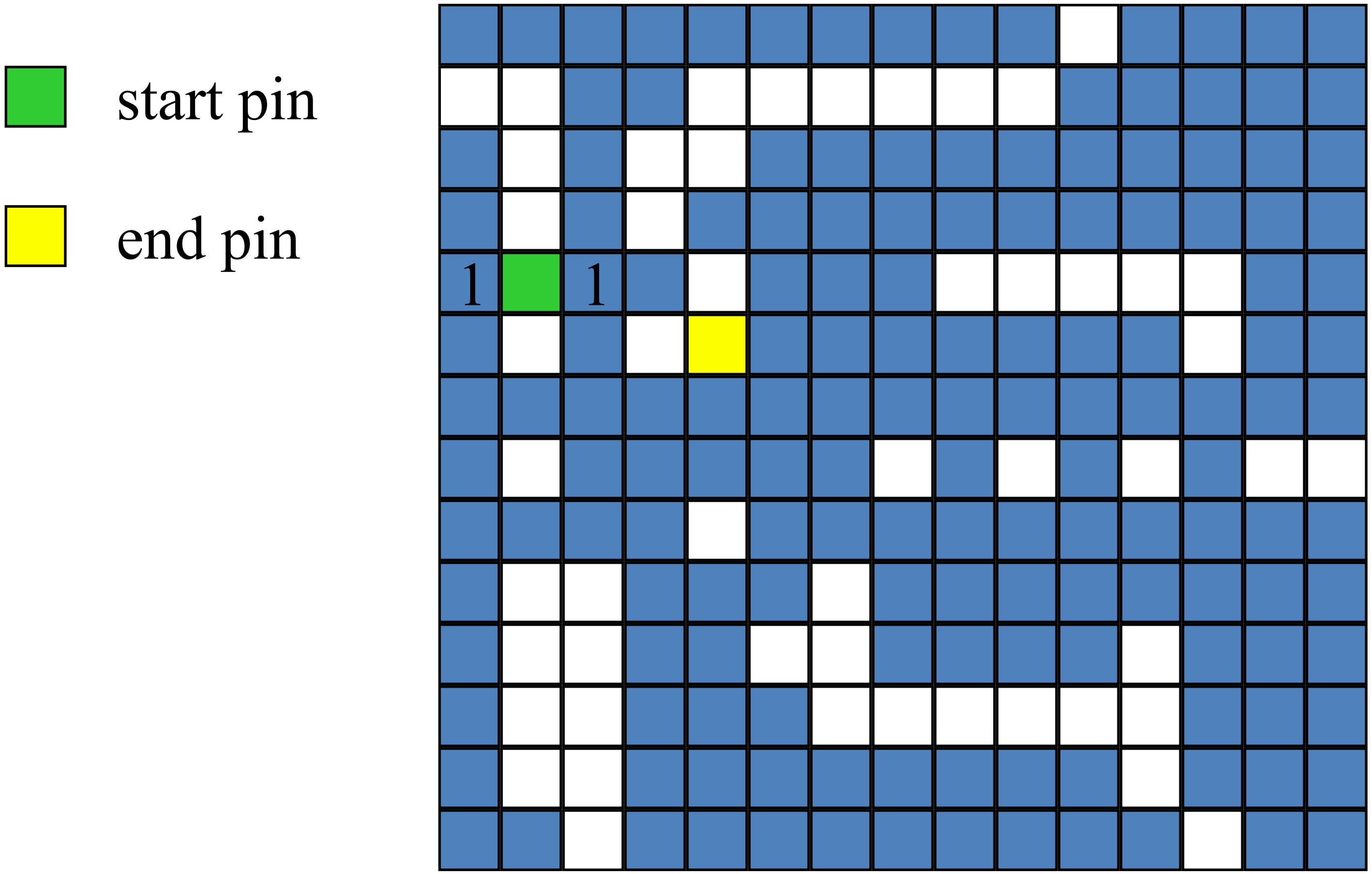


Lee’s Wire Router



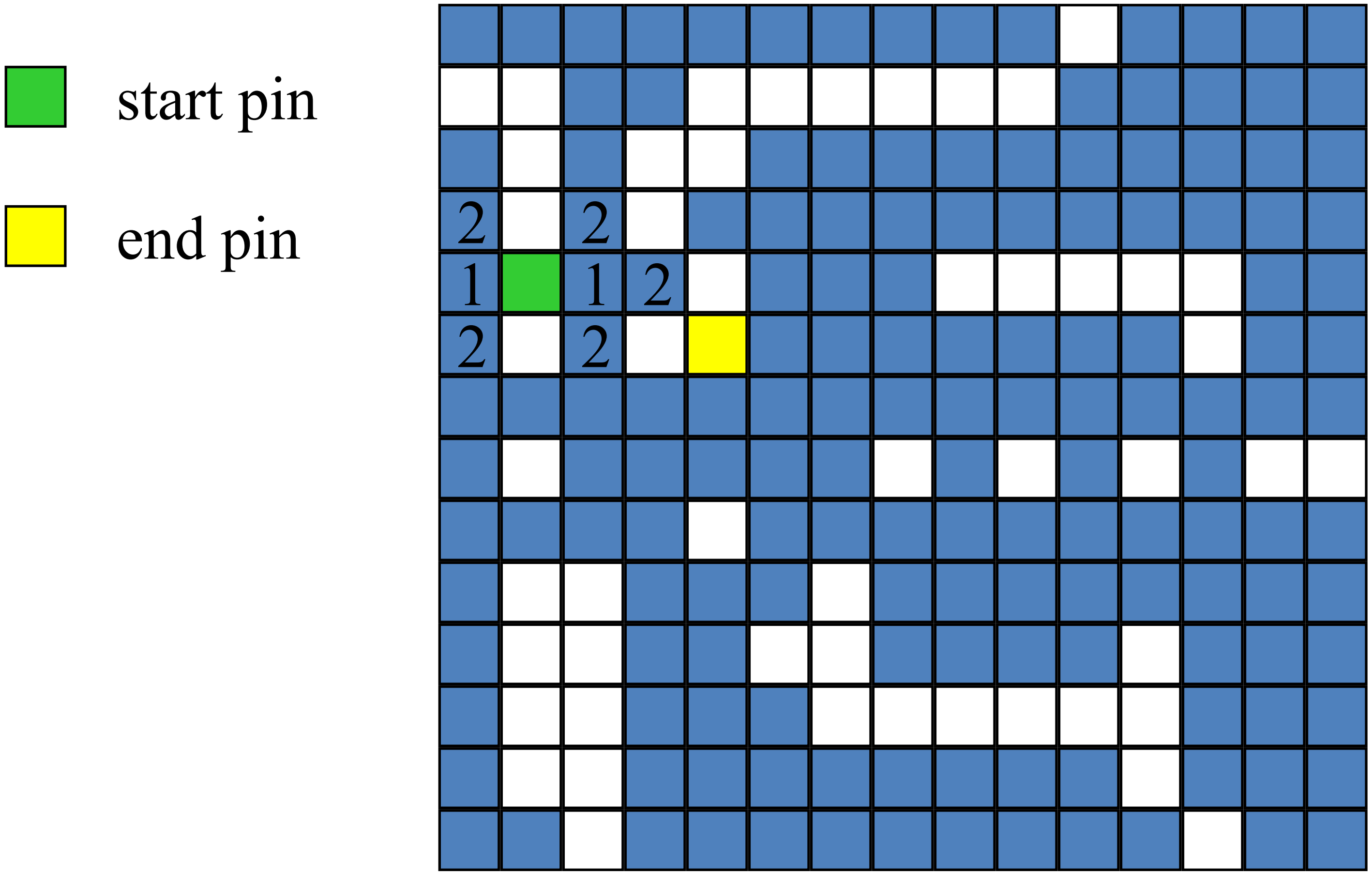
Label all reachable squares 1 unit from start.

Lee’s Wire Router



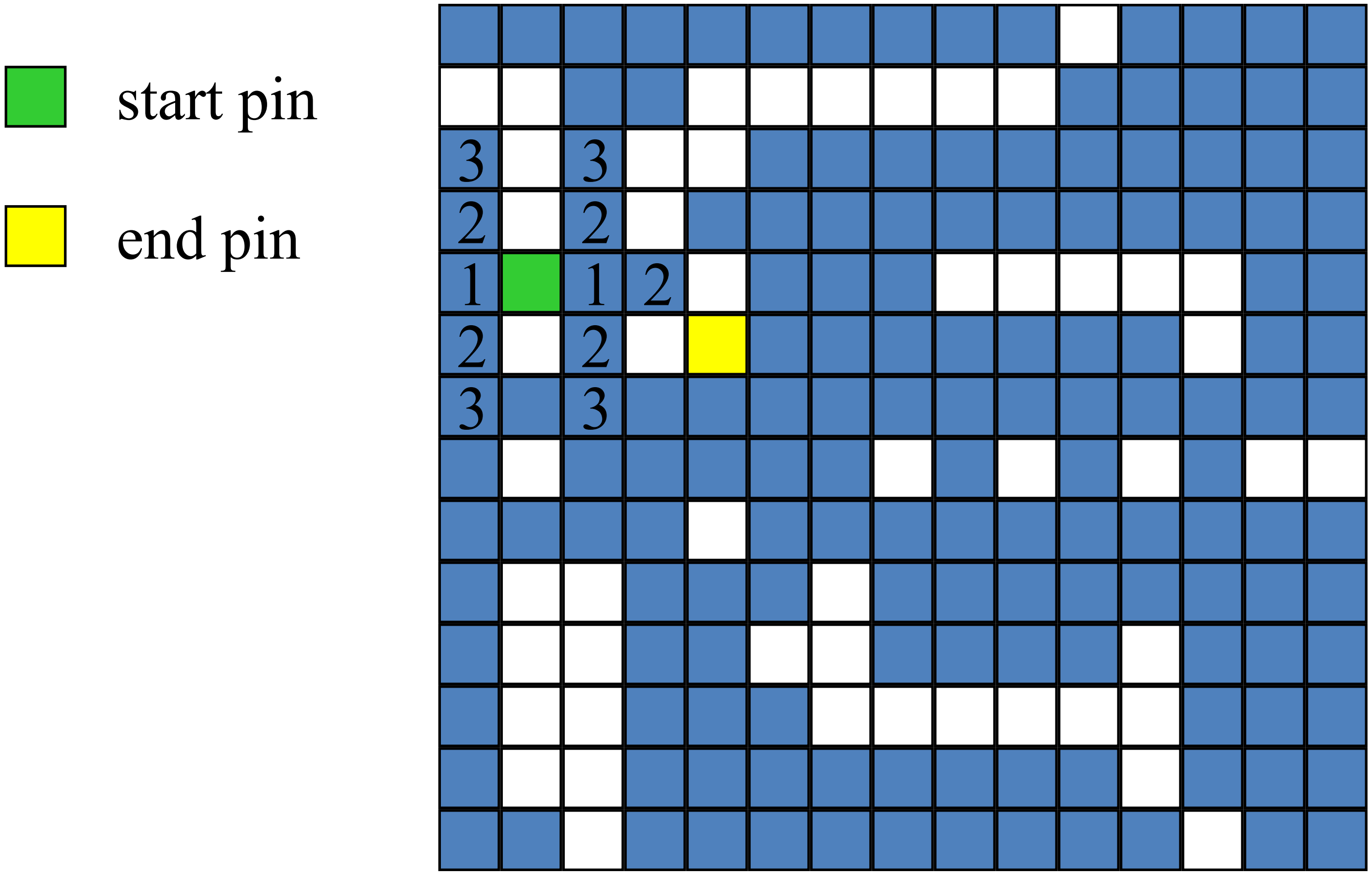
Label all reachable unlabeled squares 2 units from start.

Lee's Wire Router



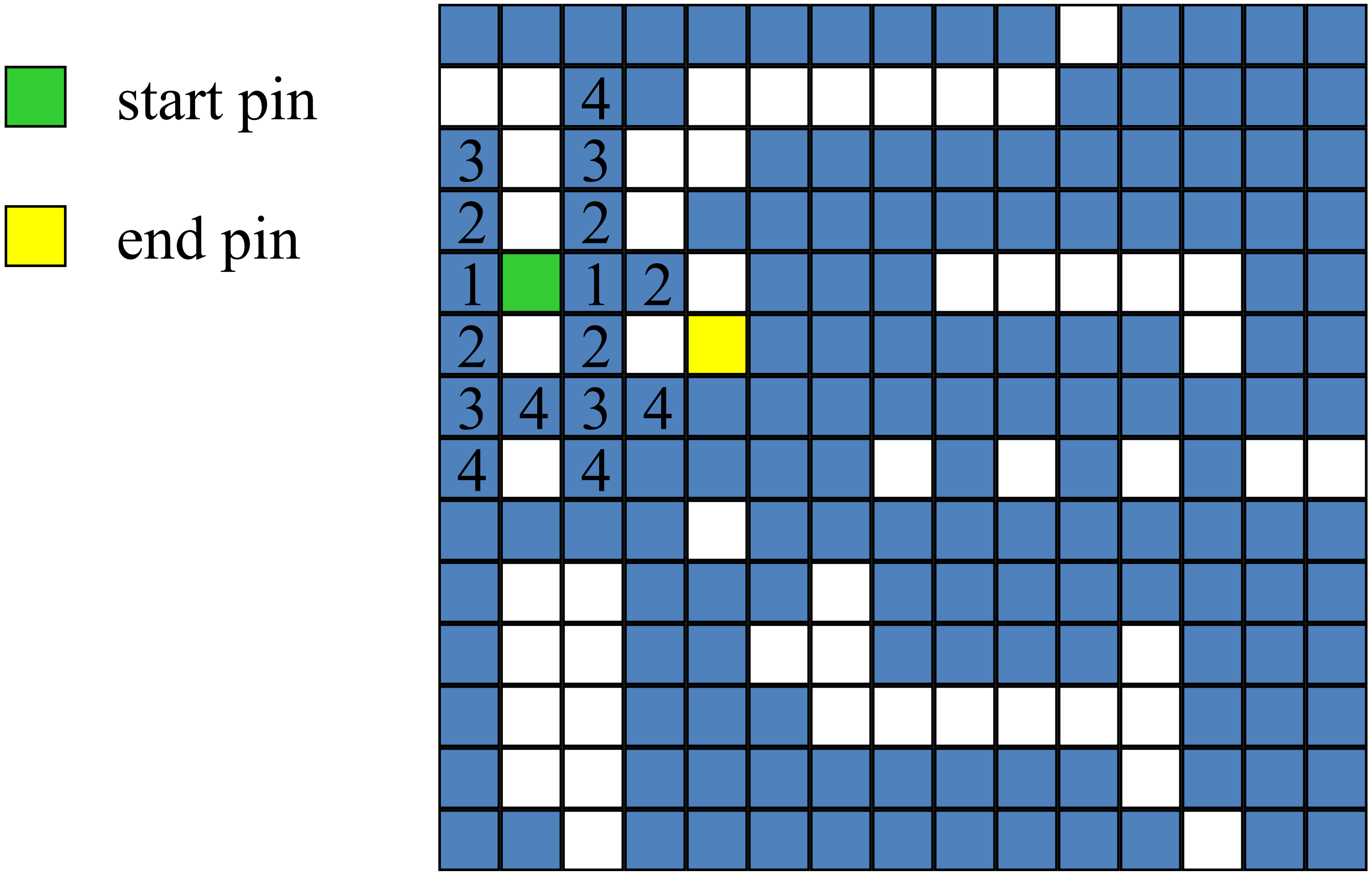
Label all reachable unlabeled squares 3 units from start.

Lee’s Wire Router



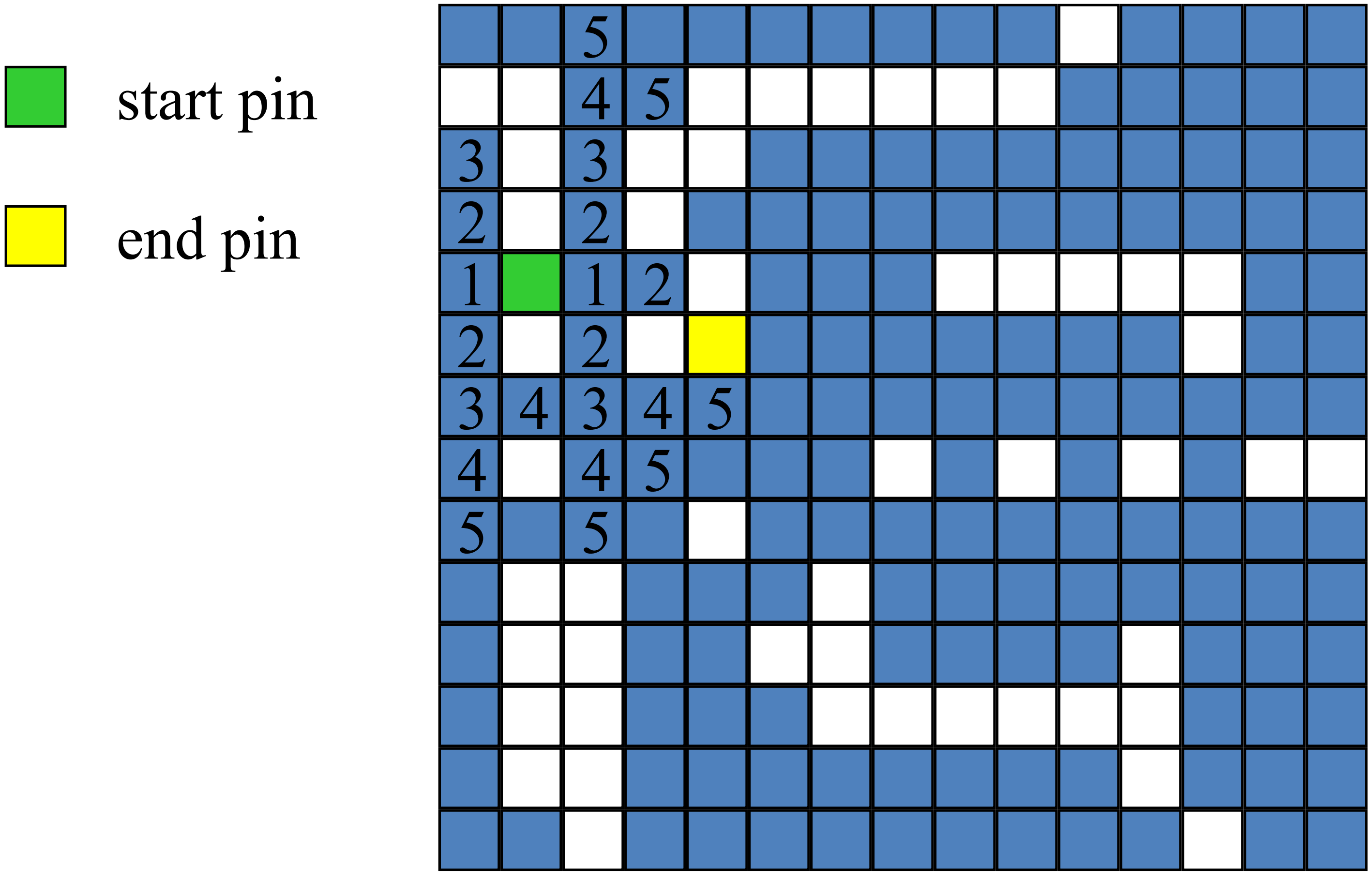
Label all reachable unlabeled squares 4 units from start.

Lee’s Wire Router



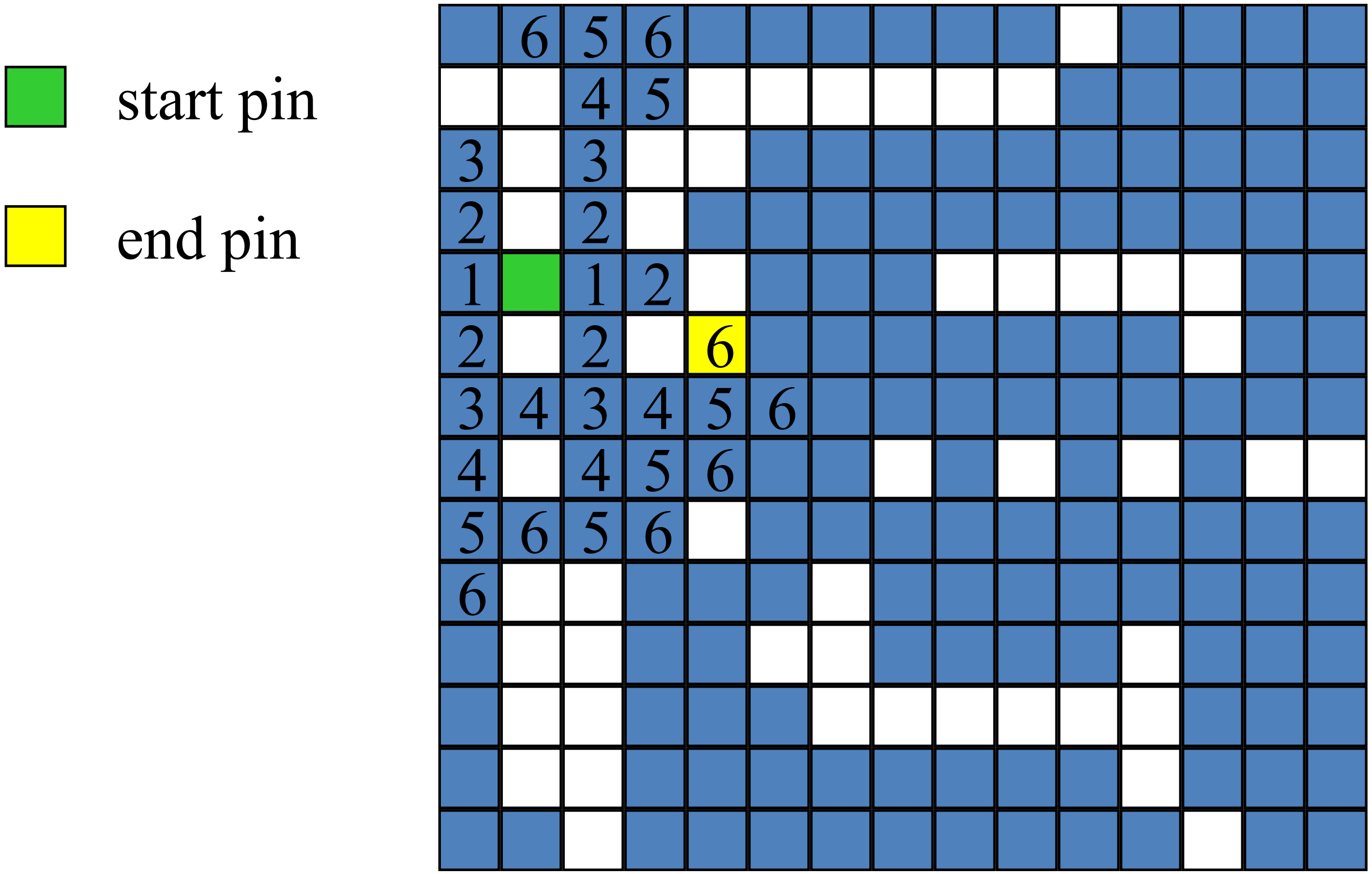
Label all reachable unlabeled squares 5 units from start.

Lee’s Wire Router



Label all reachable unlabeled squares 6 units from start.

Lee’s Wire Router



End pin reached. Traceback.

```
Queue Q;
Q.Push (startPin);
while(!Q.isEmpty())
{
    Pin p = Q.Front();
    Q.Pop();
    for all neighbours pi of p
        Visit(pi)
        Q.Push (pi);
}
```

Arithmetic Expressions

(a + b) * (c + d) + e – f/g*h + 3.25

Expressions comprise three kinds of entities.

- Operators (+, -, /, *).
- Operands (a, b, c, d, e, f, g, h, 3.25, (a + b), (c + d), etc.).
- Delimiters ((,)).

Operator Degree

Number of operands that the operator requires.

Binary operator requires two operands.

a + b

c / d

e - f

Unary operator requires one operand.

+ g

- h

Infix Form

Normal way to write an expression.

Binary operators come in between their left and right operands.

a * b

a + b * c

a * b / c

(a + b) * (c + d) + e – f/g*h + 3.25

Operator Priorities

How do you figure out the operands of an operator?

a + b * c

a * b + c / d

This is done by assigning operator priorities.

priority(*) = priority(/) > priority(+) = priority(-)

When an operand lies between two operators, the operand associates with the operator that has higher priority.

Tie Breaker

When an operand lies between two operators that have the same priority, the operand associates with the operator on the left.

a + b - c

a * b / c / d

Delimiters

Subexpression within delimiters is treated as a single operand, independent from the remainder of the expression.

(a + b) * (c – d) / (e – f)

Infix Expression Is Hard To Parse

- Need operator priorities, tie breaker, and delimiters.
- This makes computer evaluation more difficult than is necessary.
- Postfix and prefix expression forms do not rely on operator priorities, a tie breaker, or delimiters.
- So it is easier for a computer to evaluate expressions that are in these forms.

Postfix Form

The postfix form of a variable or constant is the same as its infix form.

a, b, 3.25

The relative order of operands is the same in infix and postfix forms.

Operators come immediately after the postfix form of their operands.

Infix = a + b

Postfix = ab+

Postfix Examples

Infix = a + b * c

Postfix = a b c * +

- Infix = a * b + c

  Postfix = a b * c +

- Infix = (a + b) * (c – d) / (e + f)

  - Postfix = a b + c d - * e f + /

Unary Operators

Replace with new symbols.

+ a => a @

+ a + b => a @ b +

- a => a ?

- a-b => a ? b -

Problem:

how to evaluate an expression?

postfix: A B / C – D E * + A C * –

Read the postfix left to right to evaluate it:

A B / C – D E * + A C * –

operation postfix

T_6 is the result.

Virtues of postfix:

- **no need for parentheses**
- **the priority of the operators is no longer relevant**

Idea:

✓**make a left to right scan**

✓**store operands**

✓**evaluate operators whenever occurred**

What data structure should be used?

STACK

void *Eval*(*Expression e*)

{ // evaluate the postfix expression *e*. It is assumed that the

// last token in *e* is ‘#’. A function *NextToken* is used to get

// the next token from *e*. Use *stack*.

Stack<*Token*> *stack*; //initialize *stack*

for(*Token x* = *NextToken*(*e*)**;** *x* != ‘#’**;** *x* = *NextToken*(*e*))

if (*x* is an operand) *stack*.*Push*(*x*)**;**

else { // operator

remove the correct number of operands for operator *x*

from *stack***;** perform the operation *x* and store the result

(if any) onto the stack**;**

}

}

Problem: how to evaluate an infix expression?

Solution:

1. Translate from infix to postfix;

2. Evaluate the postfix.

Infix to Postfix

Idea: note the order of the operands in both infix and postfix

infix: A / B – C + D * E – A * C

postfix: A B / C – D E * + A C * –

immediately passing any operands to the output

store the operators somewhere until the right time.

A*(B+C)*D → ABC+*D*

A*(B+C)*D

→ABC+*D*

Next token	storage	output

Attention

What Storage?

From the example, we can see the left parenthesis behaves as an operator with high priority when it is not in the stack, whereas once it get in, it behaves as one with low priority.

isp (in-stack priority)

icp (in-coming priority)

the isp and icp of all operators in Fig. 3.15 remain unchanged

isp('(')=8, icp('(')=0, isp('#')=8

Hence the rule:

Operators are taken out of stack as long as their isp is numerically less than or equal to the icp of the new operator.

```
void Postfix (Expression e)
{ // output the postfix of the infix expression e. It is assumed
  // that the last token in e is '#'. Also, '#' is used at the bottom
  // of the stack.
    Stack<Token> stack;  //initialize stack
    stack.Push('#');
```

```
for  (Token x = NextToken(e); x != '#'; x = NextToken(e))
  if (x is an operand) cout << x;
  else if (x == ')')
     { // unstack until '('
       for  (; stack.Top()!='('; stack.Pop())
            cout << stack.Top();
       stack.Pop();  // unstack '('
      }
  else { // x is an operator
      for (; isp(stack.Top()) <= icp(x); stack.Pop())
         cout << stack.Top();
      stack.Push(x);
   }
// end of expression, empty the stack
for (; !stack.IsEmpty(); cout << stack.Top(), stack.Pop());
cout << endl;
}
```

Analysis:

Computing time: one pass across the input with *n* tokens, O(*n*).

The stack will not be deeper than 1 (‘#’) + the number of operators in *e*.

Can we evaluate infix expressions directly?

infix: A / B – C + D * E – A * C

实验&作业

- 实验：P167-169: 1, 2 （任选一个Project）

  实验课上检查验收后提交源代码

- 作业：P165-166：1, 2, 3(a)。

  提交截止时间：10月23日晚22:00之前

注意：作业提交电子版，发送到助教邮箱，（可WORD/Latex编辑；也可手写拍照），建议文件格式为PDF，文件命名格式为“学号_姓名_第3章作业”