Collinearity of Points

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Problem Statement

Prove that the points $(2, -1, 3)$, $(3, -5, 1)$, and $(-1, 11, 9)$ are collinear using vectors.

Solution

Let $A = (2, -1, 3)$, $B = (3, -5, 1)$, and $C = (-1, 11, 9)$.
Construct the vectors:

$$\overrightarrow{B - A} = (3 - 2, -5 + 1, 1 - 3) = (1, -4, -2)$$

$$\overrightarrow{C - A} = (-1 - 2, 11 + 1, 9 - 3) = (-3, 12, 6)$$

Matrix Representation

Next, we construct the matrix using these vectors:

$$\text{Matrix} = \begin{pmatrix} \overrightarrow{B-A} & \overrightarrow{C-A} \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ -4 & 12 \\ -2 & 6 \end{pmatrix}$$

Row Reduction

Now, we perform row reduction:

$$\begin{pmatrix} 1 & -3 \\ -4 & 12 \\ -2 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -3 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Since the matrix has rank 1 (only one non-zero row), the points are collinear.

C-Code

```c
#include <stdio.h>

int main() {
    // Define points
    double points[3][3] = {
        {2.0, -1.0, 3.0},
        {3.0, -5.0, 1.0},
        {-1.0, 11.0, 9.0}
    };

    // Open file for writing
    FILE *file = fopen("points.txt", "w");
    if (file == NULL) {
        return 1;  // Exit if file cannot be opened
    }
```

C-Code

```c
// Write points to the file
    for (int i = 0; i < 3; i++) {
        fprintf(file, "%lf %lf %lf\n", points[i][0],
        ↪  points[i][1], points[i][2]);
    }

    fclose(file);
    return 0;
}
```

C-Code Output

2.000000 -1.000000 3.000000

3.000000 -5.000000 1.000000

-1.000000 11.000000 9.000000

Python Code

```python
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Read points from the file
points = []
with open("points.txt", "r") as file:
    for line in file:
        # Remove parentheses and split by commas
        point = line.strip()[1:-1].split(", ")
        points.append([float(coord) for coord in point])

# Convert points to a numpy array for easier handling
points = list(zip(*points))  # Unzips the list of points

# Create a 3D plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

Python Code

```python
# Plot the points
ax.scatter(points[0], points[1], points[2], color='red', s=100)

# Connect the points with a line
ax.plot(points[0], points[1], points[2], color='blue')

# Annotate the points
for i, point in enumerate(zip(*points)):
    ax.text(point[0], point[1], point[2], f'P{i+1} {point}',
    ↪  size=10, zorder=1)

# Set labels
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('3D Plot of Points')
# Show the plot
plt.show()
```
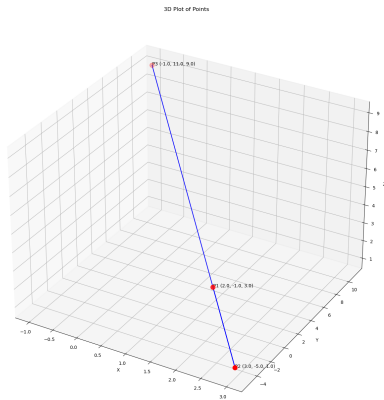
Graphical Representation

3D Plot of Points

Conclusion

The points $(2, -1, 3)$, $(3, -5, 1)$, and $(-1, 11, 9)$ are confirmed to be collinear based on the row reduction of the constructed matrix.