3.Quadratic Equation and Inequations (Inequalities)

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13) Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution. (1982-2 Marks)
14) mn squares of equal size are arranged to form a rectangle of dimensions m by n, where m and n are natural numbers. Two squares will be called 'neighbours' if they have exactly one common side. A natural number is written in each square such that the number written in any square is the arithmetic mean of the numbers written in its neighbouring squares.Show that this is possible only if all the numbers used are equal. (1982-5 Marks)
15) If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then show that

$$ac^{n\frac{1}{n+1}} + a^n c^{\frac{1}{n+1}} + b = 0.$$

(1983-2 Marks)
16) Find all real values of x which satisfy $x^2 - 3x + 2 > 0$ and $x^2 - 2x - 4 \leq 0$. (1983-2 Marks)
17) Solve for x; $5 + 2\sqrt{6}^{x^2-3} + 5 - 2\sqrt{6}^{x^2-3} = 10$. (1985-5 Marks)
18) For $a \leq 0$, determine all real roots of the equation $x^2 - 2ax - a - 3a^2 = 0$. (1986-5 Marks)
19) Find the set of all x for which $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$ (1987-3 Marks)
20) Solve $x^2 + 4x + 3 + 2x + 5 = 0$. (1988-5 Marks)
21) Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β , where α < -1 and β > 1 , then show that $1 + \frac{c}{a} + \frac{b}{a} < 0$. (1995- 5 Marks)
22) Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c, and d denote the lengths of the sides of the quadrilateral, prove that $2 \leq a^2 + b^2 + c^2 + d^2 \leq 4$. (1997-5 Marks)
23) If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta$, $\beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then prove that $\frac{b^2-4ac}{a^2} = \frac{B^2-4AC}{A^2}$ (2000-4 Marks)
24) Let a, b, c be real numbers with $a \neq 0$ and let α,β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. (2001-4 Marks)
25) If $x^2 + a - bx + 1 - a - b = 0$ where $a, b \in R$ then find the values of a for which equation has unequal real roots for all values of b. (2003-4 Marks)
26) If a, b, c are positive real numbers. Then prove that $a + 1^7 b + 1^7 c + 1^7 > 7^7 a^4 b^4 c^4$. (2004-4 Marks)
27) Let a and b be the roots of the equation $x^2 - 10cx - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$ are c, d then the value of $a = b + c + d$, when $a \neq b \neq c \neq d$, is. (2006-6 Marks)