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ECS 170

Proving Admissability

Exponential heuristic:

```
public double getHeuristic(point p){
double differenceInHeight = map.getTile(map.getEndPoint())-map.getTile(p);
double chebyshevDistance = Math.max(Math.abs(p.x-
map.getEndPoint().x),Math.abs(p.y-getEndPoint().y));
return differenceInHeight + chebyshevDistance;

}
```

Proof level 1: proving admissibility

On the chess board the Chebyshev Distance will give us the least amount of tiles we need to traverse between the start state and the goal state. Take the example of the goal state which is 4 across(x axis) and 3 on the y axis from the start state. Now, since we can go diagonally, we would be taking a step in the direction of both x and y axis. In this case, we would have finished traversing the y distance before the x distance. Thus, we would need a minimum of 4 steps in order to reach the goal state.

There are three possible cases of the start and goal tiles that we need to consider:

1. The goal tile is on the same height as the start tile
2. The goal tile is higher than the start tile
3. The goal tile is lower than the start tile

Case 1:

If the goal tile is on the same height as the start tile, the best case for a least cost path would be the one that takes the shortest amount of tiles, and the path would have to be flat(that is all the tiles in between are also on the same height). The reason for this, going down one unit height and coming back up to the same height will ultimately cost more than taking two steps on tiles that are on the same height.
Math: $1/e + e^1 < e^0 + e^0$

Thus, since the dirrerenceInHeight is 0, we would only be returning the chebyshevDistance which would be the least possible number of tiles with each tile costing 1.

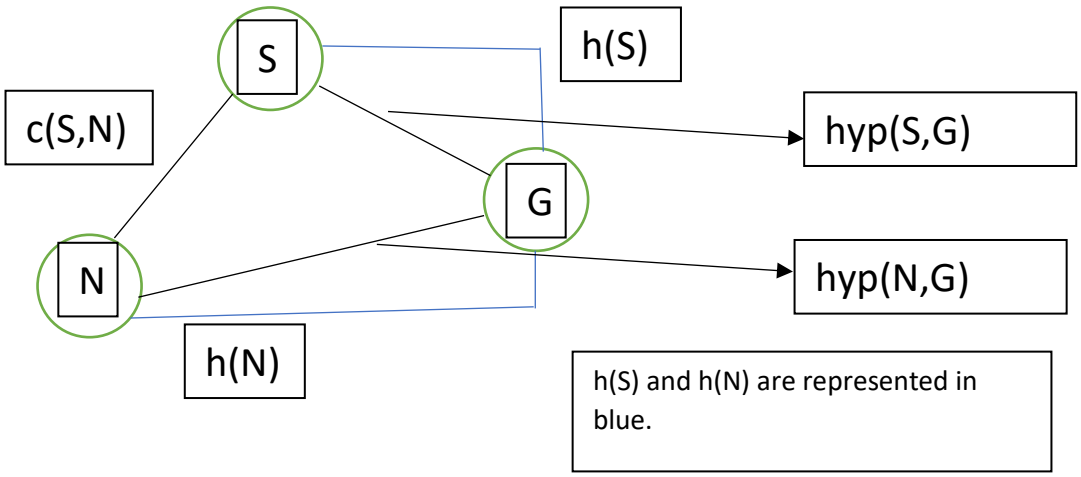
Case 2:

If the goal is higher than the start state the diferrenceInHeight would be a positive number. It will assume that the cost to cover one unit of height would be 1 and not e which is clearly and underestimate. Thus, the difference in height + chebyshev's distance to the tile < The true cost.

Case 3:

If the goal is lower than the start state the difference in height would be a negative number. It will assume that the cost to go down one unit of height would be -1 and not 1/e which is also an underestimate. Thus the difference in height + chebyshev's distance to the tile < the true cost.

Proof level 2: proving consistent



In the diagram above S is the start state, N is its neighboring state, and G is the goal state. The x distance is the chebyshev's distance(least possible number of tiles), while the y distance is the difference in height. Thus h(N) and h(S), go across(chebyshev's distance) and then up or down(difference in height). C(S,N) is the true cost for going from node S to N. hyp(S,G) and hyp(N,G) are the hypotenuses of h(S) and h(N) respectively. Another important aspect to notice is that node S is higher than G while node N is lower than G.

First let's take the triangle of S, N and G:
C(S,N) + hyp(N,G) > hyp(S,G) (Triangular inequality) --1

In the triangle of h(N):
h(N) > hyp(N,G)
Using in 1: C(S,N) + h(N) > hyp(S,G) --2

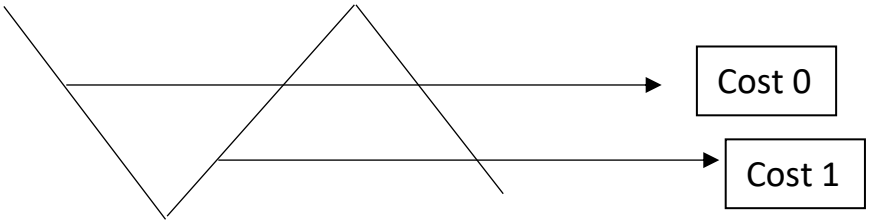
in the triangle of h(S):
Since the difference in height is negative we subtract the downward distance from the cheyshev's distance. So, h(S) < hyp(S,G).
Using in 2: C(S,N) + h(N) > h(S)

In other words h(S) < h(N) + C(S,N), hence heuristic is consistent.

Division heuristic:

```
public double getHeuristic(point p){
double chebyshevDistance = Math.max(Math.abs(p.x-
map.getEndPoint().x),Math.abs(p.y-getEndPoint().y));
return 0.5 * chebyshevDistance;
}
```

Since the division heuristic costs more to go up at higher altitudes, the best possible case for the path from the start to the goal node would traverse tiles which are closer to the ground.



To illustrate, let's start at height 0. Since no two tiles can be at height zero, the next least possible height would be height 1. This would give us a cost of 1/(1+0)=1 to go to the next tile. From the tile at height 1, the best case tile to traverse next would be at height 0. The cost for going to this tile would be 0/(1+0)=0. Hence, the cost of taking two steps from the start state is 1. In this best case scenario we will continue going down up down up (\/\/\/) until we have reached the goal. Thus, the average least possible cost for taking one step in this best case scenario would be ½=0.5.

Thus our heuristic will first find the least possible amount of tiles from going from the start state to the goal state with chebyshevDistance and then we will multiply this value of 0.5 which is the least possible amount of cost for each step. Thus, giving us an underestimate.