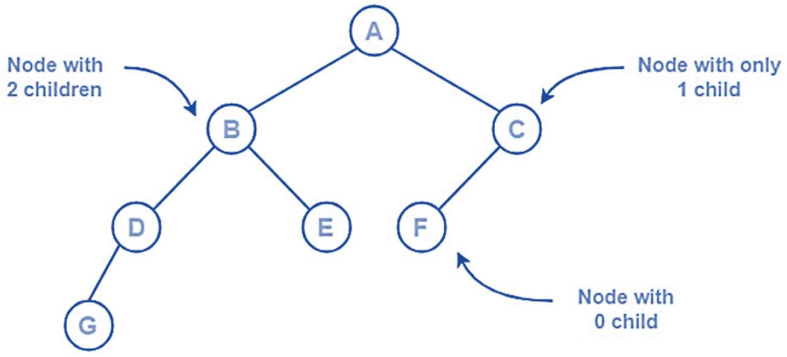
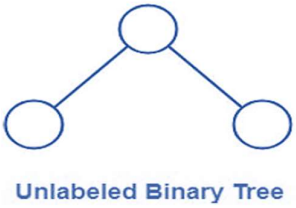
➢ Binary Tree: -

- Binary tree is a special tree data structure in which each node can have at most 2 children.
- Thus, in a binary tree, each node has either 0 child or 1 child or 2 children.



Unlabeled Binary Tree: -

A binary tree is unlabeled if its nodes are not assigned any label



Unlabeled Binary Tree

$$\text{Number of different Binary Trees possible with 'n' unlabeled nodes} = \frac{{}^{2n}C_{n}}{n+1}$$

Example: -

Consider we want to draw all the binary trees possible with 3 unlabeled nodes. Using the above formula, we have-

Number of binary trees possible with 3 unlabeled nodes

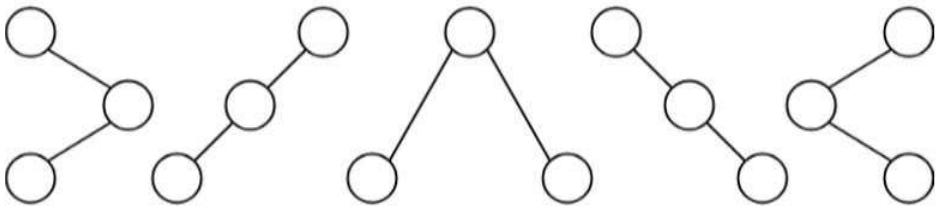
= 2 x 3 C 3 / (3 + 1)

= 6 C 3 / 4

= 5

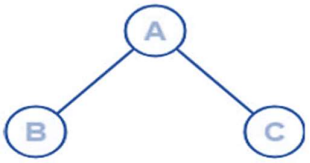
Thus,

- With 3 unlabeled nodes, 5 unlabeled binary trees are possible.
- These unlabeled binary trees are as follows-



Labeled Binary Tree: -

A binary tree is labeled if all its nodes are assigned a label.



Labeled Binary Tree

Number of different Binary Trees possible with 'n' labeled nodes

$$= \frac{{}^{2n}C_n}{n+1} \times n!$$

Example: -

Consider we want to draw all the binary trees possible with 3 labeled nodes. Using the above formula, we have-

Number of binary trees possible with 3 labeled nodes

= {2 x 3 C 3 / (3 + 1)} x 3!

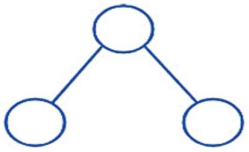
= {6 C 3 / 4} x 6

= 5 x 6

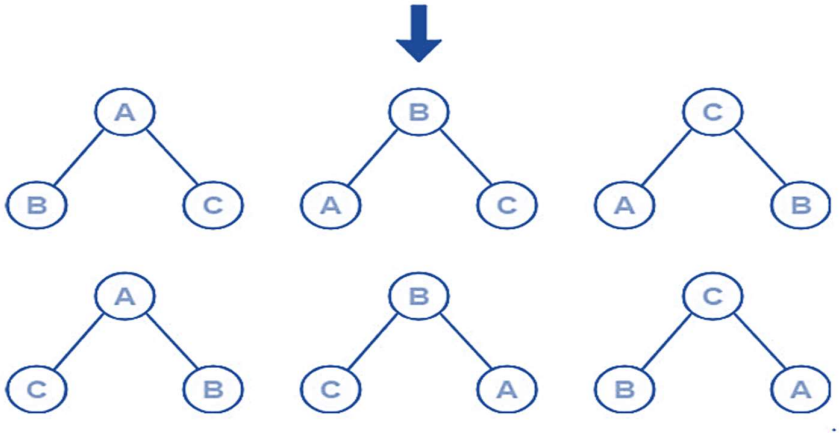
= 30

Thus,

- With 3 labeled nodes, 30 labeled binary trees are possible.
- Each unlabeled structure gives rise to 3! = 6 different labeled structures.



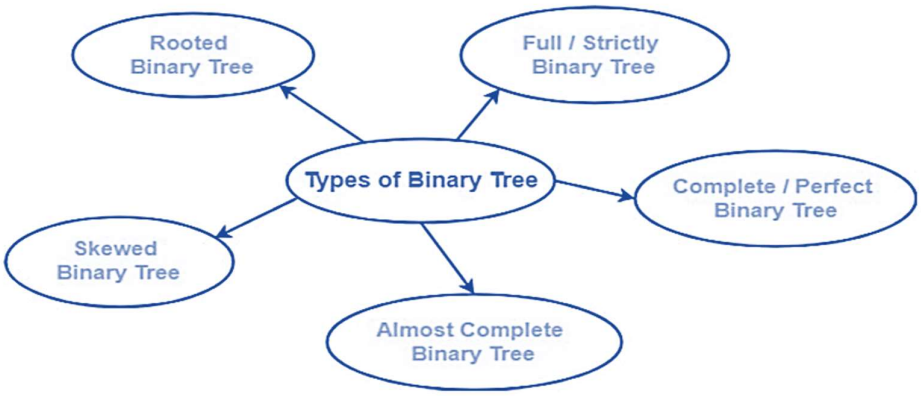
It Gives Rise to Following 6 Labeled Structures



Similarly,

- Every other unlabeled structure gives rise to 6 different labeled structures.
- Thus, in total 30 different labeled binary trees are possible.

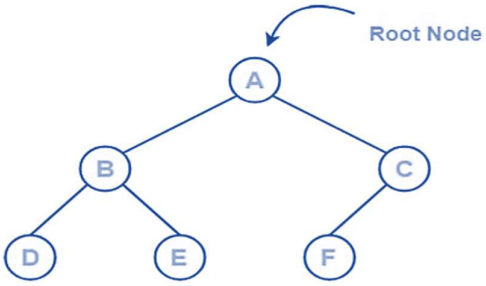
➢ Types of Binary Tree: -



1) Rooted Binary Tree: -

A rooted binary tree is a binary tree that satisfies the following 2 properties-

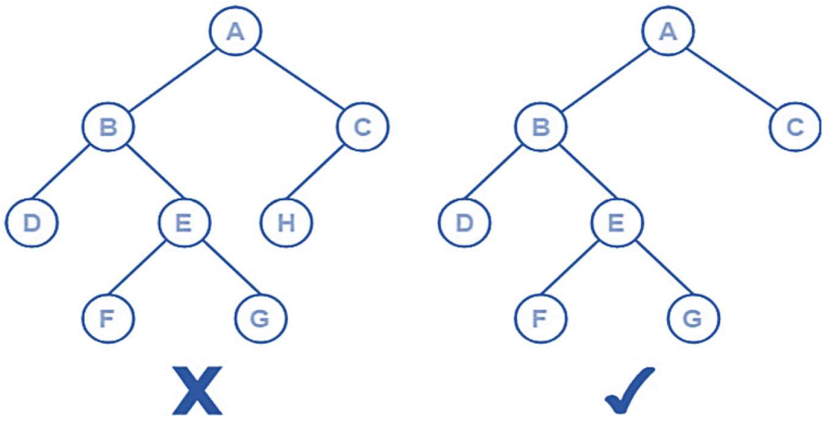
- It has a root node.
- Each node has at most 2 children.



Rooted Binary Tree

2) Full/Strictly Binary Tree: -

- A binary tree in which every node has either 0 or 2 children is called as a Full binary tree.
- Full binary tree is also called as Strictly binary tree.

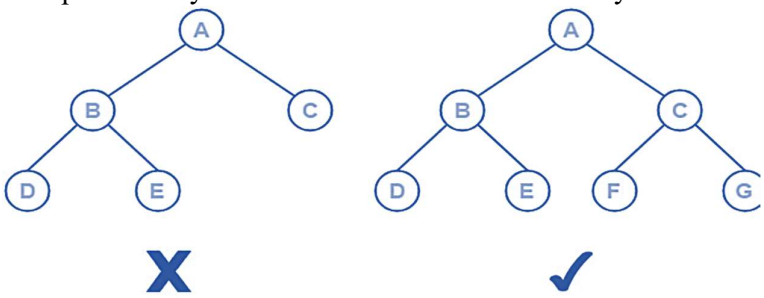


- First binary tree is not a full binary tree.
- This is because node C has only 1 child.

3) Complete / Perfect Binary Tree: -

A complete binary tree is a binary tree that satisfies the following 2 properties-

- Every internal node has exactly 2 children.
- All the leaf nodes are at the same level
- Complete binary tree is also called as Perfect binary tree.

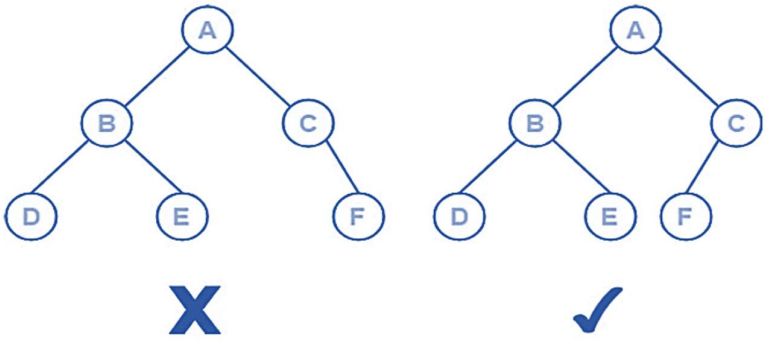


- First binary tree is not a complete binary tree.
- This is because all the leaf nodes are not at the same level.

4) Almost Complete Binary Tree: -

An almost complete binary tree is a binary tree that satisfies the following 2 properties-

- All the levels are completely filled except possibly the last level.
- The last level must be strictly filled from left to right.

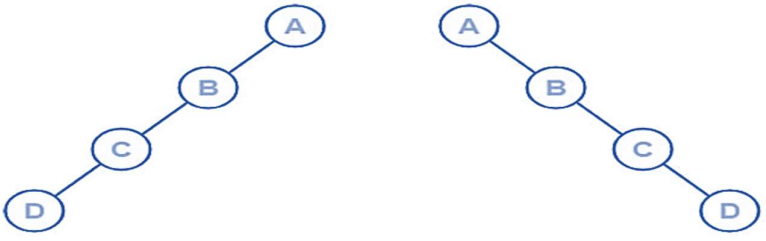


First binary tree is not an almost complete binary tree. This is because the last level is not filled from left to right.

5) Skewed Binary Tree: -

A skewed binary tree is a binary tree that satisfies the following 2 properties-

- All the nodes except one node have one and only one child.
- The remaining node has no child.



Left Skewed Binary Tree Right Skewed Binary Tree