

CS-513 | HW1

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1.) Given: $P(\text{Jerry} \cap \text{Susan}) = 8\%$

$$P(\text{Jerry}) = 20\%$$

$$P(\text{Susan}) = 30\%$$

$$\begin{aligned} 1.) P(\text{Jerry going to bank when Susan is also there}) \\ = \frac{P(\text{Jerry} \cap \text{Susan})}{P(\text{Susan})} = \frac{8}{30} = \underline{\underline{26.67\%}} \end{aligned}$$

2.) $P(\overline{\text{Susan}})$ = Prob. of Jerry that he was there on Friday & Susan was not there

$$\begin{aligned} P(J/\bar{S}) &= \frac{P(J \cap \bar{S})}{P(\bar{S})} = \frac{P(J) - P(J \cap S)}{P(\bar{S})} \\ &= \frac{20 - 8}{70} = \frac{12}{70} = 0.1714 \\ &= \underline{\underline{17.14\%}} \end{aligned}$$

3.) Both of them were at bank on Wednesday

$$\begin{aligned} P(J \cap S / J \cup S) &= \frac{P((J \cap S) \cap (J \cup S))}{P(J \cup S)} = \frac{P(J \cap S)}{P(J \cup S)} \\ &= \frac{8}{42} = 0.1905 \\ &= \underline{\underline{19.05\%}} \end{aligned}$$

1.2) Given: $P(\text{Harold}) = 80\%$

$P(\text{Sharon}) = 90\%$

~~$P(H \cup S)$~~

$$P(H \cup S) = 91\%$$

$$\therefore P(A \cap B) = P(H) + P(S) - P(H \cup S)$$

$$= 80 + 90 - 91$$

$$= 79\%$$

a) $P(\text{only Harold gets 'B'})$

$$= P(H) - P(H \cap S) = 80 - 79 = \underline{\underline{1\%}}$$

b) $P(\text{only Sharon gets 'B'})$

$$= P(S) - P(H \cap S)$$

$$= 90 - 79 = \underline{\underline{11\%}}$$

c) $P(\text{Both won't get 'B'})$

$$= P(\overline{H \cup S}) = 100 - P(H \cup S)$$

$$= 100 - 91 = \underline{\underline{9\%}}$$

1.3) Given: $P(\text{Jerry}) = 20\%$

$P(\text{Susan}) = 30\%$

$P(\text{Jerry} \cap \text{Susan}) = 8\%$

→ The events 'Jerry is at the Bank' & 'Susan is at the Bank' are independent
if \Rightarrow

$$P(\text{Jerry} \cap \text{Susan}) = P(\text{Jerry}) \times P(\text{Susan})$$

$$P(\text{Jerry} \cap \text{Susan}) = \frac{20}{100} \times \frac{30}{100}$$

$$= \frac{6}{100}$$

$$= \underline{\underline{6\%}}$$

∴ Given in question, $P(\text{Jerry} \cap \text{Susan})$ is 8% but by calculation, we get 6%.

Hence, $8\% \neq 6\%$,

"The events are not independent"

$$1.4.) \quad a.) \quad P(\text{Sum} = 6) = \frac{5}{36}$$

$$P(\text{2nd die} = 5) = \frac{1}{6}$$

$$\begin{aligned} & \cancel{P(\text{2nd die} = 5 \text{ \& Sum} = 6)} \\ & = \cancel{P(\text{Sum} = 6) \times P(\text{2nd die} = 5)} \\ & = \cancel{\frac{5}{36} \times \frac{1}{6}} \end{aligned}$$

$$\begin{aligned} P(\text{2nd die} = 5 \text{ \& Sum} = 6) &= P(\text{Sum} = 6) + P(\text{2nd die} = 5) \\ \frac{5}{36} \times \frac{1}{6} &= \frac{1}{36} \end{aligned}$$

In the above case, LHS \neq RHS

\therefore Both events are not independent

$$b.) \quad P(\text{Sum} = 7) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{first die} = 5) = \frac{1}{6}$$

$$\begin{aligned} P(\text{Sum} = 7) &= P(\text{first die} = 5) \\ \frac{1}{6} &= \frac{1}{6} \end{aligned}$$

LHS = RHS

\therefore both events are independent

$$1.5) P(\text{drill in TX}) = 60\%$$

$$P(\text{drill in AK}) = 30\%$$

$$P(\text{drill in NJ}) = 10\%$$

$$P(\text{found in TX}) = 30\%$$

$$P(\text{found in AK}) = 20\%$$

$$P(\text{found in NJ}) = 10\%$$

$$1.) P(\text{found oil}) = P(\text{drill in TX}) \times P(\text{found in TX}) \\ + P(\text{drill in AK}) \times P(\text{found in AK}) \\ + P(\text{drill in NJ}) \times P(\text{found in NJ})$$

$$= 0.6 \times 0.3 + 0.3 \times 0.2 + 0.1 \times 0.1$$

$$= 0.18 + 0.06 + 0.01$$

$$= 0.25 = \underline{\underline{25\%}}$$

2.) Probability of drill & found oil in TX

$$= \frac{P(\text{TX} \cap \text{oil})}{P(\text{oil})}$$

$$= \frac{18}{25} = \frac{72}{100} = \underline{\underline{72\%}}$$

$$\begin{aligned}
 1.6.) P(\text{Passenger did not survive}) \\
 &= P(\text{Total not survived}) \\
 &= \frac{P(\text{Total Passengers})}{2201} = \underline{\underline{67.70\%}}
 \end{aligned}$$

$$\begin{aligned}
 2.) \text{Probability of Passenger staying} \\
 \text{in 1st class} &= \frac{325}{2201} = 0.1476 \\
 &= \underline{\underline{14.76\%}}
 \end{aligned}$$

$$\begin{aligned}
 3.) P(\text{Passenger in 1st class} - \text{Passenger survived}) \\
 &= \frac{203}{711} = \underline{\underline{28.55\%}}
 \end{aligned}$$

$$4.) P(\text{staying in 1st class}) = \frac{325}{2201} = \underline{\underline{14.76\%}}$$

$$P(\text{surviving}) = \frac{711}{2201} = 0.323 = \underline{\underline{32.30\%}}$$

$$\begin{aligned}
 P(\text{Surviving \& staying in 1st class}) &= \\
 P(\text{surviving}) \times P(\text{staying in 1st class}) \\
 \frac{203}{711} &= \frac{32.30 \times 14.76}{100 \times 100}
 \end{aligned}$$

$$0.28 \neq 0.04$$

\therefore The events are not independent

$$\text{v.) } P(\text{child staying in 1st class survived}) = \frac{6}{711} = \underline{\underline{0.84\%}}$$

$$\text{vi.) } P(\text{adult passenger survived}) = \frac{654}{711} = \underline{\underline{91.98\%}}$$

iii) For survived,

$$P(\text{adult}) \times P(1^{\text{st}} \text{ class}) = P(\text{adult and } 1^{\text{st}} \text{ class})$$

$$\frac{654}{711} \times \frac{203}{711} = \frac{197}{711}$$

$$0.2626 \neq 0.2770 \quad - (1)$$

$$P(\text{child}) \times P(1^{\text{st}} \text{ class}) = P(\text{child and } 1^{\text{st}} \text{ class})$$

$$\frac{57}{711} \times \frac{203}{711} = \frac{6}{711}$$

$$0.022 \neq 0.008 \quad - (2)$$

As Equation (1) & (2) given,

The events are ~~not~~ independent events

1.7.) Confusion matrix

	AI-	Human	Total
Predicted as AI	970	30	1000
Predicted as human	70	930	1000
	1040	960	2000

$$\text{Accuracy} = \frac{TP + TN}{TP + FP + FN + TN} = \frac{970 + 930}{2000} = \underline{\underline{95\%}}$$

$$\text{Precision} = \frac{TP}{TP + FP} = \frac{970}{970 + 30} = \underline{\underline{97\%}}$$

$$\text{Recall} = \frac{TP}{TP + FN} = \frac{970}{970 + 70} = \underline{\underline{93\%}}$$

$$F1 = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

$$= \frac{2 \times 0.97 \times 0.93}{0.97 + 0.93} \approx \underline{\underline{0.949}}$$