Assignment 2

Tarun PES1201800073

A1. Equation be $A+Bx+Cx^2$

at $(1,1)$ $1 = A+B+C$

$(2,-1)$ $-1 = A+2B+4C$

$(3,1)$ $1 = A+3B+9C$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}} \underset{x}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{b}{\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}} \qquad [A \ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$2C = 4 \Rightarrow C = 2$

$B + 3C = -2 \Rightarrow B = -8$

$A+B+C = 1 \Rightarrow A = 7$

Equation of palabola is $7 - 8x + 2x^2$

A2. $$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ $R_3 \to R_3 + 5R_1$ $R_4 \to R_4 - 5R_1$

$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

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$R_3 \to R_3 + 2R_2 \quad R_4 \to R_4 + 2R_2$

$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$A = LU$

A3. Linear transformation.

$T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$

i) $T(1,0,0) = (1,0,1)$

$T(0,1,0) = (2,1,1)$

$T(0,0,1) = (-1,1,2)$

Hence Transformation $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

ii) Gaussian Elemination on T

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis → $C(T) = (1,0,1), (2,1,1)$

$C(T^T) = (1,2,-1), (0,1,1)$

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Null space : $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 0 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis for $N(T) = (-3,-1,1)$

For $N(T^T)$

$$T^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ $R_3 \to R_3 + R_1$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis $N(T^T)$ is $(-1,1,1)$

For finding eigen value, characteristic equation.

$|T - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = 0$$

Taking det $(1-\lambda)\left[(1-\lambda)(-2-\lambda) - 1\right] - 2(-1) + (-1)(\lambda-1)$

$-\lambda^3 + 3\lambda = 0$

$\lambda(\lambda^2 - 3) = 0$

$\lambda = 0, \sqrt{3}, -\sqrt{3}$

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Finding eigen vectors

$\rightarrow \lambda = -\sqrt{3}$

$$T+\sqrt{3}I = \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix}$$

$$\frac{x}{\begin{vmatrix} 2 & -1 \\ 1+\sqrt{3} & 1 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 1+\sqrt{3} & -1 \\ 0 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 1+\sqrt{3} & 2 \\ 0 & 1+\sqrt{3} \end{vmatrix}}$$

$$\frac{x}{3+\sqrt{3}} = \frac{y}{-(1+\sqrt{3})} = \frac{z}{(4+2\sqrt{3})} = k_1$$

Eigen vector for $-\sqrt{3}$ is $k_1((3+\sqrt{3}), -(1+\sqrt{3}), (4+2\sqrt{3}))$

$\Rightarrow k_1\left(\frac{-\sqrt{3}+3}{2}, \frac{-\sqrt{3}+1}{2}, 1\right)$

$\rightarrow \lambda = 0$

$$T-\lambda I = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\frac{x}{2+1} = \frac{y}{-1} = \frac{z}{1} = k_2$$

Eigen vector is $k_2(3, -1, 1)$

$\rightarrow \lambda = \sqrt{3}$

$$T-\lambda I = \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix}$$

$$\frac{x}{3-\sqrt{3}} = \frac{y}{\sqrt{3}-1} = \frac{z}{4-2\sqrt{3}} = k_3$$

$k_3(3-\sqrt{3}, \sqrt{3}-1, 4-2\sqrt{3})$

$k_3\left(\frac{\sqrt{3}+3}{2}, \frac{\sqrt{3}+1}{2}, 1\right)$

iv) $A = QR$ factorization.

$a = (1,0,1)$

$b = (2,1,1)$

$c = (-1,1,-2)$

$q_1 = \frac{a}{\|a\|} = \frac{(1,0,1)}{\sqrt{2}}$ $q_2 = \frac{B}{\|b\|}$ PES1201800073

$B = b - (q^T b) q_1$

$(2,1,1) - \left(\frac{2}{\sqrt{2}} + 0 + \frac{1}{\sqrt{2}}\right)\left(\frac{1,0,1}{\sqrt{2}}\right)$

$(2,1,1) - \frac{3}{2}(1,0,1)$

$= (1/2, 1, -1/2) \Rightarrow 1/2(1, 2, -1)$

$q_2 = \frac{(1,2,-1)}{2\left(\sqrt{1/4+1+1/4}\right)} = \frac{(1,2,-1)}{\sqrt{6}}$

$q_3 = \frac{C}{\|C\|}$

$C = c - (q^T c) q_1 - (q_2^T c) q_2$

$c - \left(\frac{-1}{\sqrt{2}} \frac{-2}{\sqrt{2}}\right)\left(\frac{1,0,1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{6}} + \frac{2}{\sqrt{6}} + \frac{2}{\sqrt{6}}\right)\left(\frac{1,2,-1}{\sqrt{6}}\right)$

$\Rightarrow (0,0,0)$

$q_3 = (0,0,0)$

$$R = \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ 0 & q_2^T b & q_2^T c \\ 0 & 0 & q_3^T c \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & \sqrt{3}/2 & \sqrt{3}/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = QR = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & 2/\sqrt{6} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & \sqrt{3}/2 & \sqrt{7}/2 \\ 0 & 0 & 0 \end{bmatrix}$$

A4.

x	-4	1	2	3
y	4	6	10	8

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Matrix form: $\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ \cancel{x} \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$

$$A^T A x^n = A^T b$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{d} \end{bmatrix} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

$\hat{c} = \frac{193}{27}$ $\hat{d} = \frac{20}{29}$

$$y = \frac{193}{29} + \frac{20x}{29}$$

A5. Matrix representing the plane

$$A = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 4 \end{bmatrix}$$

$$P = \frac{AA^T}{A^TA} = \frac{1}{27} \begin{bmatrix} 1 & 1 & 3 & 0 & 4 \\ 1 & 1 & 3 & 0 & 4 \\ 3 & 3 & 9 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 12 & 0 & 16 \end{bmatrix}$$

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$Q = I - P$

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$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \frac{1}{27}\begin{bmatrix} 1 & 1 & 3 & 0 & 4 \\ 1 & 1 & 3 & 0 & 4 \\ 3 & 3 & 9 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 12 & 0 & 16 \end{bmatrix}$$

$$Q = \frac{1}{27}\begin{bmatrix} 26 & -1 & -3 & 0 & 4 \\ -1 & 26 & \cancel{8}\,{-3} & 0 & -4 \\ -3 & -3 & 18 & 0 & -12 \\ 0 & 0 & 0 & 1 & 0 \\ -4 & -4 & -12 & 0 & 11 \end{bmatrix}$$

$$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$$

All sub deteminents should be positive

1) $|a| > 0 \qquad a \in (0, \infty)$

2) $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} > 0 \qquad a^2 - 4 > 0 \qquad a \in (-\infty, -2) \cup (2, \infty)$

3) $\begin{vmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{vmatrix} > 0 \qquad (a+4)(a-2)^2 > 0 \qquad a \in (-4, \infty)$

$a \in (2, \infty)$

ii) $$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$

given equation $\quad 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$

comparing we get

$a_{11} = a_{22} = a_{33} = 2$

$a_{12} = -1$

$a_{13} = 0$

$a_{23} = -1$

$$B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ & & 2 \end{bmatrix}$$

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<u>A7:</u> $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

$$A^TA = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} = B$$

$|B - \lambda I| = 0$

$(81-\lambda)(9-\lambda) - (27)^2 = 0$

$\Rightarrow 729 - 90\lambda + \lambda^2 - 729 = 0$

$\lambda^2 - 90\lambda = 0$

$\lambda = 0, 90$

Eigen values are 0, 90

i) $\lambda = 0$ $\quad A = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

$81x = 27y \qquad \frac{x}{1} = \frac{y}{3}$

$k(1,3)$ eigen vector

ii) $\lambda = 90$

$A = \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix}$

$9x = -27y \qquad \frac{x}{-3} = \frac{y}{1}$

$k_2(-3,1)$ eigen vectors

Matrix $V = \begin{bmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$

$$AA^T = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix}$$

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$$= \begin{bmatrix} -10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix}$$

$|(AA^T) - \lambda I| = 0$

$(10-\lambda)(\lambda^2 - 80\lambda) + 800\lambda = 0$

$\lambda^2(\lambda - 90) = 0$

$\lambda = 90, 0$

$\sigma = 3\sqrt{10} \quad \sigma = 0$

$$\Sigma = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 3\sqrt{10} \end{bmatrix}$$

$AV = U\Sigma$

$$\begin{bmatrix} 0 & \sqrt{10} \\ 0 & -2\sqrt{10} \\ 0 & -2\sqrt{10} \end{bmatrix} = U \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 3\sqrt{10} \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 0 & 1/3 \\ 0 & 0 & -2/3 \\ 0 & 0 & -2/3 \end{bmatrix}$$

$A = U\Sigma V^T$

$$\begin{bmatrix} 0 & 0 & 1/3 \\ 0 & 0 & -2/3 \\ 0 & 0 & -2/3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 3\sqrt{10} \end{bmatrix} \begin{bmatrix} 1/\sqrt{10} & 3/\sqrt{10} \\ -3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$$