

Write an R program to solve the following problems using **Chebyshev's Inequality**:

1. Suppose 1,000 applicants show up for a job interview, but there are only 70 positions available. To select the best 70 people amongst the 1,000 applicants, the employer gives an aptitude test to judge their abilities. The mean score on the test is 60, with a standard deviation of 6. If an applicant scores an 84, can they assume they are getting a job?
2. A class's test scores have a mean of 80 and a standard deviation of 10. Use Chebyshev's inequality to determine the probability that a student's score is between 60 and 100.
3. The mean score of a group of students in a test is 75 with a standard deviation of 8. Find the minimum probability that a randomly selected student's score is between 67 and 83

**Markov's Inequality programs:**

4. A company knows that the average time an employee spends on social media during work hours is **20 minutes per day**. Using **Markov's Inequality**, find the maximum probability that an employee spends **at least 1 hour (60 minutes)** per day on social media. Write an R program to calculate this bound.
5. The average number of defective items produced by a machine per day is **5**. Using **Markov's Inequality**, estimate the upper bound on the probability that the machine produces **at least 15 defective items** in a single day. Implement an R program to compute the result.
6. Suppose the **average income** in a town is **\$50,000**. Use **Markov's Inequality** to calculate the maximum probability that a randomly chosen individual has an income of at least **\$200,000**. Write an R program that prints the result clearly.

**Chernoff bounds**

7. A fair coin is tossed **1,000 times**. Expected number of heads = 500. Use Chernoff bounds to estimate the probability that the number of heads is at least **600**. Write an R program to compute the bound.
8. A web server receives **independent requests**, each succeeding with probability  $p=0.9$ . Over **500 requests**, Expected successful requests = 450. Use Chernoff bounds to compute the probability that at least **490** requests succeed. Write an R program to compute this probability bound.
9. A Company is analyzing its **customer support system**. The data collected shows:

## 1. Call Duration

- a. The average duration of customer support calls is **15 minutes**.
- b. Use **Markov's inequality** to estimate the maximum probability that a call lasts longer than **1 hour (60 minutes)**. **Customer Satisfaction Scores**

## 2. Customer Satisfaction Scores (Chebyshev's Inequality)

- a. Customer satisfaction scores (out of 100) have a mean of **70** and a standard deviation of **8**.
- b. Using **Chebyshev's inequality**, Estimate the minimum probability that a customer's score lies between **54 and 86**.

## Resolved Tickets

- a. Each support agent resolves a ticket correctly with probability  $p=0.9$ .
- b. In a day, an agent handles **500 tickets**.
- c. Using **Chernoff bounds** Find:
  - i. (a) The probability that the agent resolves fewer than **400 tickets** correctly.
  - ii. (b) The probability that the agent resolves more than **480 tickets** correctly.

- Write an **R program** that computes probability bounds for the above scenarios.
- Apply your program to solve all three parts.
- Print results with clear explanations in plain English (e.g., *"At most 25% of calls can last more than 1 hour"*).